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SEMI-ACTIVE VIBRATION DAMPING BY MODULATION OF JOINT FRICTION

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ABSTRACT

The semi-active concept for vibration suppression in a clamped-free structural configuration is investigated. A fixed-free beam with the semi-active joint, designed to modulate friction, and hence the clamping load level, is considered. The vibration of the beam resulting from assumed initial conditions is examined in the time domain for a range of clamping load levels. The results obtained are studied and a control strategy is proposed to dissipate the energy efficiently and in a controlled way.

INTRODUCTION

Joints are the major source of damping in structures having jointed connections due to friction. Friction in these joints if controlled can increase the inherent damping of the structures and thereby control the vibration response. A major nonlinear characteristic of classical dry friction is the stick-slip motion which is described as intermittent change from no movement, called sticking, to very small slipping, at the contact surface. The behavior of the stick-slip motion depends strongly on both characteristics of static friction during the stick period and kinetic friction during the slip period. A vibrating beam or plate with clamped boundaries will experience slipping at the supports for certain combinations of the amplitude of vibration, clamping force, and coefficients of friction at the clamps. To induce slipping, it is necessary for the vibration amplitude to be of sufficient magnitude to create large loads at the supports in the beam or plate. Slipping will occur when these loads meet or exceed the friction force at the boundaries. The consequence of this intermittent sticking and slipping is that there is a net decrease in the vibration energy as well as increased damping in the system. In addition to that, with each slipping event, there is a slight change in the structural orientation from the one attained at the end of previous slipping event. Once there is enough energy dissipation, the elastic forces at the boundaries become less than the friction force and there will be no more damping. The structure will continue to vibrate with reduced amplitude.

In order to further dissipate energy, the friction force at the joint needs to be lowered. The joint needs to be designed in such a way that it offers controlled interfacial slip by actively controlling the friction forces at the joint. It is to be noted that unfastening of the joint can be done in steps, with limited freedom to slip in each step to avoid large permanent displacement of the structure once the steady state is achieved. Additionally, with the boundary frictional moment lowered to an extent resulting in large slipping velocity, analytical model neglecting large amplitude and velocity of rotation would fall short of delivering realistic results. By allowing small slipping, a well-controlled and stable damping in the system can be achieved with little sacrifice in the static stiffness of the structure. This type of control where the passivity of control is modified by an active mechanism is termed semi-active control. The appeal of semi-active control is that performance levels rivaling fully active control can be achieved with a fraction of the input power required of active control.

Results have been published on the energy dissipation and frequency responses of structures equipped with joints allowing controlled interfacial slip but most consider slipping in translation ([1], [2], [3], [4]). Beards and Williams [5] studied the rotational slip in joints of a rectangular frame in which diagonal strut was allowed to slip in rotation at one joint. The frictional torque was assumed to be a harmonic function having same frequency as that of structure. Ferri and Heck [6] observed that semi-active dry friction damping can provide effective damping and shock isolation with simple controllers. Ferri and Heck [7] considered a system consisting of two elastic beams connected by single joint. It was obtained that joints designed with amplitude or rate-dependent frictional forces can offer substantial improvement in performance over joints with constant normal forces. Gaul and Nitsche [8] [9] studied the same system of two beams, however, with advanced friction model dynamic friction model proposed by Canudas de Wit, et al. [10]. The friction model suited for micro-slip range. The same system model was revisited by Buaka et al [11], however it was realized that coulomb model is sufficient to understand the concept for macro-slip range. Experimental investigation suggested that excitation force should be sufficiently large to overcome the force of stiction and to cause relative movement at the contact surface in such a way that energy dissipation can occur.

So far, research related to vibration damping of elastic structure mainly concentrated on trusses, frames or oversimplified model of mass-damper system. The vibration damping for beam or plates by friction in joints concentrated on axial slipping. A model of two pinned beams connected to each other by a pin joint has been studied by numerous researchers. Recently, the author [12] studied the vibration damping in a clamped-free beam configuration by actively inducing the slipping at the joint. The response of the structure was studied for a range of clamping load levels. The purpose of the present study is to further investigate the proposed concept and design a strategy to suppress the vibration of a beam using actively-controlled slipping events.

