

Crossover vs. Mutation: A Comparative Analysis of the Evolutionary Strategy of Genetic Algorithms Applied to Combinatorial Optimization Problems

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Abstract

Since their first formulation, genetic algorithms (GA) have been one of the most widely used techniques to solve combinatorial optimization problems. The basic structure of the GA is known by the scientific community, and thanks to their easy application and good performance, GAs are the focus of a lot of research works annually. Although throughout history there have been many studies analyzing various concepts of GAs, in the literature there are few studies that analyze objectively the influence of using blind crossover operators for combinatorial optimization problems. For this reason, in this paper a deep study on the influence of using them is conducted. The study is based on a comparison of nine techniques applied to four well-known combinatorial optimization problems. Six of the techniques are GAs with different configurations, and the remaining three are evolutionary algorithms that focus exclusively on the mutation process. Finally, to perform a reliable comparison of these results, a statistical study of them is made, performing the normal distribution z-test.

Keywords: Genetic Algorithm, Crossover Operator, Combinatorial optimization, Routing Problems, N-Queens Problem, Bin Packing Problem

1. Introduction

Genetic Algorithms (GA) are one of the most successful meta-heuristics for solving combinatorial optimization problems. Thanks to their easy application and good performance, GAs have been used to solve many complex problems framed in various fields, as for example transport [1, 2], software engineering [3, 4], or industry [5, 6]. GAs were proposed in 1975 by Holland [7], in an attempt to imitate the genetic process of living organisms, and the law of the evolution of species. Anyway, their practical use to solve complex optimization problems was shown later, by Goldberg [8] and De Jong [9].

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Throughout history, many researches have been focused on the study of genetic algorithms. These studies can be grouped into 3 different categories:

- Practical applications of GAs: These studies are focused on the application of GAs for solving specific problems. Among these three categories, this is the most common in the literature. Two subcategories can be identified in this first group of works: variations of a classic GA [10, 11, 12] or hybridization of a GA with some other technique [13, 14, 15].
- Development of new operators: These researches present new specific operators, such as crossover [16, 17], or mutation functions [18, 19]. Normally, these operators are heuristic, and they are applied to a particular problem, in which they get a great performance.
- Analysis of the algorithm behaviour: These works focus on the theoretical and practical analysis of GAs. This kind of research analyze, for example, behavioural characteristics of the algorithm, as the convergence [20]; the efficiency of certain phases of the algorithm, such as crossover [21, 22] or mutation phases [23, 24]; or the influence of adapting some parameters, as the crossover and mutation probability [25, 26, 27]. These works attempt to overcome the drawbacks of traditional genetic algorithms, and are the source of new problem-solving techniques, such as: the adaptive genetic algorithms [28, 29], or the parallel genetic algorithms [30, 31].

In this paper a deep study on the influence of using blind crossover operators in GAs for solving combinatorial optimization problems is conducted. This study is developed by means of a comparison between GAs with this kind of operators and EAs based only on mutation operators. Thus, this work could be framed into the third category. Previously, other studies in the literature have had a similar purpose, for example [32], where the authors tried to validate the hypothesis that the crossover phase of genetic algorithms is not efficient when it is applied to routing problems. In that work, the authors develop several versions of the basic GA with some blind crossover operators (for example, Order Crossover (OX) [33], or Modified Order Crossover (MOX) [34]), and they apply these techniques to the Traveling Salesman Problem [35]. Performances of these GAs are compared with the one of an evolutionary algorithm (EA) based solely on mutations. The comparison is based on the quality of the solution and the runtime. Furthermore, the comparison also takes into account the percentage of deviation from the average values of each parameter.

On the other hand, in [22] the efficiency of six different versions of the classic GA applied to the Degree Constrained Minimal Spanning Tree Problem [36] is compared. Each version has its own crossover function. In that work, the only data shown for each version of the GA is the average value of the results obtained, so, the comparison is performed based only on this criterion. Moreover, the authors do not perform the comparison of the results obtained by a conventional GA and an EA. For this reason, with this study is not possible to quantify the real influence of the crossover phase in the optimization capacity of a GA.

Together with the above studies, in literature there are many others that are not comparable with the study presented in this paper. The main reason is that they are

focused on other types of problems [21], or because they analyzed only the crossover process of a traditional GA [37, 38, 39].

The motivation of this work stems from the absence in the literature of a study that proves objectively the efficiency of using blind crossover operators in GAs for combinatorial optimization problems. Although [32] studies on this topic, it is only applicable to routing problems, and it is only tested with one problem, the TSP. In addition, the comparison of the results done in [32] is not as deep as the made in the present work. On the other hand, as it has been mentioned, the study presented in the above mentioned [22] is not truly conclusive to prove the real influence of the crossover process in a GA.

Therefore, the goal of this paper is to perform an objective study on the efficiency of blind crossover operators in basic GAs with respect to blind mutation operators in basic EAs. In order to reach this goal, an exhaustive comparison between different versions of genetic and evolutionary algorithms is presented. This comparison includes the following criteria: quality of the results, runtime, and convergence behavior of each of the techniques reviewed. Furthermore, to perform a reliable comparison of these results a statistical study is made. For this purpose, the normal distribution z -test is performed. For the experimentation, four different problems have been used: the Traveling Salesman Problem (TSP), the Capacitated Vehicle Routing Problem (CVRP) [40], the N-Queens Problem (NQP), and the one-dimensional Bin Packing Problem (BPP) [41].

The rest of the paper is structured as follows. In Section 2 the description of the experimentation is presented. In Section 3, the tests for the TSP are shown. After that, the experiments performed with the CVRP (Section 4) are displayed, followed by those conducted with the NQP, and BPP (Section 5 and Section 6, respectively). Finally, the work is finished with the conclusions of the study and further work (Section 7).

2. Description of the experimentation

In this section a description of the experimentation is made. First, in Section 2.1, the problems used for the tests are introduced. Then, in Section 2.2, the details of the techniques developed are described, including the functions of the different steps of the algorithms. Finally, in Section 2.3 the experimentation setup is presented.

2.1. Description of the problems

For this study four different combinatorial problems have been used. Two of them are optimization problems of routing, the TSP, and the CVRP. In addition, to verify that the results of this study are valid for other types of problems apart from the routing ones, two constraint satisfaction problem has also been used in the experimentation, the NQP and the BPP. These problems were chosen because they are well-known, easy to implement and understand; and since they are easily replicable. In this way, any researcher can perform these same tests, either to check the results, or to perform them with other crossover functions, or different parameters.

The first problem used is the TSP. The TSP is one the most famous and widely studied problems throughout history in operations research and computer science. It has a great scientific interest, and it is used in a large number of studies [42, 43, 44]. This problem can be defined on a complete graph $G = (V, A)$ where $V = \{v_1, v_2, \dots, v_n\}$ is the set of

vertexes which represents the nodes of the system, and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the set of arcs which represents the interconnection between nodes. Each arc has an associated distance cost d_{ij} . The objective of the TSP is to find a route that visits every customer once (and only once), that is, a Hamiltonian cycle in the graph G , and that minimizes the total distance traveled. In a formal way, the TSP can be formulated as follows [45]:

$$\text{Minimize : } f(X) = \sum_{i=0} \sum_{i \neq j, j=0} d_{ij} x_{ij} \quad \forall i, j \in V \quad (1)$$

$$\text{Where : } x_{ij} \in \{0, 1\}, \quad \forall \{i, j\} \in A \quad (2)$$

$$\text{Subject to constraints : } \sum_{i=0, i \neq j} x_{ij} = 1, \quad \forall j \in V \quad (3)$$

$$\sum_{j=0, i \neq j} x_{ij} = 1, \quad \forall i \in V \quad (4)$$

$$\sum_{i \in S, j \in S, i \neq j} x_{ij} \geq 1, \quad \forall S \subset V \quad (5)$$

Being x_{ij} in Equation (2) a binary variable which is 1 if the arc (i, j) is used in the solution. Furthermore, V is the set of nodes of the system and d_{ij} is the distance between the nodes i and j . The objective function, (1), is the sum of all the arcs in the solution used, i.e., it is the total distance of the route. Constraints (3) and (4) indicate that each node has to be visited and abandoned only once, while the formula (5) guarantees the absence of sub-tours and indicates that any subset of nodes S has to be abandoned at least 1 time. This restriction is vital, because it avoids the presence of cycles.

Finally, all the solutions are encoded following the path representation [46]. In this way, each individual X is encoded by a permutation of numbers, which represents the path. In Figure 1.A can be seen a possible 9-node instance of the TSP, and in Figure 1.B a possible solution. This solution would be encoded as $X = (1, 2, 4, 6, 8, 9, 7, 5, 3)$, and its fitness would be $f(X) = d_{12} + d_{24} + d_{46} + d_{68} + d_{89} + d_{97} + d_{75} + d_{53} + d_{31}$.

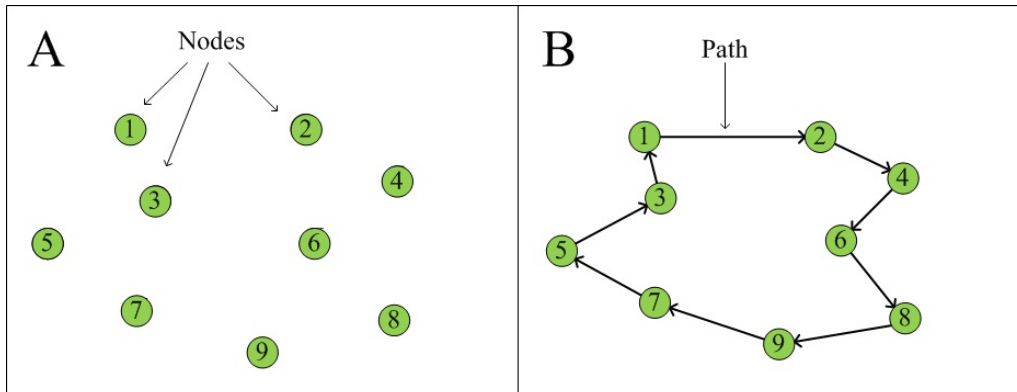


Figure 1: Example of TSP instance and possible solution

The second selected problem is the CVRP. Due to its complexity and, above all, its applicability to real life, the CVRP is also used in many researches every year [47, 48]. The problem can be defined on a complete graph $G = (V, A)$ where $V = \{v_0, v_1, \dots, v_n\}$ is the set of vertexes and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the set of arcs. The vertex v_0 represents the depot and the rest are the customers, each of them with a demand q_i . A fleet of vehicles K is available with a limited capacity Q for each vehicle. The objective of the CVRP is to find a number of routes with a minimum cost such that *i*) each route starts and ends at the depot, *ii*) each client is visited exactly by one route and *iii*) the total demand of the customers visited by one route does not exceed the total capacity of the vehicle that performs it [49]. This problem could be formulated as follows [40]:

$$\text{Minimize : } f(X) = \sum_{i=0} \sum_{i \neq j, j=0} d_{ij} x_{ij} \quad \forall i, j \in V \quad (6)$$

$$\text{Where : } y_i^r \in \{0, 1\}, \quad \forall r \in K \quad (7)$$

$$\text{And : } x_{ij} \in \{0, 1\}, \quad \forall \{i, j\} \in A; i \neq j \quad (8)$$

$$\text{Subject to constraints : } \sum_{i=0, i \neq j} x_{ij} = 1, \quad \forall j \quad (9)$$

$$\sum_{j=0, i \neq j} x_{ij} = 1, \quad \forall i \in V \quad (10)$$

$$\sum_i x_{ij} \geq |S| - v(S), \quad \{S : S \subseteq V/\{1\}, |S| \geq 2\} \quad (11)$$

$$\sum_{i \in S} q_i y_i^r \leq Q, \quad \forall r \in K \quad (12)$$

The formula (6) is the objective function, which is the total distance traveled by all the routes. The variable (7) is a binary variable which is 1 if the vehicle r satisfies the demand of the client i , and 0 otherwise. The binary variable (8) is 1 if the arc (i, j) is used in the solution. Formulas (10) and (11) ensure that every customer is visited by one route only and exactly once. Finally, clause (11) serves to eliminate sub-tours, where $|S|$ is the number of customers and $r(S)$ the minimum number of vehicles to serve all. Finally, the restriction (12) ensures that the sum of all the demands of a route does not exceed the maximum vehicle capacity.