To examine the dynamics of the system, an analytical model of a beam under free excitation is developed in the following section, which is then evaluated by numerical simulation. It is assumed that one end of the beam is free and the other end is clamped in a fixture that permits rotational slip when the beam applies sufficient moments at the support to overcome friction. It is found that during random vibration of the beam, the rotational slip will vary as the end sticks and slips. After this motion has taken place for a period of time, however, a steady state condition prevails. This steady state situation is shown to be simply related to the beam parameters and the maximum allowed friction force. Next, the results obtained are studied and a control strategy is proposed to dissipate the energy efficiently and in a controlled way. The beauty of the concept lies in the use of single actuating mechanism to dissipate energy with semi-active technique as compared to the use of distributed active mechanism. The preliminary design of the piezo-actuated joint capable of performing the required control action is specified.

JOINT DESIGN

The semi-active joint created using piezoelectric stack actuator is shown in Figure 1 and is described by the author in reference [12]. The joint consists of a hollow cylindrical piezo stack actuator mounted over the pin 1 which can be pushed in and out by means of a spring mechanism. The spring is mounted on the plate attached to the rear end of actuator and is connected to pin 1 at the other end. The beam rotates freely about the pin 2 and the interface between the two is assumed frictionless. The only interface where friction exists is in the interface between the pin 1and the pin 2. In the absence of actuation, i.e., default configuration, spring is relaxed and pin 1 pushes against pin 2 such that frictional moment is sufficient to prevent beam from slipping. When voltage is applied, the actuator extends, causing spring to expand which then pushes pin 2 away from pin 1, reducing the normal load on friction interface. The voltage in the actuator, thus, can be used to control the clamping load, and the resulting rotational slip. When the frictional moment is larger than the beam elastic moment at the root, the joint is fully locked with no slip. When the frictional moment is either equal to or less than the support moment, there will be slipping at the joint.

The actuator size, properties and power requirements depends on the beam geometry and amount of elastic energy stored. A pin-free beam under free vibration has zero bending moment at the support in the direction normal to the plane of rotation while a clamped free beam has the same moment varying harmonically. In other terms, if the resistive moment at the support is more than the one due to elasticity of the system, beam is clamped or the joint is locked. If the resistive moment is zero, it is pinned. The definition clearly states out that support moment if varied from zero to a value greater than the critical one, boundary condition shifts from clamped to that of the pinned. The critical value is the magnitude of the elastic moment at the boundary. The actuator blocking force necessary for perfect clamping with no slipping is given by

$$f_b \geq \frac{|M_e|}{\mu r_e} \tag{1}$$

where M_e is the elastic moment at the support, μ is the friction coefficient at the interface, and r_e is the effective radius of pin 2.

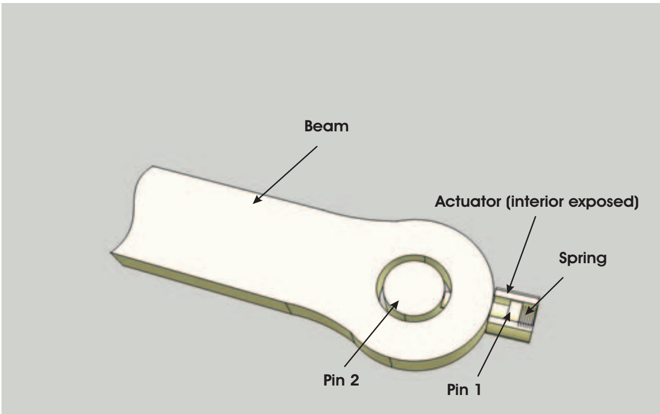

Figure 1. Semi-active joint

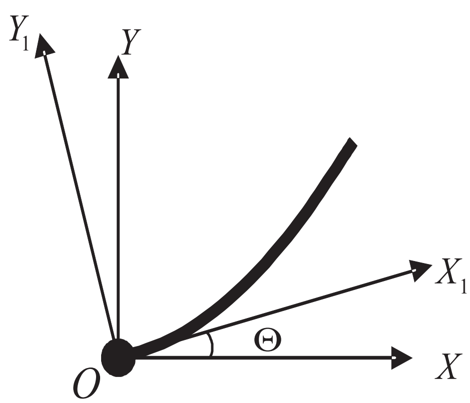

Figure 2. A beam-joint structure

PROBLEM FORMULATION

A rotating uniform beam of length L, Young's modulus E, cross section moment of inertia I , clamped to a rigid hub is shown in Figure 2. The specific mass per unit length of length of the beam is ρ. The hub has moment of inertia I_h and radius r_h. It is considered that the reference frame $[O,X,Y]$ is the inertial frame with the corresponding unit vectors $\hat{e}_1, \hat{e}_2$. The reference system is set at the same point O as the inertial frame with the unit vectors $\hat{e}^1_1, \hat{e}^1_2$ which are directed along the tangent, and the normal to beam at the point O. θ denotes the angle between $\hat{e}^1_1$ and $\hat{e}_1$.

The governing equations are developed for a beam clamped at in a fixture that allows slipping when elastic forces at the boundary exceed a certain value. The other end of the beam is free. The nonlinearities accounted for here are due to rotational slip and friction damping at the boundaries.
In order to ensure that conditions for slipping at boundary are accounted for properly, the boundary condition will be derived, along with the governing equations, using a variational approach. Hamilton principle states that

$$\delta \int_{t_1}^{t_2} (T - V + W_{nc}) dt = 0 \tag{2}$$

where δ is the variational operator, T is the kinetic energy, V is the potential energy and W_{nc} is the work done by non-conservative forces. The limits of integration, t_1 and t_2, define an arbitrary time interval.

Consider the kinematics of any point on the beam. Its position vector may be written as

$$\vec{r} = x\hat{e}^1_1 + w\hat{e}^1_2 \tag{3}$$

Its time derivative can then be written as

$$\dot{\vec{r}} = -w\dot{\theta}\hat{e}^1_1 + (\dot{w} + x\dot{\theta})\hat{e}^1_2 \tag{4}$$

From above equations,

$$\delta\vec{r} = -w\delta\theta\hat{e}^1_1 + (\delta w + x\delta\theta)\hat{e}^1_2 \tag{5}$$

The kinetic energy can be written as

$$T = \frac{1}{2} I_H \dot{\theta}^2 + \frac{1}{2}\rho \int_{r_h}^{r_h+L} \left(\dot{\vec{r}}\right)^2 dx \tag{6}$$

Taking variation of kinetic energy

$$\delta T = I_H \dot{\theta}\delta\dot{\theta} + \rho \int_{r_h}^{r_h+L} \dot{\vec{r}}\delta\dot{\vec{r}} dx \tag{7}$$

After neglecting higher powers of $w, \dot{w}$, and θ, and assuming inertia of the hub to be negligible compared to that of beam, we get

$$\delta T = -I_H\ddot{\theta} - \rho \int_{r_h}^{r_h+L} \left(x^2\ddot{\theta}\delta\theta + x\ddot{w}\delta\theta + \ddot{w}\delta w + x\ddot{\theta}\delta w\right)dx \quad (8)$$

The elastic energy of the beam can be written as

$$V = \frac{1}{2}EI \int_{r_h}^{r_h+L} \left(w''\right)^2 dx \quad (9)$$

where superscript denote partial differentiation with respect to x.

Taking variation of elastic energy and integrating by parts

$$\delta V = EIw''\delta w'\Big|_{r_h}^{r_h+L} - EIw'''\delta w\Big|_{r_h}^{r_h+L} + EI \int_{r_h}^{r_h+L} w^{IV}\delta w dx \quad (10)$$

The contribution due to non-conservative forces is in the form of friction at the support at root of the beam. The model of the frictional damping accounts for sticking and slipping depending on the magnitude of elastic force at the boundary.

$$W_{nc} = M_r s \quad (11)$$

where M_r is the frictional moment in the area of contact.

$$\delta W_{nc} = M_r \delta s \quad (12)$$

Bringing the variational operator inside the integrand of Eqn. 2 gives

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc})dt = 0 \quad (13)$$

Substituting energy and work done terms from Eqns. 8, 10 and 12

$$-\rho \int_{r_h}^{r_h+L} \left(x^2\ddot{\theta}\delta\theta + x\ddot{w}\delta\theta + \ddot{w}\delta w + x\ddot{\theta}\delta w\right)dx - EIw''\delta w'\Big|_{r_h}^{r_h+L} + EIw'''\delta w\Big|_{r_h}^{r_h+L} - EI\int_{r_h}^{r_h+L} w^{IV}\delta w dx + M_r\delta s = 0 \quad (14)$$

Combining the terms that multiply $\delta\theta$

$$-\rho \int_{r_h}^{r_h+L} \left(x^2\ddot{\theta} + x\ddot{w}\right)dx + M_r = 0 \quad (15)$$