In the case of CVRP, the path representation is also used for the individuals encoding [50]. In this case, the routes are also represented as a permutation of nodes. To distinguish the routes of one solution, they are separated by zeros. In Figure 2.A an example of a CVRP is shown. On the other hand, in Figure 2.B a solution composed by three different routes is depicted. On this occasion, this solution would be encoded as $X = (3, 1, 5, \mathbf{0}, 2, 4, \mathbf{0}, 7, 9, 8, 6)$, and its fitness would be $f(X) = d_{03} + d_{31} + d_{15} + d_{50} + d_{02} + d_{24} + d_{40} + d_{07} + d_{79} + d_{98} + d_{86} + d_{60}$.

The third problem is the NQP. This problem is a generalization of the problem of putting eight non attacking queens on a chessboard [51], which was introduced by M. Bezzel in 1848 [52]. The NQP consist of placing N queens on a $N \times N$ chess board, in order that they cannot attack each other, i.e., on every row, column and diagonal, only one queen can be placed. This problem is a classical combinatorial design problem (constraint

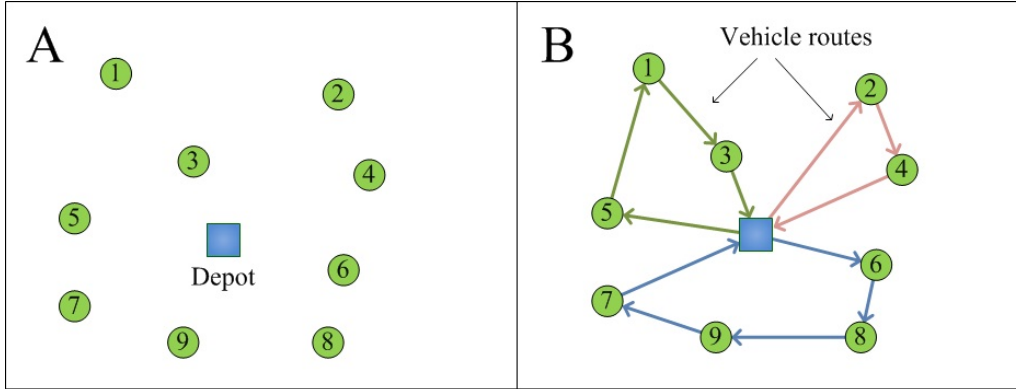


Figure 2: Example of CVRP instance and possible solution

satisfaction problem), which can also be formulated as a combinatorial optimization problem [53]. Although NQP is often used as benchmarking problem, it has also some real applications [54]. In this study, NQP has been formulated as a combinatorial optimization problem, where a solution X is coded as a N -tuple (q_1, q_2, \dots, q_n) , which is a permutation of the N -tuple $(1, 2, \dots, N)$. Each q_i represents the row occupied by the queen positioned in the i th column. Using this representation, vertical and horizontal collisions are avoided, and the complexity of the problem becomes $O(N!)$. Thus, the fitness function is defined as the number of diagonal collisions along the board. i th and j th queens collide diagonally if:

$$|i - q_i| = |j - q_j| \quad \forall i, j : \{1, 2, \dots, N\}; i \neq j \quad (13)$$

The objective is to minimize the number of conflicts, being zero the ideal fitness. An example of an individual for a 6-queens chess board could be seen in Figure 3. According to the encoding explained, the individual represented in this figure would be encoded as $f(X) = (2, 1, 4, 6, 5, 3)$. In addition, its fitness would be 3, since there are three diagonal collisions (2-1, 1-4, and 6-5). This same formulation has been widely used in literature [55, 56].

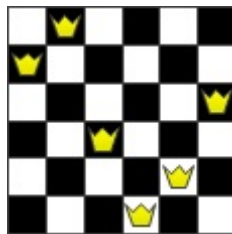


Figure 3: Example of a 6x6 instance for the NQP

Finally, the last used problem is the BPP. In distribution and production the fact of packing items into boxes or bins is a daily task. Depending on the shape and size of the items, as well as the form and capacity of bins, a wide amount of different packing

problems can be formulated. The BPP is the simplest problem in this field [41, 57], and it is frequently used in the literature as benchmarking problem [58, 59, 60]. The BPP consists in a set of items $I = \{i_1, i_2, \dots, i_n\}$, each with an associated size s_i , and an infinite number of bins B of an equal capacity q . The objective of the BPP is to pack all the items into a minimum number of bins. Therefore, the objective function is the number of bins, which has to be minimized. In this way, given n items and n bins, the BPP can be formulated as follows:

$$\text{Minimize : } f(X) = \sum_{i=0}^n y_i \quad (14)$$

$$\text{Where : } y_i \in \{0, 1\}, \quad \forall i \in \{1, \dots, n\} \quad (15)$$

$$\text{And : } x_{ij} \in \{0, 1\}, \quad \forall \{i, j\} \in \{1, \dots, n\} \quad (16)$$

$$\text{Subject to constraints : } \sum_{i=0}^n x_{ij} = 1, \quad \forall j \in \{1, \dots, n\} \quad (17)$$

$$\sum_{j=0}^n s_i x_{ij} \leq q, \quad \forall i \in \{1, \dots, n\} \quad (18)$$

Being x_{ij} in Equation (15) a binary variable which is 1 if item j is put in bin i , and y_i a variable which is 1 if bin i is used.

In this study, the solutions of this problem are encoded as a permutation of items. To count the number of bins needed for one solution, the size of the items is accumulated in a variable, *sumSize*. When *sumSize* exceeds q , the number of bins is increased in 1, and *sumSize* is restarted. For example, in a simple instance of 10 items, each one with a size of 40, and $q=120$. One possible solution could be $X = (i_9, i_6, i_1, i_2, i_4, i_{10}, i_8, i_3, i_7, i_5)$, and its fitness would be 4.

2.2. General description of the developed techniques

For the experiments, nine different techniques have been implemented and compared. The first six techniques (GA_{v1} , GA_{v2} , GA_{v3} , GA_{v4} , GA_{v5} , and GA_{v6}) are conventional GAs with different configurations. The remaining three techniques are EAs (EA_{v1} , EA_{v2} , and EA_{v3}). The structure used for both GAs is the represented in Algorithm 1, and it is considered the conventional one. On the other hand, the flowchart of the EAs is the same, eliminating the parent selection process and crossover phase.

The parametrization of the GAs has been made based on the concepts outlined in many previous studies [61, 62, 63]. According to these researches, the crossover is considered the main operator of genetic algorithms, while the mutation is a secondary operation. In this way, GA_{v1} and GA_{v2} have a crossover probability (p_c) of 90%, and a mutation probability (p_m) of 10%. In addition, GA_{v3} and GA_{v4} have a $p_c = 75\%$ and $p_m = 25\%$. Finally, GA_{v5} and GA_{v6} have $p_c = 50\%$ and $p_m = 50\%$. On the other hand, all the EAs have a $p_c = 0\%$ and a p_m of 100%. For GA_{v1} , GA_{v2} and EA_{v1} , a population composed by 50 randomly created individuals is used. Additionally, for GA_{v3} , GA_{v4} , and EA_{v2} , the population has 75 individuals generated randomly. Finally, for GA_{v5} , GA_{v6} , and EA_{v3} , a population composed by 100 random individuals is used. In relation to the parents selection criteria, the well-known binary tournament has been

Algorithm 1: Pseudocode of both GAs

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1 Initialization of initial population
2 repeat
3   | Parents selection process
4   | Crossover phase
5   | Mutation phase
6   | Survivor selection process
7 until termination criterion reached;
8 Return the fitness of the best individual found

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used. Regarding the survivor function, it is 50% elitist-random (which means that the half of the population is composed by the best individuals, and the remaining ones are selected randomly). About the ending criteria, the execution of each technique finishes when there are $n + \sum_{k=1}^n k$ generations without improvements in the best solution, where n is the size of the problem.

In order to make a fair and rigorous comparison, the good practices proposed in [64] have been followed to develop all the techniques. These practices dictate that to test the ability of optimization of a meta-heuristic, and to perform a rigorous comparison between different techniques, it is appropriate to use neutral operators throughout the implementation of them. In other words, heuristic operators that use characteristics of the problem and optimize by themselves have to be avoided. Otherwise, by using heuristic operators, the optimization capacity of the technique is influenced by the performance of these operators, and it could not be possible to determine, objectively, which is the real efficiency of the meta-heuristic.

With respect to TSP, the well-known 2-opt [65], and the insertion function [66] have been used as mutation function. The first one is a classic operator which selects randomly two arcs of the solution. Then, these edges are removed from the route, and two new arcs are created, avoiding subtours. On the other hand, the second operator selects, and extracts one random node of a solution, and inserts it in another random position. Regarding crossover functions, the OX [33], Order Based Crossover (OBX)[67], MOX [34], and the Half Crossover (HX) [32] have been used. These same mutation and crossover functions have been used for the NQP and BPP.