Combining the terms that multiply δw

$$\rho\ddot{w} + EIw^{IV} + \rho x\ddot{\theta} = 0 \quad r_h \le x \le r_h + L \quad (16)$$

with the boundary conditions that of a clamped free beam

$$\begin{aligned} &W(r_h,t) = 0 \\ &\frac{\partial W(r_h,t)}{\partial x} = 0 \\ &\frac{\partial^2 W(r_h+L,t)}{\partial x^2} = 0 \\ &\frac{\partial^3 W(r_h+L,t)}{\partial x^3} = 0 \end{aligned} \quad (17)$$

Friction damping at the support will be modeled as a contact surface of effective radius r_e , equal to 2/3 of hub radius r_h , with a clamping force N. It is assumed that as long as the magnitude of the boundary moment is less than a critical magnitude $\mu N r_e$, there is no slipping and moment applied by the beam is equal and opposite to that of the frictional support. As the boundary moment is increased to $\mu N r_e$, however, the contact surface slips and no further increase in boundary moment is allowed. The stick/slip boundary will then act as a force limiter. The behavior is characterized by three possible states: (1) the moment applied by the support is equal to $\mu N r_e$ when the beam end is slipping in the negative θ-direction; (2) zero velocity (no slip) when the magnitude of moment applied by the beam is less than $\mu N r_e$, and (3) force applied by the support equal to $-\mu N r_e$ when the beam end is slipping in the positive θ-direction. Note that a force applied by the support is considered positive when it acts in the positive θ-direction .

It is important to note that during slipping, slip θ varies in a manner that depends on the applied force and system parameters. During slipping, when the moment applied by the beam reduces so that its magnitude is less than $\mu N r_e$ slipping decelerates and stops, which means that the end of the beam must remain where it is until the next slipping event. It is found that this stopped position is not necessarily the one we started with. We find that complex behavior of slip θ plays an essential role in the dynamics of the system.

To obtain the critical value of frictional moment, Eqn. 16 is substituted in Eqn. 15,

$$M_r = -\left. EIw'' \right|_{x=r_h} + r_h \left. EIw''' \right|_{x=r_h} \tag{18}$$

The right side of equation is the moment applied by the beam end and the left side is the frictional moment applied by the support. In the beginning, when there is no slipping with beam moment less than the critical value $\mu N r_e$, boundary moment acts in the direction opposite to the beam moment and balances it. The response, then is that of a simple cantilever beam and may be written in terms of a single equation

$$\rho \ddot{w} + EIw^{IV} = 0 \tag{19}$$

If the applied force is sufficient to increase and hence, so that the force applied by the beam in the right side of the equation has a magnitude greater than the frictional moment, the beam will slip relative to the support. During slipping, the support limits the magnitude of the boundary moment to $\mu N r_e$. The direction in which the force acts will be determined by the sign of the slip velocity. The force takes on the value.

$$M_r = -\mu N r_e Sign(\dot{\theta}) \tag{20}$$

During vibration, if the moment applied by the beam has a magnitude that is smaller than $\mu N r_e$ then slip velocity decelerates until it becomes zero and then θ must maintain the value that it had at the previous instant. During the deceleration, boundary moment acts opposite to the slip velocity direction having magnitude that of a critical one. Once the beam sticks, support friction moment balances the moment applied by the beam and the response once again is directed by dynamics of cantilever beam as in Eqn. 19.

UNCONTROLLED SLIPPING

The case of a passive joint with a fixed clamping load level is studied first. The load magnitude is to be chosen such that frictional moment at the joint is slightly lower than the maximum value of the elastic moment for the beam under vibration leading to the occurrence of intermittent slipping events. Using the analytical model presented, numerical simulations of the beam response were conducted and presented by the author in reference [12] for a range of clamping load levels. The results presented in this section correspond to a normal load of magnitude 400 N. The properties and dimensions of the beam and the hub are given in Table 1.

Table 1. Dimensions and properties of the beam and hub

E (GPa)	20.7
b (mm)	8
h (mm)	10
$\rho(Kg.m^{-1})$	0.6264
L (m)	1
r_h (m)	0.01

The beam is set to undergo free vibration by giving an excitation prescribed in terms of initial conditions. The initial deformed shape profile is that of the first mode of the cantilever beam with tip displacement 0.027 m and the initial velocity distribution is taken to be zero. The nonlinear set of equations is solved using a Matlab code developed to simulate the dynamic response of the structure. By spatial discretization, the continuous system was reduced to the integration of a differential-algebraic system. The time step was adjusted to the order of 10^{-4} to avoid numerical error associated with numerical solution of differential equations.