The OX builds the children by choosing a sub-route of one of the parents and maintaining the order of the nodes of the remaining parent. First, two cut points are randomly selected, identical for both parents, and the segments between the cut points are preserved in the children. Then, starting from the second breakpoint, the remaining nodes are inserted in the same order they appear in the other parent (starting also from the second cut point), considering that the nodes that has already been inserted have to be omitted. When the end of the string is reached, it continues through the beginning of this. An example of this type of crossover could be as follows:

$$P = (12345678) \rightarrow P = (12|345|678) \rightarrow H1 = (**|345|**) \rightarrow H1 = (87|345|126)$$

$$M = (24687531) \rightarrow M = (24|687|531) \rightarrow H2 = (**|687|**) \rightarrow H2 = (45|687|123)$$

In the OBX, some random positions are selected in a parent tour. The order of the nodes in the selected positions is imposed on the other parent. For example, considering the same parents (P and M), and supposing that the second, third, and sixth positions are selected, the nodes placed in these positions have to be inserted in the same order in the corresponding offspring. In this case, in M these nodes are 4, 6, and 5, and they have to be inserted in the first child in this same order. The rest of the route remains in the same order and position as in P .

$$P = (12345678) \rightarrow H1 = (123***78) \rightarrow H1 = (12346578)$$

The other child would be the next one, considering that the nodes in the second, third and sixth positions of P are 2, 3, and 6:

$$M = (24687531) \rightarrow H2 = (*4*875*1) \rightarrow H2 = (24387561)$$

In the case of MOX, a random cut point is selected. This cutpoint divides each parent into two sections. The nodes placed on the left part of the cut point impose their position on the other parent. Then, the remaining nodes are inserted in the children in the same order they appear in the other parent. An example of the working way of this crossover function could be as follows:

$$P = (1234|5678) \rightarrow H1 = (*2*4*6*8) \rightarrow H1 = (72543618)$$

$$M = (2468|7531) \rightarrow H2 = (24***31) \rightarrow H2 = (24567831)$$

The HX is a particular case of the traditional crossover, in which the cut point is made always in the middle of the path. In this way, first, a cut is made in the central position of the parents. Then, the order of nodes placed in the left part remains in the same order in the offspring. The remaining nodes are added in the same position that they can be found in the other parent. An example of the HX could be as shown below:

$$P = (1234|5678) \rightarrow H1 = (1234****) \rightarrow H1 = (12346875)$$

$$M = (2468|7531) \rightarrow H2 = (2468****) \rightarrow H2 = (24681357)$$

On the other hand, for CVRP, the implemented crossover functions are the Short Route Crossover (SRX), the Random Route Crossover (RRX), and the Large Routes Crossover (LRX). These operators are a particular case of the traditional crossover, in which the cut point is made always in the middle of the chromosome. The operation of the first of them is the following: first of all, half of the routes (the shortest ones) of one of the parents are inserted in the child. After that, the nodes already selected are removed from the other parent, and the remaining nodes are inserted in the child in the same order (taking into account the vehicle capacity). Assuming a 17-node instance (including the depot), an example could be the following:

$$P = (1, 2, 3, 4, \mathbf{0}, 9, 10, 11, 12, \mathbf{0}, 13, 14, 15, 16, \mathbf{0}, 5, 6, 7, 8)$$

$$M = (1, 12, 6, 3, \mathbf{0}, 2, 4, 7, 11, \mathbf{0}, 5, 14, 16, 9, \mathbf{0}, 8, 13, 10, 15)$$

The resulting offsprings could be as follows:

$$H1 = (1, 2, 3, 4, \mathbf{0}, 9, 10, 11, 12, \mathbf{0}, 6, 7, 5, 14, \mathbf{0}, 16, 8, 13, 15)$$

$$H2 = (1, 12, 6, 3, \mathbf{0}, 2, 4, 7, 11, \mathbf{0}, 9, 10, 13, 14, \mathbf{0}, 15, 16, 5, 8)$$

RRX works similar to the SRX. In this case, the routes selected in the first step of the process are selected randomly, instead of choosing the best ones. Ultimately, in the case of LRX, the selected routes are the longest ones. Regarding the mutation functions for CVRP, the Vertex Insertion Function and the Swapping function have been used. The first one selects one random node from one randomly chosen route of the solution. This node is extracted, and inserted in another randomly selected route, respecting the capacity constraints. On the other hand, in the Swapping function two nodes are selected at random from two random routes to swap their positions, respecting also the capacity constraints. These functions have been used for the CVRP in some other works, such as [68].

2.3. Experimentation setup

In this section the aspects common to all the experimentations are introduced. to begin with, all GA_{v1}, GA_{v2} and EA_{v1} were run on an Intel Core i5 2410 laptop, with 2.30 GHz and a RAM of 4 GB. The rest of the techniques were executed on an Intel Core i7 3930 computer, with 3.20 GHz and a RAM of 16GB. Java was used as programming language. For every problem 10 different instances have been used, and for each of them 50 runs have been executed. For each experimentation, the results, average runtime (in seconds), and convergence behaviour of every technique are shown. In addition, the standard deviation of each of them is also shown. In addition, for every problem three different experimentations have been performed. In each experimentation, the performance of one EA is compared with the one of two different GAs. The three experimentations differ in the configuration of the techniques.

Additionally, in order to make a fair and rigorous comparison, the normal distribution z -test has been performed for all experiments. Thanks to this statistical test, it can be shown whether the differences in the results obtained by each technique are significant or not. The z statistic has the following form:

$$z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}}$$

where:

\overline{X}_1 : Average of an EA,

σ_1 : Standard deviation of an EA,

\overline{X}_2 : Average of the other technique,

σ_2 : Standard deviation of the other technique,

n_1 : Sample size for an EA,

n_2 : Sample size for the other technique,

The z value can be positive (+), neutral (*), or negative (-). The positive value of z indicates that the EA is significantly better. In the opposite case, the EA obtains substantially worse solutions. If z is neutral, the difference is not significant. The confidence interval has been stated at 95% ($z_{0.05} = 1.96$). Besides showing the symbolic value of z (+, -, or *), its numerical value is also displayed. Thus, the difference in results may be seen more easily. Finally, as it has been mentioned, the z -test has been performed for the results quality, runtime and convergence behaviour.

3. Experimentation with the TSP

In this section the experimentation with the TSP is shown. All the instances have been picked from the well-known TSPLIB benchmark [69]. In order to make this experimentation more understandable, Table 1 summarizes the characteristics of the nine algorithms used for the tests.

Alg.	Population	p_c	p_m	Crossover operator	Mutation operator
GA_{v1}	50 individuals	90%	10%	OX	2-opt
GA_{v2}	50 individuals	90%	10%	OBX	2-opt
EA_{v1}	50 individuals	0%	100%	No crossover performed	2-opt
GA_{v3}	75 individuals	75%	25%	HX	Insertion
GA_{v4}	75 individuals	75%	25%	MOX	Insertion
EA_{v2}	75 individuals	0%	100%	No crossover performed	Insertion
GA_{v5}	100 individuals	50%	50%	OBX	2-opt
GA_{v6}	100 individuals	50%	50%	OX	2-opt
EA_{v3}	100 individuals	0%	100%	No crossover performed	2-opt

Table 1: Summary of the characteristics of all the techniques developed for the TSP

In Table 2 the results and average runtimes can be found. On the other hand, in Table 3 the convergence behaviour of each technique is displayed. For this purpose, the average number of generations needed to reach the final solution is used. In Table 4 the results of the z -test are shown.

Several conclusions can be drawn by analyzing the results shown. First of all, looking at Table 2 it can be seen that, for the three experimentations, all the EAs performs better than the other two techniques in all the instances. According to Table 4, in the first

TSP		GA_{v1}				GA_{v2}				EA_{v1}			
Instance		Results		Time (s)		Results		Time (s)		Results		Time (s)	
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
St70	675	711.1	25.5	8.9	2.3	714.4	18.3	4.6	1.3	705.0	13.9	1.5	0.5
Eilon75	535	574.7	11.8	12.2	3.1	580.9	14.0	7.5	2.2	570.1	10.9	2.2	0.6
Eil76	538	575.6	11.4	13.0	3.0	586.1	13.1	6.7	1.9	574.0	11.2	2.0	0.7
KroA100	21282	22129.5	557.5	22.5	6.0	22376.5	546.5	14.3	4.3	22117.0	454.2	3.9	0.8
KroB100	22140	23133.2	561.3	24.5	5.6	23332.5	420.9	13.1	4.0	23098.1	401.6	4.0	0.8
KroC100	20749	21822.5	706.7	21.1	3.4	21924.4	479.2	15.4	4.4	21642.5	544.7	4.1	1.2
KroD100	21294	22347.9	573.2	24.4	7.5	22550.2	463.5	15.9	5.2	22239.8	383.4	4.2	1.0
Eil101	629	680.1	11.3	42.6	9.9	685.8	13.2	22.8	6.5	680.0	9.2	4.4	0.9
Pr107	44303	46282.2	1528.9	36.4	13.7	46470.5	1401.2	23.1	7.9	45587.8	936.4	5.8	1.8
Pr124	59030	60407.6	722.2	47.0	11.0	60678.3	1170.1	26.4	6.5	60384.6	927.8	7.5	1.2
Instance		GA_{v3}				GA_{v4}				EA_{v2}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
St70	675	744.6	21.9	3.6	1.1	725.2	20.5	3.4	0.8	713.0	10.9	0.5	0.1
Eilon75	535	604.2	26.6	4.6	1.1	603.9	16.5	4.9	1.2	579.6	14.1	0.7	0.1
Eil76	538	619.5	19.9	4.8	0.9	597.5	26.1	5.4	1.1	583.7	8.5	0.7	0.1
KroA100	21282	22416.4	518.4	13.3	3.0	22375.6	533.5	10.0	2.6	22202.0	539.4	1.7	0.3
KroB100	22140	23425.4	421.7	11.8	2.6	23542.6	612.1	10.1	1.9	23024.2	458.9	1.7	0.3
KroC100	20749	22304.0	634.9	11.8	2.5	22302.1	733.7	10.2	3.3	21539.1	468.3	1.8	0.3
KroD100	21294	22592.3	434.4	13.0	2.6	22797.8	629.6	8.9	2.1	22370.8	525.3	1.6	0.2
Eil101	629	718.9	17.6	16.4	3.7	712.7	15.3	17.7	3.9	687.4	11.1	1.6	0.3
Pr107	44303	46810.9	1100.7	17.1	3.1	46661.2	1242.7	13.2	3.4	45319.4	694.5	2.4	0.5
Pr124	59030	61421.5	1500.9	27.1	6.1	61148.1	1286.2	18.0	3.0	60380.6	669.8	3.6	0.5
Instance		GA_{v5}				GA_{v6}				EA_{v3}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
St70	675	716.1	19.8	2.8	0.6	712.4	11.6	4.9	1.0	705.3	10.3	1.3	0.2
Eilon75	535	582.8	11.9	4.0	1.1	576.2	9.9	7.7	1.5	569.0	7.4	1.9	0.6
Eil76	538	582.0	12.9	4.0	1.2	576.5	13.4	8.5	2.7	572.7	10.3	1.8	0.3
KroA100	21282	22366.4	522.9	5.5	1.5	22279.4	614.0	13.9	5.7	21838.6	294.3	3.6	0.6
KroB100	22140	23123.7	371.7	7.0	2.8	23134.9	375.8	12.7	3.9	22964.2	529.3	3.6	0.5
KroC100	20749	22005.9	584.2	6.0	1.8	21718.2	456.8	10.7	2.4	21468.0	400.8	3.8	0.8
KroD100	21294	22404.3	317.7	7.7	2.6	22163.7	356.9	13.3	4.2	22039.1	441.6	3.5	0.5
Eil101	629	696.9	16.3	11.4	2.1	689.5	12.1	24.6	5.1	675.3	9.8	4.3	0.6
Pr107	44303	46276.0	1153.6	10.5	4.2	45542.7	1053.5	25.5	9.4	45145.4	590.4	4.9	1.2
Pr124	59030	60450.1	675.1	13.7	3.2	60020.6	564.1	26.7	8.3	59962.0	740.7	6.9	0.9