The time history of the slip rate for a normal load of 400 N is shown in Figure 3. The figure also shows dying out of slip phenomenon slowly due to continued dissipation leaving the beam eventually reach a roughly steady state condition. In one cycle, slip rate goes from positive to negative or vice-versa causing the amplification or reduction in angular orientation of the beam caused due to slip. For different loads, the peak slip rate attained is given in Table 2. The residual slip angle or the ultimate change in orientation is obtained when the slip rate reduces by almost 80% from the peak value and is given in Table 2.

Figure 4 shows the time history of slip angle at the same clamping load level. A decaying response with flattened maxima and minima is obtained. The flatness of the plot results from the occurrence of stiction when the beam boundary moment is less than the frictional moment. After slipping occurs, slip angle no longer remains zero, and it can be expected to stick at some angle and can take on both positive as well as negative values. Finally, when the dissipation dies out, there is permanent sticking and the beam continues to vibrate with the displaced orientation de-

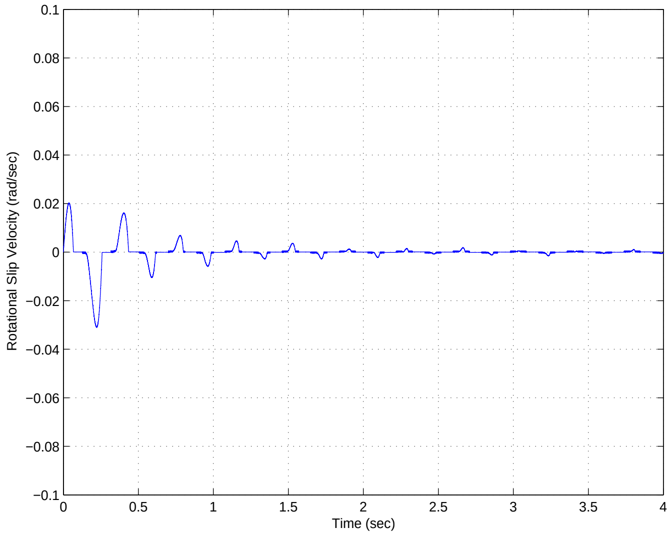

Figure 3. Time history of slip rate at 400 N

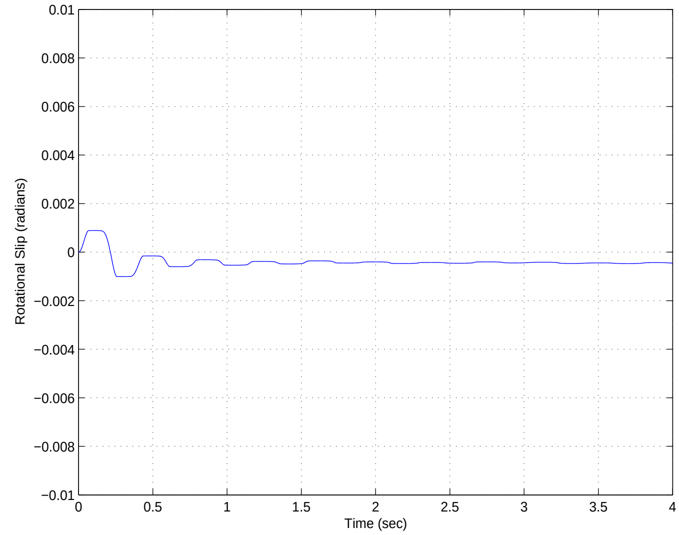

Figure 4. Time history of slip at 400 N

Table 2. Peak slip velocity, residual slip angle, and energy dissipated at various load levels

$Load$ (N)	$PeakSlip$ $Velocity(\dot{\theta})$ $(radsec^{-1})$	$ResidualSlip$ $Angle(\lvert\theta\rvert)$ (rad)	$Energy$ $Dissipated(\%)$
475	$6.3x10^{-3}$	$5.75x10^{-5}$	9.75
450	$1.35x10^{-2}$	$1.35x10^{-4}$	18.22
425	$2.22x10^{-2}$	$2.18x10^{-4}$	25.42
400	$3.01x10^{-2}$	$3.83x10^{-4}$	31.77