Table 2: Results and runtimes of the nine techniques applied to the TSP. For each instance, the results, average runtime, and their standar deviations are shown.

experimentation, these differences are significant only in two cases compared to GA_{v1} . On the other hand, these improvements are significant in all but one instances respect to GA_{v2} . In the second experimentation, the EA_{v2} gets significantly better results in all the instances compared with the GA_{v3} , and in nine instances (out of ten) compared with GA_{v4} . Finally, for the last experimentation, the EA_{v3} significantly outperforms GA_{v5} in the 100% of the instances, and in the 60% (6 out of 10) regarding GA_{v6} . For this reason, taking into account that EAs never gets worse results than the other two alternatives in the three experiments, the following conclusion can be stated:

- Conclusion 1: According to the experimentation performed, using blind crossover operators in genetic algorithms does not offer significant improvements in the results for the TSP.

This conclusion could be explained in the following way. The main purpose of the

TSP	GA_{v1}		GA_{v2}		EA_{v1}	
Instance	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
St70	6093.1	1530.5	5590.7	2192.0	6162.6	1530.5
Eilon75	7920.3	2715.6	7279.4	2761.5	8439.5	5852.9
Eil76	8248.8	2663.1	6635.9	2481.2	7461.0	1900.5
KroA100	9568.5	3549.2	9980.5	3831.0	12345.7	2404.4
KroB100	10419.9	3158.8	10090.6	3655.4	13775.1	3594.4
KroC100	9224.9	3853.9	9686.7	3364.6	13614.0	3623.4
KroD100	9495.2	3736.6	9901.1	3919.0	13086.0	3855.8
Eil101	18646.2	5144.4	15209.5	5494.0	15003.4	3936.2
Pr107	13115.3	6858.9	12489.0	5737.5	18683.9	6795.6
Pr124	13662.3	4851.2	11033.7	4303.2	18917.7	4239.8
Instance	GA_{v3}		GA_{v4}		EA_{v2}	
St70	4400.2	1446.8	2431.6	770.6	3895.7	677.8
Eilon75	4868.1	1342.0	3123.5	940.5	4575.0	4712.5
Eil76	4954.5	1098.8	3352.6	922.7	4712.5	1269.3
KroA100	8382.2	2206.6	7714.8	3258.0	8682.8	2441.2
KroB100	7341.8	1863.3	6516.6	2593.8	9087.0	1879.9
KroC100	8304.0	1034.9	3100.2	1299.2	9824.8	1881.3
KroD100	8183.0	1886.2	7005.9	2796.8	8798.4	1485.5
Eil101	10241.1	2563.7	7260.9	1876.6	8744.9	2062.3
Pr107	8986.3	2021.3	4540.6	1455.5	12741.4	3343.7
Pr124	11880.6	2304.7	9462.5	3687.2	15258.5	2877.1
Instance	GA_{v5}		GA_{v6}		EA_{v3}	
St70	4188.5	1503.2	4748.8	1500.6	6134.2	1136.0
Eilon75	5792.6	2290.8	7020.5	1952.7	8631.4	2727.7
Eil76	7618.1	2425.2	7637.6	3274.4	7521.0	2056.8
KroA100	3316.9	2245.0	7032.5	4883.1	11817.8	2574.5
KroB100	5491.8	4151.6	5950.4	3318.7	11619.8	2321.5
KroC100	3928.9	2597.3	4360.2	2136.4	12817.6	3363.8
KroD100	6494.7	3838.2	6484.9	3566.6	11216.1	2434.8
Eil101	10718.5	2877.3	14500.3	5100.6	14450.1	2938.9
Pr107	6753.0	4931.3	12775.2	11806.8	16436.2	5161.2
Pr124	6387.7	3342.2	8251.1	4914.7	18022.6	3610.0

Table 3: Convergence behaviour of the nine techniques applied to the TSP.

crossover phase is to obtain new individuals making combinations of the existing ones. Although these operations were designed for the exploitation of the solution space, several studies in the literature discuss this fact [39, 70]. On the other hand, as it has been shown in several works before [71, 72], blind crossovers between different individuals can be useful to make large jumps along the solution space. For this reason, blind crossover operators applied to the TSP contribute to increase the exploration capability of the algorithm, instead of helping to the exploitation.

This way, it could be said that for the TSP, using blind crossovers helps a broad exploration of the solution space, but does not help to make an exhaustive search of promising regions. This is so because it is improbable that the resulting offspring from blind crossovers can improve their parents. In addition, this fact is accentuated when the execution is near to the convergence. To get a deeper search, the existence of a function that makes little jumps in the solution space becomes necessary. The mutation function

TSP	EA_{v1} vs. GA_{v1}			EA_{v1} vs. GA_{v2}		
Instance	Results	Convergence	Time	Results	Convergence	Time
St70	* (1.46)	* (-0.19)	+ (21.61)	+ (4.76)	* (-1.51)	+ (14.99)
Eilon75	+ (2.00)	* (-0.56)	+ (22.02)	+ (4.30)	* (-1.26)	+ (16.53)
Eil76	* (0.69)	* (1.70)	+ (24.85)	+ (4.93)	* (-1.86)	+ (16.53)
KroA100	* (0.12)	- (-4.58)	+ (21.72)	+ (2.58)	- (-3.69)	+ (16.48)
KroB100	* (0.36)	- (-4.95)	+ (25.63)	+ (2.84)	- (-5.08)	+ (15.57)
KroC100	* (1.42)	- (-5.86)	+ (28.95)	+ (2.74)	- (-5.61)	+ (17.57)
KroD100	* (1.10)	- (-4.72)	+ (18.89)	+ (3.64)	- (-4.09)	+ (15.73)
Eil101	* (0.05)	+ (3.97)	+ (26.94)	+ (2.56)	* (0.21)	+ (19.56)
Pr107	+ (2.73)	- (-4.07)	+ (15.56)	+ (3.70)	- (-4.92)	+ (14.90)
Pr124	* (0.13)	- (-5.76)	+ (25.15)	* (1.39)	- (-9.92)	+ (20.12)
Instance	EA_{v2} vs. GA_{v3}			EA_{v2} vs. GA_{v4}		
St70	+ (9.13)	+ (2.23)	+ (25.43)	+ (3.71)	- (-10.08)	+ (19.84)
Eilon75	+ (5.77)	* (0.42)	+ (24.66)	+ (7.91)	- (-2.13)	+ (24.96)
Eil76	+ (11.61)	* (1.01)	+ (30.08)	+ (3.55)	- (-6.12)	+ (32.01)
KroA100	+ (2.02)	* (-0.64)	+ (22.42)	* (1.61)	* (-1.68)	+ (27.20)
KroB100	+ (4.55)	- (-4.66)	+ (30.87)	+ (4.55)	- (-5.60)	+ (27.28)
KroC100	+ (6.85)	- (-5.00)	+ (17.92)	+ (6.19)	- (-20.79)	+ (28.08)
KroD100	+ (2.29)	* (-1.81)	+ (24.46)	+ (3.68)	- (-4.00)	+ (30.91)
Eil101	+ (10.70)	+ (3.10)	+ (29.10)	+ (9.46)	- (-3.89)	+ (28.19)
Pr107	+ (8.10)	- (-6.76)	+ (22.22)	+ (6.66)	- (-15.90)	+ (33.10)
Pr124	+ (4.47)	- (-6.47)	+ (33.47)	+ (3.74)	- (-8.76)	+ (27.14)
Instance	EA_{v3} vs. GA_{v5}			EA_{v3} vs. GA_{v5}		
St70	+ (3.42)	- (-7.30)	+ (16.77)	+ (3.23)	- (-5.20)	+ (24.96)
Eilon75	+ (6.96)	- (-5.63)	+ (11.85)	+ (4.11)	- (-3.39)	+ (25.38)
Eil76	+ (3.98)	* (0.21)	+ (12.57)	* (1.58)	* (0.21)	+ (17.43)
KroA100	+ (6.21)	- (-17.59)	+ (8.31)	+ (4.57)	- (-6.12)	+ (12.7)
KroB100	+ (1.74)	- (-9.10)	+ (8.45)	* (1.85)	- (-9.89)	+ (16.36)
KroC100	+ (5.36)	- (-14.78)	+ (7.89)	+ (2.91)	- (-15.00)	+ (19.28)
KroD100	+ (4.74)	- (-7.34)	+ (11.21)	* (1.55)	- (-7.74)	+ (16.38)
Eil101	+ (8.03)	- (-6.41)	+ (22.98)	+ (6.44)	* (0.06)	+ (27.85)
Pr107	+ (6.16)	- (-9.59)	+ (9.06)	+ (2.32)	- (-2.00)	+ (15.37)
Pr124	+ (3.44)	- (-16.72)	+ (14.46)	* (0.44)	- (-11.33)	+ (16.77)

Table 4: z -test for TSP. '+' indicates that EA is better. '-' depicts that it is worse. '*' indicates that the difference between the two algorithms is not significant (at 95% confidence level)

can handle this goal, and it can also contribute to perform a broad search of promising regions [73, 74]. Thus, an EA can conduct a deep and wide search, obtaining similar (or better) results to the GAs.

Regarding the runtimes, the EAs also outperform their corresponding algorithms in all the instances and experimentations. In addition, in this case these improvements are significant in all of the cases. Besides this, the differences in the runtimes become wider as the size of the instance grows. This is particularly important in real-time applications, where the runtime is a key factor. For these reasons, the following conclusion can be deduced:

- Conclusion 2: In relation to the experimentation performed, the use of blind crossover operators increases significantly the execution time of an evolutionary algorithm applied to the TSP.