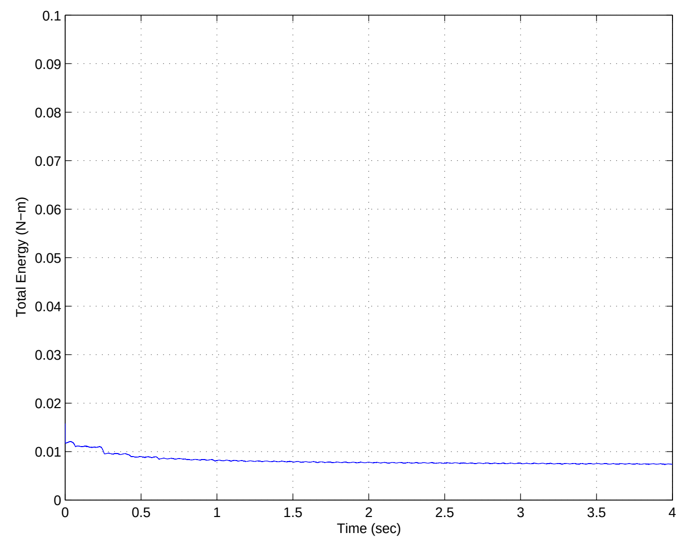

Figure 5. Time history of elastic energy at 400 N

pending on slip.

The energy dissipation evolution is shown in Figure 5. A small bump up in the plot can be seen which is due to the numerical discretization error. There is a change in the direction of the frictional force when the slip velocity passes through zero. For numerical computation, it is very difficult to define the absolute zero. So, a finite value of $1x10^{-6}$ rad/sec has been chosen to represent the value below which sticking occurs. Table 2 compares the amount of energy dissipation at different load levels in the same time in which slip rate goes down by 80%.

Based on these results, conclusions are made and a control strategy is proposed in the next section.

CONTROL STRATEGY

The semi-active control law for maximizing the damping was developed by Dupont and Stokes [13] using friction model of the form

$$f = N\varepsilon, \;\; \dot{\varepsilon} = \gamma\left(\dot{\theta}\right) \tag{21}$$

For the Coulomb friction model, maximum damping control law transforms into

$$N = \begin{cases} N_{\max} & if\ \dot{\theta} \neq 0 \\ 0 & if\ \dot{\theta} = 0 \end{cases} \qquad (22)$$

Above control law does not specify magnitude of the normal load as well as its variation with time. As discussed earlier, a systematic strategy is required to control the frictional resistance in the joint to achieve desired damping without augmentation in angular orientation while maintaining intermittent slipping at the joint. A constant frictional force renders the structure locked with no subsequent damping before it even begins to slip or after a period of time depending on the magnitude. The clamping load level is required to be controlled in a certain fashion to maintain slipping once equilibrium between maximum elastic moment and friction moment at the boundary is achieved. From previous studies, it is known that in order to design a fixture for a randomly excited beam or plate such that slipping is a suitably rare event, it is very difficult to accurately specify the required clamping force. However, based on parametric study done above with different sets of load values, it is clear that one can specify the friction force needed based on limiting slip rate. The two conflicting requirements decide slip rate specification, one that it should be low enough for our analytical model to be valid and other to prevent a large permanent angular displacement of structure due to deflection at each slipping event. The analytical model developed neglects higher order terms related to $\dot{\theta}$ since large angle rigid motion would change the projection of the beam axis onto the undeformed axis substantially. On the other hand, it should be large enough to have enough energy dissipation in the structure. Seemingly, the control law should be based on the fact that whenever the moment at the root of the beam becomes less than or equal to the amount that friction can provide, stack actuator reduces the normal force. It is to be noted that a large decrement of normal force in a single step would result in larger angular orientation of the beam at the attainment of steady state. Table 3 shows the frictional load as a percentage of boundary loads for different normal load levels at which results are obtained earlier. A normal load value of 450N is chosen to begin with which corresponds to frictional moment 42.3% lower than the boundary moment value and for that peak residual slip is $1.35\text{x}10^{-4}$ and 18.2% reduction in vibration energy.

The normal load is kept constant, keeping the frictional moment unchanged until there is nearly-permanent sticking and negligible dissipation available. The nearly permanent sticking is considered as one when the slip rate falls below 80%. The damping rate at this juncture is negligibly small and hence the boundary moment would take immense time to reach frictional moment level at the joint. Hence, the control law is chosen such

Table 3. Comparison of peak elastic moment and frictional moment at various load levels

Load (*N*)	*BoundaryMoment*(*BM*) *Amplitude*(*Nm*)	*FrictionalMoment* (%)*BM*
475	1.3	39.10
450	1.3	42.30
425	1.3	45.38
400	1.3	48.72

that clamping load is reduced as soon as the peak slip rate falls below 80% of peak slip rate in the first cycle after the clamping load is reduced. The load reduction strategy has been kept same that is around 40 % lower than the peak boundary load. Sensors are required to monitor the boundary moment along with the elastic energy of the system by sensing the state of the system.