This difference in runtime between the GAs and the EAs can be explained easily, in the same manner as explained in previous works [28]: comparing the working way

of the crossover and mutation operators, the former are complex operations in which two individuals combine their characteristics. On the other hand, a mutation is a small modification of a chromosome, and requires considerably less time than the previous ones. Thereby, the fact that an EA substitutes the crossover phase in exchange for performing more mutations is perfectly reflected in runtime, giving a great advantage to an EA in this aspect.

Finally, if the data presented in Table 3 is analyzed, first, it can be seen that both GA_{v1} and GA_{v2} present a better convergence behaviour compared to EA_{v1} . More specifically, GA_{v1} is better than EA_{v1} in the 80% of the cases, and GA_{v2} in all but one. In addition, comparing with the EA_{v1} , these differences are significantly better for the GA_{v1} in 60% of the instances, while in 30% are not significant. In the remaining cases, the differences are substantially better for the EA_{v1} . Regarding GA_{v2} , these data is, respectively, 60%, 40%, and 0%. Regarding the second experimentation, GA_{v4} shows a significantly better convergence behaviour than EA_{v2} in the 100% of the instances. On the other hand, the GA_{v3} outperforms EA_{v2} in the 60% on the cases, being these differences significant in four instances (out of 10). By the way, EA_{v2} significantly outperforms GA_{v3} in two instances. Finally, regarding the last experimentation, the GA_{v5} and GA_{v6} present a substantially better convergence in the 90%, and 80% of the instances, respectively. In the remaining instances, the EA_{v3} shows a non-significant better performance. Taken into account all these data, the following conclusion can be drawn:

- Conclusion 3: Considering these tests conducted for the TSP, the algorithms that use blind crossover operators demonstrate a better convergence behaviour, needing less generations to find their final solution.

This improvement in the convergence behaviour can be explained as follows. As mentioned above, blind crossover operators can be a great help to make a broad exploration of the solution space. Comparing with the mutation functions, a blind crossover can make more sudden jumps in the solution space. On the other hand, mutations are simple operations which move along the solution space little by little, conducting small jumps. For this reason, and depending on the problem complexity, with the crossover functions a broader and faster exploration can be made, and the final solution can be found in less generations.

Furthermore, as has been mentioned above, mutations are an excellent option to explore the solution space. In addition, as can be seen in the results shown in Table 2, mutations can also take care of the exploitation capacity of the technique. So, using them, similar (or better) solutions can be found.

In conclusion, all the GAs converge faster than their corresponding EA. Thus, comparing with the EAs, all the versions of the GA need less generations to reach the final solution. Anyway, this fact does not mean a better performance. As can be seen in the results presented, the EAs obtains similar, or significantly better results for all the TSP instances (needing a substantially smaller runtime).

4. Experimentation with the CVRP

In this section the experimentation with the CVRP is displayed. In this case, instances have been picked from the CVRP set of Christofides and Eilon (<http://neo.lcc.uma.es/vrp>)

Alg.	Population	p_c	p_m	Crossover operator	Mutation operator
GA_{v1}	50 individuals	90%	10%	SRX	Vertex Insertion Function
GA_{v2}	50 individuals	90%	10%	RRX	Vertex Insertion Function
EA_{v1}	50 individuals	0%	100%	No crossover performed	Vertex Insertion Function
GA_{v3}	75 individuals	75%	25%	LRX	Swapping Function
GA_{v4}	75 individuals	75%	25%	SRX	Swapping Function
EA_{v2}	75 individuals	0%	100%	No crossover performed	Swappingn Function
GA_{v5}	100 individuals	50%	50%	RRX	Vertex Insertion Function
GA_{v6}	100 individuals	50%	50%	LRX	Vertex Insertion Function
EA_{v3}	100 individuals	0%	100%	No crossover performed	Vertex Insertion Function

Table 5: Summary of the characteristics of all the techniques developed for the CVRP

¹). As in previous section, with the aim of making the experimentation more understandable, Table 5 summarizes the characteristics of the nine algorithms developed.

In Table 6 the results and average runtime can be found. Moreover, the convergence behaviour is shown in Table 7. Finally, Table 8 displays the statistical z -test performed for the CVRP.

The conclusions that can be drawn looking at these tables are similar to those mentioned in the previous section. In this case, regarding the quality of the results, and according to the data shown in Table 6, EA_{v1} outperforms GA_{v1} in 80% of the instances, and GA_{v2} in all of them. In addition, looking at Table 8 these improvements are significant in the 60% of the cases compared to GA_{v1} . On the other hand, in 30% the differences are not significant, and in the remaining ones EA_{v1} gets substantially worse results. Regarding GA_{v2} , these percentages are, respectively, 90%, 10%, and 0%.

Furthermore, EA_{v2} performs better than GA_{v3} in the 90% of the instances, and GA_{v4} in the 80%. In the case of GA_{v3} , the EA_{v2} obtains significantly better results in nine instances. In the remaining instance, GA_{v3} outperforms EA_{v2} but not substantially. Moreover, EA_{v2} improves significantly GA_{v4} in the 50% of the instances. In addition, in the 40% these improvements are not substantially. Additionally, in the remaining instances, EA_{v2} gets significantly worse results.

Finally, regarding the third experimentation, EA_{v3} outperforms GA_{v5} and GA_{v6} in 80% of the cases. In addition, these improvements are significant in the 60% of the instances regarding both versions of the GAs. On the other hand, EA_{v3} gets worse

¹Last update: January 2013

CVRP		GA_{v1}				GA_{v2}				EA_{v1}			
Instance		Results		Time (s)		Results		Time (s)		Results		Time (s)	
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
En22k4	375	389.0	9.8	1.8	0.5	410.9	23.2	2.5	1.1	404.8	19.5	1.1	0.3
En23k3	569	622.7	28.9	2.1	0.9	629.9	41.6	2.3	1.0	602.7	30.8	1.6	0.6
En30k3	534*	559.6	29.2	3.9	1.2	582.7	43.3	5.0	2.0	545.8	41.6	2.0	0.7
En33k4	835	907.4	31.9	6.0	1.8	932.8	30.6	7.0	2.3	911.9	24.9	2.2	0.7
En51k5	521	641.0	38.3	13.8	5.4	694.3	53.4	18.2	7.9	628.4	37.4	4.5	1.4
En76k7	682	850.0	45.7	44.4	16.4	899.5	63.3	55.1	16.5	822.3	42.9	10.0	3.3
En76k8	735	920.6	59.3	40.9	19.1	952.2	44.6	52.3	17.5	886.9	37.6	8.3	2.8
En76k14	1021	1186.9	35.6	33.4	14.0	1219.6	47.4	38.1	12.6	1171.0	36.2	6.5	2.2
En101k8	815	1061.4	54.8	107.5	33.9	1110.9	71.6	126.3	35.3	1016.7	49.9	15.7	5.1
Pr101k14	1071	1320.0	46.5	88.1	29.6	1370.7	73.1	114.9	34.3	1270.6	41.4	14.8	4.6
Instance		GA_{v3}				GA_{v4}				EA_{v2}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
En22k4	375	388.0	14.8	1.6	0.4	386.1	10.3	2.3	0.4	392.8	13.9	0.8	0.1
En23k3	569	622.5	31.1	2.7	1.1	615.7	37.9	2.5	1.2	601.8	38.4	0.9	0.2
En30k3	534*	608.1	58.0	3.3	1.3	557.6	18.3	4.0	1.0	547.0	28.9	1.4	0.4
En33k4	835	917.0	24.9	3.4	1.3	901.3	29.2	3.1	1.0	903.4	23.7	1.2	0.4
En51k5	521	716.0	50.1	8.6	2.8	631.7	34.3	8.5	3.2	623.9	31.1	2.4	0.9
En76k7	682	847.8	48.5	35.1	13.5	835.4	56.3	26.0	10.8	809.6	40.8	4.8	1.5
En76k8	735	914.8	54.4	32.4	13.9	895.2	37.9	24.4	7.0	870.2	54.4	5.1	1.5
En76k14	1021	1198.9	46.1	24.3	8.7	1188.8	45.1	33.8	10.5	1167.9	28.8	4.5	1.9
En101k8	815	1034.2	57.8	86.9	24.3	1021.6	72.9	67.2	26.0	1007.0	49.4	8.0	2.1
Pr101k14	1071	1309.8	51.0	75.6	16.7	1288.5	45.3	59.3	25.0	1253.2	36.5	8.7	2.4
Instance		GA_{v5}				GA_{v6}				EA_{v3}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
En22k4	375	400.2	29.6	1.9	0.3	411.8	31.0	1.9	0.4	390.4	15.0	1.7	0.2
En23k3	569	604.2	37.8	2.9	0.8	608.9	32.8	2.9	1.2	613.5	40.9	2.1	0.8
En30k3	534*	550.6	37.9	3.4	1.3	573.5	42.0	4.0	1.8	549.8	36.2	2.0	0.9
En33k4	835	914.9	33.3	3.8	1.4	904.8	24.7	3.9	1.4	901.2	24.8	1.6	0.3
En51k5	521	655.9.6	43.9	7.9	3.6	668.0	52.6	9.3	3.2	636.9	41.2	4.9	1.6
En76k7	682	833.1	42.0	23.2	8.1	821.8	38.5	28.8	9.5	815.2	29.2	7.7	3.7
En76k8	735	907.3	31.9	23.0	6.2	908.0	30.3	24.3	7.3	895.1	29.5	8.3	2.5
En76k14	1021	1188.3	43.8	19.0	8.3	1171.3	23.9	22.6	7.6	1178.1	32.5	7.5	3.0
En101k8	815	1001.4	57.2	71.9	23.9	1031.2	53.7	59.6	17.7	1006.9	57.7	14.1	4.8
Pr101k14	1071	1309.8	55.3	44.3	14.0	1320.0	47.2	48.2	19.2	1285.2	53.9	12.3	4.6

Table 6: Results and runtime of the nine techniques applied to the CVRP. For each instance, the results, average runtime, and their standar deviations are shown. *For En30k3, 534 has been used as best solution, instead of 503, according to the study made in [75]

results in the 20% of the instances in relation to both GAs, but this differences are not substantial in any case.

With all this, the following finding can be stated:

- Conclusion 4: According to the tests conducted for the CVRP, the use of blind crossover operators does not offer significant improvements in the results.

This conclusion can be explained in the same way that Conclusion 1 was explained in Section 3. Regarding the runtime, as in TSP, all the EAs need less time than their corresponding GAs in all the instances, being these improvements significant in all of the cases for the first two experimentations. In the third experimentation, the differences are substantial in the 90% of the instances. In addition, as in the previous problem, these differences become higher as the size of the instance grows. For this reason, the following

CVRP	GA_{v1}		GA_{v2}		EA_{v1}	
Instance	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
En22k4	3020.9	2216.7	5099.3	4273.6	2358.7	1874.9
En23k3	6717.1	5062.1	6162.3	4303.3	5813.9	3771.3
En30k3	9392.5	4583.3	9204.8	4963.3	7926.0	4872.9
En33k4	11042.1	4743.8	11628.4	5454.5	4614.0	3104.7
En51k5	15848.3	6991.7	18453.5	9183.2	10387.4	4816.7
En76k7	31420.8	13044.2	39220.3	14444.4	19357.7	7601.9
En76k8	27460.1	14326.1	36647.7	14385.6	16032.6	7197.8
En76k14	20042.1	10435.7	23084.1	8948.0	12133.8	6145.9
En101k8	51525.9	17393.5	55627.8	15426.4	25925.0	8783.4
Pr101k14	39834.9	14442.8	47396.1	14656.6	21276.6	6997.8
Instance	GA_{v3}		GA_{v4}		EA_{v2}	
En22k4	3227.0	2286.6	2551.0	1395.4	2352.9	1384.5
En23k3	8341.3	4495.1	5519.0	4585.9	4128.9	2740.6
En30k3	7837.7	5142.3	7806.9	3114.9	7668.0	3385.8
En33k4	6563.4	4333.6	6919.3	3760.7	4606.3	3169.3
En51k5	10472.0	5002.5	14226.3	7316.2	9727.1	5062.7
En76k7	27919.0	12521.7	25863.9	12369.9	19385.4	7286.1
En76k8	26178.2	13442.7	23249.8	7909.5	19027.0	7234.8
En76k14	16498.5	9190.2	16464.2	7082.0	11310.7	6886.4
En101k8	48219.9	15013.7	42115.4	17572.3	27595.8	8480.1
Pr101k14	38812.3	10129.9	33882.8	16091.8	23878.2	7960.6
Instance	GA_{v5}		GA_{v6}		EA_{v3}	
En22k4	2368.5	1464.5	2175.0	1989.2	1554.5	1313.2
En23k3	6543.4	4060.4	7632.6	5486.8	6300.8	2979.3
En30k3	8121.9	4806.2	8707.9	5987.3	7977.2	5820.4
En33k4	7586.2	4555.1	7107.0	4242.0	4942.9	1917.0
En51k5	10322.0	7118.3	11619.2	5673.0	9013.9	4690.5
En76k7	21857.2	8862.2	23312.1	9214.4	15688.0	9329.0
En76k8	19507.3	6989.5	19086.4	7402.4	14941.9	4849.8
En76k14	12945.4	7692.4	14955.9	6730.7	10477.0	5971.6
En101k8	44202.0	16688.4	42967.6	11510.6	27313.3	9049.3
Pr101k14	23547.1	9207.9	24205.8	12040.8	17700.1	7241.0

Table 7: Convergence behaviour of the nine techniques applied to the CVRP.

conclusion can be deduced:

- Conclusion 5: In the same way as with the TSP, the use of crossover phase for the CVRP increases significantly the execution time of an evolutionary algorithm.

The reasons of this increase in the runtime are the same as explained in the previous section for the TSP. Anyway, regarding the convergence behaviour, the results displayed in Table 7 are different in relation to the previously studied problem. Analyzing these outcomes it can be observed how the EAs show better convergence behaviour in all the instances and experimentations. Additionally, these improvements are significant in 80% of the cases compared to GA_{v2} and GA_{v3} , in 70% regarding GA_{v2} , GA_{v4} , and GA_{v6} , and in 60% compared to GA_{v5} . This means that the EAs reach the final solution in less generations than the other alternatives. The following finding can be extracted from these observations:

CVRP		EA_{v1} vs. GA_{v1}			EA_{v1} vs. GA_{v2}		
Instance	Results	Convergence	Time	Results	Convergence	Time	
En22k4	- (-5.09)	* (1.61)	+ (7.96)	* (1.41)	+ (4.15)	+ (8.64)	
En23k3	+ (3.35)	* (1.01)	+ (2.76)	+ (3.71)	* (0.43)	+ (3.99)	
En30k3	* (1.91)	* (1.55)	+ (9.45)	+ (4.33)	* (1.30)	+ (9.88)	
En33k4	* (-0.78)	+ (8.01)	+ (13.24)	+ (3.73)	+ (7.90)	+ (13.69)	
En51k5	* (1.65)	+ (4.54)	+ (11.63)	+ (7.14)	+ (5.50)	+ (12.09)	
En76k7	+ (3.12)	+ (5.64)	+ (14.47)	+ (7.14)	+ (8.60)	+ (18.90)	
En76k8	+ (3.42)	+ (5.03)	+ (11.89)	+ (7.90)	+ (9.06)	+ (17.45)	
En76k14	+ (2.20)	+ (4.61)	+ (13.36)	+ (5.76)	+ (7.13)	+ (17.44)	
En101k8	+ (4.26)	+ (9.29)	+ (18.86)	+ (7.63)	+ (11.83)	+ (21.88)	
En101k14	+ (5.60)	+ (8.17)	+ (17.26)	+ (8.42)	+ (11.37)	+ (20.43)	
Instance	EA_{v2} vs. GA_{v3}			EA_{v2} vs. GA_{v4}			
En22k4	* (-1.67)	+ (2.31)	+ (13.71)	- (-2.73)	* (0.71)	+ (25.72)	
En23k3	+ (2.96)	+ (5.65)	+ (11.38)	* (1.83)	* (1.83)	+ (9.29)	
En30k3	+ (6.66)	* (0.19)	+ (9.87)	+ (2.19)	* (0.21)	+ (17.06)	
En33k4	+ (2.80)	+ (2.57)	+ (11.43)	* (-0.33)	+ (3.32)	+ (12.47)	
En51k5	+ (11.04)	* (0.74)	+ (14.90)	* (1.19)	+ (3.57)	+ (12.97)	
En76k7	+ (4.26)	+ (4.16)	+ (15.77)	+ (2.62)	+ (3.19)	+ (13.74)	
En76k8	+ (4.09)	+ (3.31)	+ (13.80)	+ (2.66)	+ (2.78)	+ (19.06)	
En76k14	+ (4.03)	+ (3.19)	+ (15.72)	+ (2.76)	+ (3.68)	+ (19.41)	
En101k8	+ (2.52)	+ (8.45)	+ (22.87)	* (1.17)	+ (5.26)	+ (16.04)	
En101k14	+ (6.38)	+ (8.19)	+ (28.03)	+ (4.29)	+ (3.94)	+ (14.24)	
Instance	EA_{v3} vs. GA_{v5}			EA_{v3} vs. GA_{v6}			
En22k4	+ (2.08)	+ (2.92)	+ (3.92)	+ (4.39)	* (1.84)	+ (3.16)	
En23k3	* (-1.18)	* (0.34)	* (1.87)	* (-0.62)	* (1.50)	* (1.47)	
En30k3	* (0.10)	* (0.13)	+ (6.26)	+ (3.02)	* (0.61)	+ (7.02)	
En33k4	+ (2.33)	+ (3.78)	+ (10.86)	* (0.72)	+ (3.28)	+ (11.35)	
En51k5	+ (2.23)	* (1.08)	+ (5.38)	+ (3.29)	+ (2.50)	+ (8.69)	
En76k7	+ (2.47)	+ (3.39)	+ (12.30)	* (0.96)	+ (4.11)	+ (14.63)	
En76k8	+ (1.98)	+ (3.79)	+ (15.54)	+ (2.15)	+ (3.31)	+ (14.66)	
En76k14	* (1.32)	* (1.79)	+ (9.21)	* (-1.19)	+ (3.51)	+ (13.06)	
En101k8	* (-0.47)	+ (6.29)	+ (16.76)	+ (2.17)	+ (7.56)	+ (17.54)	
En101k14	+ (2.25)	+ (3.52)	+ (15.35)	+ (3.44)	+ (3.27)	+ (12.85)	

Table 8: z -test for CVRP. '+' indicates that EA is better. '-' depicts that it is worse. '*' indicates that the difference between the two algorithms is not significant (at 95% confidence level)

- Conclusion 6: Contrary to what happens for the TSP, and according to the experimentation conducted, the use of blind crossover operators does not improve the convergence behaviour of an evolutionary algorithm applied to the CVRP.

This change in the behavior of the EA compared to that observed for the previous problems can be justified as follows: Crossover operators are complex functions that combine the characteristics of two individuals of the population. These functions are easy to design and implement if the problem has not many constraints (for example, TSP and NQP). Anyway, if the problem has a complex formulation, or its restrictions are numerous, the development of a crossover function can be very hard. For this reason, many operators designed for this type of problems include problem dependent heuristics [76, 77], or they do not consider some of the constraints of the problem [78, 79]. In any case, these operators are difficult to implement and understand, and they increase considerably the complexity of the algorithm and its runtime.

Thus, blind operators are rarely used in solving these complex problems. In addition,