RESULTS

As seen from Figure 6, the controller lowers the normal load at the frictional interface in the joint each time slip rate falls below 80% of peak slip rate. It can be seen to occur at around 2nd, 4th, 6th, 8th, 10th and 12th seconds, which are equally spaced. There is a sudden jump in slip rate at these time instants which soon dissipates, followed by the occurrence of next event. Figure 7 shows slip evolution with time. A large increase in slip can be seen between 2nd and 8th seconds and later between 10th and 14th seconds. The reason behind this behavior is that actuator lowered the normal load at around 2nd, 4th and 6th seconds, all of which events occurred when beam elastic moment at the support was in the same direction causing the beam to in slip in the same direction at all the three times,thus increasing the orientation of the beam. The large increase in slip between 10th and 14th seconds results due to the same reason. The time instant at which actuator kicks in to lower the normal load is one of the key factors in deciding final orientation due to residual slip apart from magnitude of frictional moment. The control law could be supplemented to prevent consecutive actuator events occurring at the instants for which elastic moment at the joint has the same direction. There is decrease of frictional moment as seen in Figure 8 at each of the above mentioned time instants. The time history of energy dissipation due to controlled slipping events is shown in Figure 9. There is a significant amount of dissipation of 73% vibration energy in just 13 seconds with just a single actuator using semi-active technique. Figure 10 shows beam tip response.