NQP	GA_{v1}				GA_{v2}				EA_{v1}			
Instance	Results		Time (s)		Results		Time (s)		Results		Time (s)	
Instance	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
8-Queens	0.0	0.0	0.1	0.0	0.1	0.2	0.1	0.0	0.0	0.0	0.1	0.0
20-Queens	1.6	0.8	0.1	0.1	1.5	0.7	0.1	0.1	0.8	0.5	0.1	0.0
50-Queens	6.6	1.6	0.6	0.1	6.4	1.6	0.3	0.1	5.1	1.4	0.3	0.1
75-Queens	13.7	2.2	0.8	0.3	13.1	2.5	0.7	0.4	9.2	2.3	0.6	0.1
100-Queens	15.4	2.3	6.2	1.5	15.2	2.6	4.7	1.3	11.5	2.3	2.9	0.7
125-Queens	25.5	3.4	5.2	1.5	24.3	3.6	3.9	1.2	17.0	3.1	3.6	0.8
150-Queens	32.0	3.9	9.5	3.4	27.7	3.9	7.6	2.2	21.9	3.2	6.6	1.4
200-Queens	43.2	5.9	69.9	7.9	38.2	4.5	38.0	8.1	26.6	3.9	32.5	7.9
250-Queens	56.4	7.1	63.8	19.8	52.1	5.2	45.5	12.5	38.0	5.3	42.5	10.7
300-Queens	69.9	7.9	123.3	41.3	65.2	6.5	109.5	25.6	45.6	5.3	94.6	19.3
Instance	GA_{v3}				GA_{v4}				EA_{v2}			
8-Queens	0.0	0.0	0.1	0.1	0.0	0.0	0.1	0.1	0.0	0.0	0.1	0.1
20-Queens	1.4	1.0	0.1	0.1	1.3	0.8	0.1	0.1	0.8	0.6	0.1	0.1
50-Queens	5.9	1.8	0.2	0.1	5.6	1.3	0.2	0.1	4.6	1.5	0.1	0.1
75-Queens	10.9	2.1	0.7	0.1	10.0	2.5	0.8	0.1	8.7	1.6	0.5	0.1
100-Queens	14.7	3.3	2.2	0.6	15.3	2.8	1.8	0.5	12.1	2.0	1.5	0.3
125-Queens	19.8	2.9	4.2	1.1	18.3	2.7	4.8	1.1	17.2	2.5	3.1	0.5
150-Queens	23.7	3.7	8.1	2.7	22.2	3.2	9.3	2.0	21.3	3.0	5.8	1.0
200-Queens	33.3	4.4	26.7	7.2	30.4	4.3	27.1	6.1	26.9	4.8	18.7	4.0
250-Queens	43.5	5.6	52.6	12.0	41.6	5.2	56.4	13.1	37.1	4.5	44.8	9.1
300-Queens	57.8	5.7	98.6	33.6	50.4	6.5	118.6	28.5	45.9	4.9	77.6	19.7
Instance	GA_{v5}				GA_{v6}				EA_{v3}			
8-Queens	0.0	0.0	0.1	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0
20-Queens	1.3	0.6	0.1	0.1	1.1	0.5	0.1	0.1	0.8	0.6	0.1	0.1
50-Queens	5.2	1.6	0.2	0.1	4.9	1.2	0.2	0.1	4.2	1.4	0.1	0.1
75-Queens	10.0	2.0	0.9	0.1	8.7	1.9	0.8	0.1	7.6	2.3	0.6	0.1
100-Queens	12.7	2.7	2.6	0.3	13.4	2.6	2.5	0.6	11.8	2.1	2.1	0.4
125-Queens	17.8	2.1	6.3	0.9	15.6	3.1	5.1	1.0	14.4	2.7	4.7	1.0
150-Queens	21.2	4.3	8.2	2.7	21.2	2.7	8.6	1.9	19.5	3.3	7.7	1.6
200-Queens	30.3	3.5	28.6	3.9	30.5	3.8	25.8	5.3	27.0	4.2	22.9	5.0
250-Queens	36.9	3.7	59.1	11.6	36.2	3.0	62.5	10.9	32.1	4.1	52.8	10.0
300-Queens	46.7	7.0	93.5	21.9	46.9	4.6	111.3	27.2	42.5	6.6	89.7	16.8

Table 9: Results and runtime of the nine techniques applied to the NQP. For each instance, the results, average runtime, and their standar deviations are shown.

its performance is usually not good. An evidence of this last statement is shown in this study: all GA techniques that prioritize the use of blind crossover operators are outperformed by the technique that gives more importance to the mutation phase, in terms of exploration and exploitation.

5. Experimentation with the NQP

In this section the experimentation with the NQP is detailed. The characteristics of the nine techniques implemented are the same as the algorithms used for the TSP. Table 1 summarizes them.

In Table 9 the results and average runtime can be found. The name of each instance describes the number of queens and the size of the chessboard. In this case, the optimum

NQP	GA_{v1}		GA_{v2}		EA_{v1}	
Instance	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
8-Queens	3.3	3.2	4.9	4.6	2.8	2.9
20-Queens	36.9	26.2	37.9	28.3	39.1	21.8
50-Queens	210.0	126.6	191.3	112.3	151.9	71.4
75-Queens	183.4	98.8	195.6	93.5	224.8	89.0
100-Queens	818.0	385.7	791.7	333.7	575.8	255.8
125-Queens	589.1	202.6	599.7	217.8	578.9	173.9
150-Queens	636.0	290.9	788.1	299.1	723.1	209.1
200-Queens	1181.6	417.3	1560.7	563.6	1854.1	606.3
250-Queens	1649.7	615.9	1717.2	564.3	1853.6	567.6
300-Queens	2279.4	897.3	2402.7	843.5	2821.7	683.3
Instance	GA_{v3}		GA_{v4}		EA_{v2}	
8-Queens	3.0	1.8	2.1	1.3	1.8	1.2
20-Queens	18.5	10.3	18.2	8.9	23.9	10.9
50-Queens	128.0	54.3	131.2	48.4	116.0	38.1
75-Queens	207.4	82.0	176.6	65.7	210.7	74.3
100-Queens	416.3	208.9	252.7	112.3	346.7	105.9
125-Queens	511.7	187.0	451.1	151.0	462.4	105.8
150-Queens	712.5	292.0	654.1	190.8	614.1	159.1
200-Queens	1363.1	458.5	1351.0	345.4	1208.5	330.6
250-Queens	1714.2	480.1	1461.5	433.2	1827.3	432.4
300-Queens	2465.8	948.1	2222.1	636.2	2250.8	675.3
Instance	GA_{v5}		GA_{v6}		EA_{v3}	
8-Queens	1.6	1.2	2.3	1.5	1.4	1.1
20-Queens	21.2	9.1	21.7	6.8	21.1	10.8
50-Queens	89.2	42.4	80.0	31.4	91.6	38.1
75-Queens	154.3	65.1	159.0	55.3	172.3	65.8
100-Queens	240.8	89.4	315.4	124.2	325.2	102.9
125-Queens	329.8	98.8	420.3	127.1	569.5	145.9
150-Queens	590.2	254.0	484.4	164.0	672.8	199.4
200-Queens	847.0	237.5	903.6	250.1	1193.7	337.6
250-Queens	1344.2	399.0	1491.5	329.7	1470.1	360.6
300-Queens	1974.5	975.1	1829.6	567.3	2137.0	492.9

Table 10: Convergence behaviour of the nine techniques applied to the NQP.

of each instance is not shown, since it is known that it is 0 for all of them. In addition, Table 10 displays the convergence behaviour of each algorithm. On the other hand, the z -test made for this problems is shown in Table 11.

The conclusions that can be drawn analyzing these tables are similar to those obtained in previous sections. First of all, as can be seen in Table 9, the EAs obtain better results than their corresponding GAs in all but one of the instances. In the remaining case (8-Queens instance), they get the same outcomes. In addition, these improvements are significant in 90% of the instances compared to GA_{v1} , GA_{v2} , GA_{v3} , GA_{v4} and GA_{v6} , being the 8-Queens instance the only where the differences are not significant. Additionally, these improvements are substantial in the 80% of the cases regarding GA_{v5} , being not significant in the remaining 20%. For these reasons, Conclusions 1 and 4 are also applicable for the NQP.

The same happens with runtime. The EAs are never overcome by any of the

NQP		EA_{v1} vs. GA_{v1}			EA_{v1} vs. GA_{v2}		
Instance	Results	Convergence	Time	Results	Convergence	Time	
8-Queens	* (0.00)	* (0.80)	* (0.00)	* (1.41)	+ (2.70)	* (0.00)	
20-Queens	+ (5.08)	* (-0.57)	+ (15.00)	+ (5.07)	* (-0.75)	+ (6.32)	
50-Queens	+ (4.76)	+ (2.84)	+ (10.00)	+ (4.03)	+ (2.10)	+ (2.16)	
75-Queens	+ (9.60)	- (-2.19)	+ (4.14)	+ (7.88)	* (-1.58)	* (1.06)	
100-Queens	+ (8.23)	+ (3.69)	+ (13.04)	+ (7.37)	+ (3.63)	+ (7.82)	
125-Queens	+ (12.98)	* (0.26)	+ (6.30)	+ (10.85)	* (0.52)	* (1.10)	
150-Queens	+ (13.86)	* (1.71)	+ (5.40)	+ (13.86)	* (1.25)	+ (2.61)	
200-Queens	+ (16.48)	- (-6.45)	+ (23.56)	+ (13.64)	- (-2.50)	+ (3.39)	
250-Queens	+ (14.54)	* (-1.72)	+ (6.67)	+ (14.54)	* (1.20)	* (1.29)	
300-Queens	+ (18.06)	- (-3.39)	+ (4.44)	+ (16.47)	- (-2.72)	+ (3.27)	
Instance	EA_{v2} vs. GA_{v3}			EA_{v2} vs. GA_{v4}			
8-Queens	* (0.00)	+ (3.92)	* (0.00)	* (0.00)	* (1.19)	* (0.00)	
20-Queens	+ (3.63)	- (-2.54)	* (0.00)	+ (3.53)	- (-2.81)	* (0.00)	
50-Queens	+ (3.93)	* (1.27)	+ (5.00)	+ (3.56)	* (1.74)	+ (5.00)	
75-Queens	+ (5.89)	* (-0.21)	+ (10.00)	+ (3.09)	- (-2.43)	+ (15.00)	
100-Queens	+ (4.76)	+ (2.09)	+ (7.37)	+ (6.57)	- (-4.30)	+ (3.63)	
125-Queens	+ (4.80)	* (1.61)	+ (6.43)	+ (2.11)	* (-0.43)	+ (9.94)	
150-Queens	+ (3.56)	+ (2.09)	+ (5.64)	+ (1.45)	* (1.13)	+ (11.06)	
200-Queens	+ (6.94)	* (1.93)	+ (6.86)	+ (3.84)	+ (2.10)	+ (8.14)	
250-Queens	+ (6.29)	* (-1.23)	+ (3.66)	+ (4.62)	- (-4.22)	+ (5.14)	
300-Queens	+ (11.19)	* (1.30)	+ (3.81)	+ (3.92)	* (-0.21)	+ (8.36)	
Instance	EA_{v3} vs. GA_{v5}			EA_{v3} vs. GA_{v6}			
8-Queens	* (0.00)	* (0.86)	* (0.00)	* (0.00)	+ (3.42)	* (0.00)	
20-Queens	+ (4.16)	* (0.05)	* (0.00)	+ (2.71)	* (0.33)	* (0.00)	
50-Queens	+ (3.32)	* (-0.29)	+ (5.00)	+ (2.68)	* (-1.66)	+ (5.00)	
75-Queens	+ (5.56)	* (-1.37)	+ (15.00)	+ (2.60)	* (-1.09)	+ (10.00)	
100-Queens	* (1.86)	- (-4.37)	+ (7.07)	+ (3.38)	- (-0.42)	+ (3.92)	
125-Queens	+ (7.02)	- (-9.61)	+ (8.40)	+ (2.06)	- (-5.45)	+ (2.00)	
150-Queens	+ (2.21)	* (-1.80)	* (1.12)	+ (2.81)	- (-5.15)	+ (2.56)	
200-Queens	+ (4.26)	- (-5.93)	+ (6.35)	+ (4.36)	- (-4.88)	+ (2.81)	
250-Queens	+ (6.14)	* (-1.65)	+ (2.90)	+ (5.70)	* (0.30)	+ (4.63)	
300-Queens	+ (3.06)	* (-1.05)	* (0.97)	+ (3.86)	- (-2.89)	+ (4.77)	

Table 11: z -test for NQP. '+' indicates that EA is better. '-' depicts that it is worse. '*' indicates that the difference between the two algorithms is not significant (at 95% confidence level)

genetic algorithms used, obtaining significantly better runtimes in 90% and 60% of cases regarding GA_{v1} and GA_{v2} , in 80% of the instances compared to GA_{v3} and GA_{v4} , and in 60% and 80% in relation to GA_{v5} and GA_{v6} , respectively. Therefore, Conclusion 2 and Conclusion 5 are also applicable for this problem.