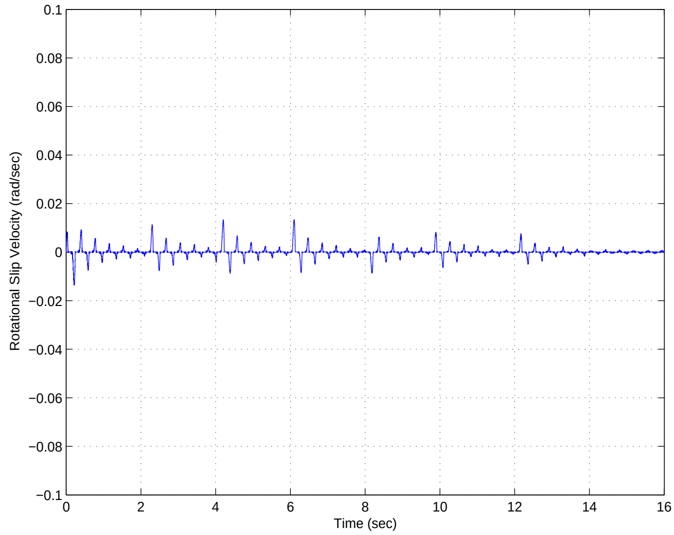

Figure 6. Slip rate evolution for controlled dissipation

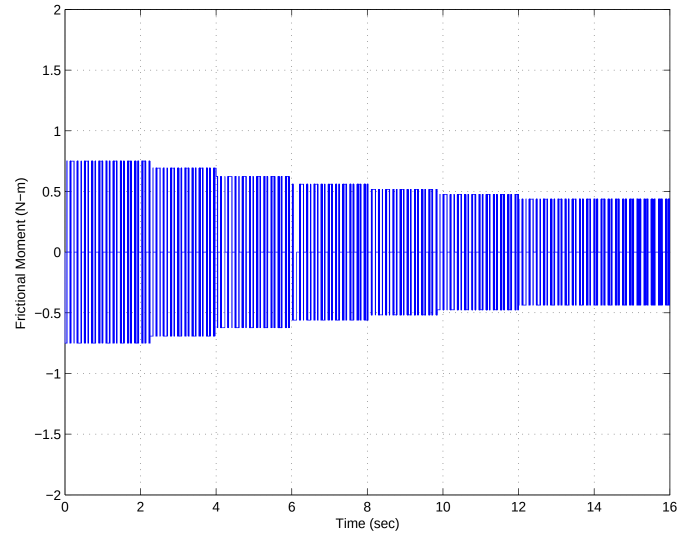

Figure 8. Frictional moment variation under controlled dissipation

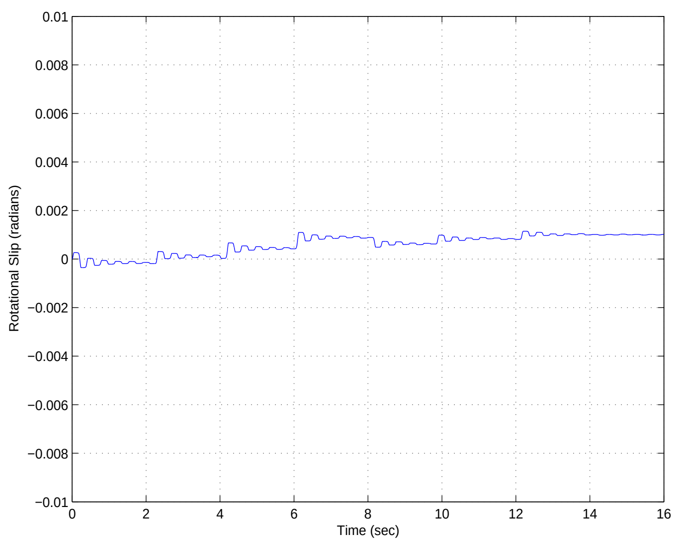

Figure 7. Time history of slip angle under controlled dissipation

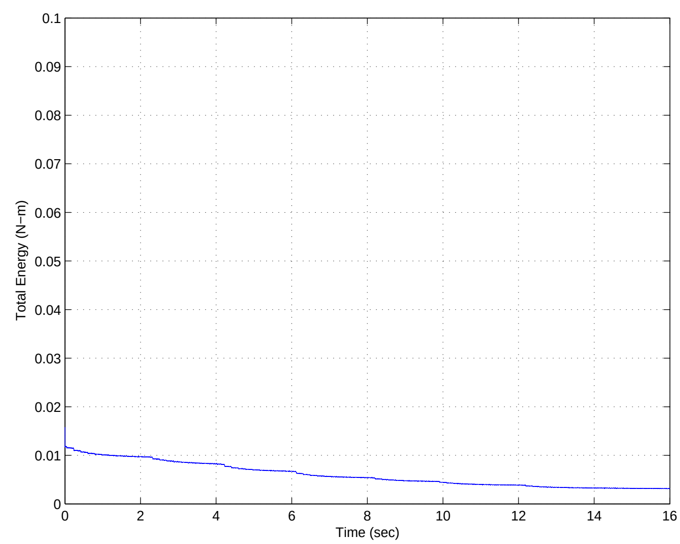

Figure 9. Energy dissipated under controlled dissipation

CONCLUSIONS

A controlled approach for vibration suppression in a clamped structure through semi-active joint is presented. The control approach requires initial assessment of energy content of the structure through mounted/embedded sensors and thereby choosing the clamping load to begin with. The slip rate at the joint can be measured at the joint using encoder and thereby frictional resistance is updated. An efficient vibration damping in structure with minimal effort using a single piezo-actuator based on friction modulation is demonstrated.

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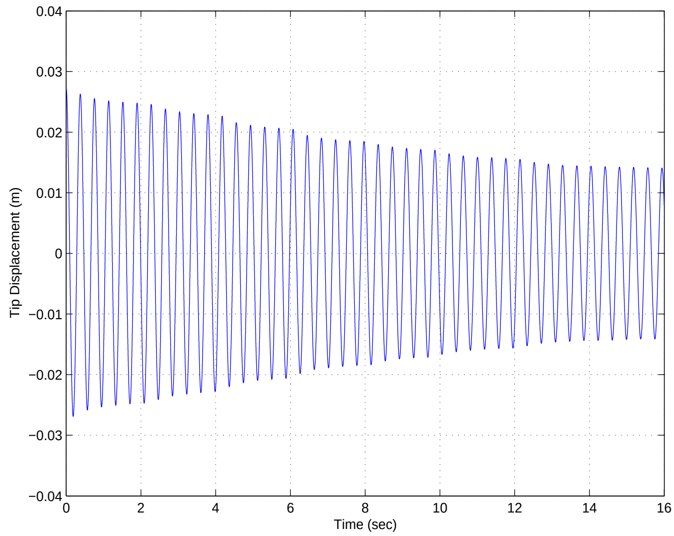

Figure 10. Beam tip response under controlled dissipation

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