Finally, regarding the convergence behaviour, the results obtained are more similar to those seen for the TSP. Looking at the data displayed in Table 9, the EA_{v1} has a better convergence behaviour in 40% of the instances, and the GA_{v1} and GA_{v2} in the other 60%. According to Table 11, comparing to GA_{v1} , the differences in the results are significantly better for the EA_{v1} in 20% of the instances, and significantly worse in 30% of them. In the remaining cases the differences are not substantial. On the other hand, comparing to GA_{v2} , these percentages are, respectively, 30%, 20%, and 50%.

Regarding the second experimentation, the EA_{v2} gets a better convergence compared to GA_{v3} and GA_{v4} in the 40% of the instances. In the remaining 60%, the EA_{v2} has

been overcome by at least one of the GAs. Regarding the GA_{v3} , the differences are not significant in the 60% of the cases. In addition, the EA_{v2} has showed a substantial better convergence behaviour in 30% of the instances. In the remaining 10% the GA_{v3} has significantly outperformed the behaviour of EA_{v2} . On the other hand, compared to GA_{v4} , these percentages are different, being 50%, 10%, and 40% respectively.

In relation to the third experimentation, the EA_{v3} has shown a better convergence than GA_{v5} and GA_{v6} in the 20% of the cases, being overcome in the remaining 80%. Compared to GA_{v5} , the difference in the behaviour is not significant in the 70% of the cases. Furthermore, they are substantially better for the GA in the remaining 30%. On the other hand, the EA_{v3} has significantly improved the convergence of GA_{v6} in the 10% of the instances. In addition, in the 40% of the cases the differences are not substantial. Ultimately, in the remaining 50%, GA_{v6} has shown a significant better convergence behaviour.

For this reason, the following finding can be drawn:

- **Conclusion 7:** According to the tests conducted, the use of blind crossover operators in the development of genetic algorithms for the NQP entails an improvement in the convergence behavior of the technique.

The NQP is a problem with a simple formulation. For this reason, the convergence behaviour of the GAs is much better than the shown for the CVRP, since the crossover phase helps to the exploration capacity of the technique. In this way, the results obtained in this aspect are similar to those obtained for the TSP.

6. Experimentation with the BPP

In this section the experimentation with the BPP is shown. The characteristics of the nine techniques developed are the same as the ones used for the TSP. Table 1 summarizes them.

In Table 12 the results and average runtime can be found. Each instance has been picked from the Scholl/Klein benchmark². These cases are named $NxCyWz_a$, where x is, 2 (100 items), 3 (200 items) or 4 (500 items); y is 1 (capacity of 100), 2 (capacity of 120) and 3 (capacity of 150); z is 1 (items size between 1 and 100) and 2 (items size between 20 and 100); and a is A or B as benchmark indexing parameter. Additionally, Table 13 shows the convergence behaviour of each technique. Furthermore, the z -test made for the BPP is shown in Table 14.

The conclusions that can be obtained in this case are very similar to those drawn for the NQP. As can be seen in Table 12, the EAs obtain better or same (in two cases only) results in the 100% of the instances, being significantly better in the 90% of the cases. Therefore, Conclusions 1 and 4 can be also applied for this problem. Regarding runtimes, as already seen in the previous experimentations, all the EAs outperform their corresponding GAs. In this case, the EAs obtain significantly better runtimes in the 100% of the instances. In this way, Conclusion 2 and Conclusion 5 are also valid for the BPP.

²<http://www.wiwi.uni-jena.de/entscheidung/binpp/index.htm>.

BPP		GA_{v1}				GA_{v2}				EA_{v1}			
Instance		Results		Time (s)		Results		Time (s)		Results		Time (s)	
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
N2C1W1.A	48	53.4	0.7	0.35	0.12	53.7	0.7	0.08	0.03	53.1	0.7	0.02	0.01
N2C1W1.B	49	54.3	0.7	0.29	0.08	54.4	0.8	0.09	0.02	53.3	0.5	0.02	0.01
N3C2W2.A	107	121.4	1.5	1.84	0.33	121.8	1.4	0.47	0.16	120.2	1.3	0.07	0.02
N3C2W2.B	105	117.7	1.8	1.93	0.54	118.2	2.2	0.39	0.20	116.7	1.1	0.06	0.03
N3C3W1.A	66	73.9	0.8	1.48	0.42	73.6	0.8	0.42	0.18	73.2	0.9	0.07	0.03
N3C3W1.B	71	80.4	0.9	1.46	0.37	79.8	0.7	0.46	0.24	79.2	0.9	0.06	0.02
N4C1W1.A	240	277.9	2.4	7.79	2.90	275.4	2.4	5.84	1.85	273.4	1.7	0.37	0.12
N4C1W1.B	262	300.4	3.2	7.48	3.12	298.8	1.4	5.93	2.15	295.8	2.2	0.45	0.21
N4C1W1.C	241	277.9	2.6	7.67	2.69	276.8	2.7	6.15	2.05	273.6	1.6	0.49	0.18
N4C2W1.A	210	245.8	2.9	7.08	2.41	244.8	2.1	6.02	1.99	242.6	1.9	0.51	0.24
Instance		GA_{v3}				GA_{v4}				EA_{v2}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
N2C1W1.A	48	53.2	0.9	0.37	0.10	53.4	0.8	0.06	0.02	52.8	0.6	0.01	0.01
N2C1W1.B	49	54.0	0.5	0.25	0.12	54.1	0.7	0.08	0.02	53.5	0.6	0.01	0.01
N3C2W2.A	107	121.0	1.3	1.93	0.41	122.0	1.5	0.51	0.19	120.4	1.5	0.06	0.02
N3C2W2.B	105	117.4	1.5	2.12	0.77	117.9	1.9	0.40	0.22	116.8	1.0	0.05	0.01
N3C3W1.A	66	74.2	1.0	1.82	0.57	73.3	0.5	0.58	0.21	73.0	0.6	0.08	0.02
N3C3W1.B	71	80.1	0.7	1.39	0.28	79.5	1.1	0.49	0.32	78.9	1.0	0.06	0.03
N4C1W1.A	240	276.3	2.7	7.91	2.49	274.3	2.1	6.12	2.09	273.5	1.4	0.43	0.26
N4C1W1.B	262	299.8	3.4	8.27	3.93	299.4	1.8	6.29	2.77	295.3	2.0	0.51	0.28
N4C1W1.C	241	278.2	2.9	8.93	3.00	277.1	2.2	7.00	2.22	272.9	1.9	0.68	0.25
N4C2W1.A	210	245.2	3.1	8.11	2.91	245.1	2.1	5.99	2.42	242.9	2.1	0.89	0.32
Instance		GA_{v5}				GA_{v6}				EA_{v3}			
Instance	Optimum	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
N2C1W1.A	48	52.9	0.8	0.41	0.09	52.7	0.9	0.21	0.12	52.7	0.6	0.02	0.01
N2C1W1.B	49	53.8	0.7	0.31	0.12	53.5	0.7	0.32	0.11	52.8	0.7	0.02	0.01
N3C2W2.A	107	119.2	1.1	1.95	0.77	120.1	1.6	1.84	0.70	118.9	1.1	0.08	0.03
N3C2W2.B	105	117.2	1.9	1.99	0.71	117.4	2.7	1.72	0.81	116.4	1.4	0.09	0.04
N3C3W1.A	66	73.8	0.7	2.11	0.70	72.8	0.9	2.21	1.00	72.8	0.7	0.10	0.03
N3C3W1.B	71	80.1	1.2	2.01	0.54	78.4	1.1	1.87	0.91	78.1	0.5	0.12	0.04
N4C1W1.A	240	278.1	2.8	7.89	2.71	276.0	2.8	6.84	2.08	273.5	1.9	0.47	0.12
N4C1W1.B	262	298.4	3.7	8.21	3.03	297.1	2.1	6.94	2.72	295.1	2.8	0.53	0.32
N4C1W1.C	241	277.1	2.2	9.00	3.09	275.9	2.4	8.95	2.71	273.0	2.1	0.81	0.37
N4C2W1.A	210	242.4	3.1	8.15	3.12	244.1	2.6	7.99	2.40	241.7	1.5	0.97	0.42

Table 12: Results and runtimes of the nine techniques applied to the BPP. For each instance, the results, average runtime, and their standar deviations are shown.

Concerning the convergence behavior, the results obtained are similar to those obtained for the NQP. The EAs have a better convergence in the 43.33% cases (13 out of 30), while the GAs performs better in the remaining 56.67%. In addition, comparing to GA_{v1} , the differences are significantly better for the EA_{v1} in 10% (1 out of 10) of the cases, and significantly worse in 20% (2 out of 10). In the remaining 7 instances these differences are insignificant. Furthermore, regarding GA_{v2} , these percentages are 30%, 0%, and 70% respectively. In relation to the second experimentation, the EA_{v2} shows a substantial better behaviour in 10% of the instances, and substantially worse in 25%. In the rest of the instances, the differences are not substantial. Finally, for the third experimentation these percentages are, respectively, 20%, 40%, and 40%. Thereby, looking at Table 14 it can be said that Conclusion 7 is also applicable for the BPP.

BPP	GA_{v1}		GA_{v2}		EA_{v1}	
Instance	Avg.	St.d.	Avg.	St.d.	Avg.	St.d.
N2C1W1.A	134.4	85.0	143.8	76.8	128.7	88.8
N2C1W1.B	64.8	24.6	112.8	81.4	86.9	34.7
N3C2W2.A	332.1	144.1	384.9	153.7	301.2	185.7
N3C2W2.B	356.4	116.7	345.1	128.0	314.8	111.0
N3C3W1.A	298.7	102.4	310.8	117.0	332.1	98.6
N3C3W1.B	366.0	176.8	410.2	218.4	385.8	158.4
N4C1W1.A	1542.3	312.7	1569.7	583.9	1328.6	586.9
N4C1W1.B	1663.4	497.8	1682.4	597.7	1538.7	486.8
N4C1W1.C	1364.8	599.4	1473.1	757.2	1499.4	584.7
N4C2W1.A	1340.0	573.0	1495.5	674.6	1616.4	473.5
Instance	GA_{v3}		GA_{v4}		EA_{v2}	
N2C1W1.A	151.7	80.8	132.2	81.2	112.7	90.7
N2C1W1.B	87.4	43.1	95.6	42.2	100.3	56.1
N3C2W2.A	232.7	101.8	299.4	81.4	285.7	91.3
N3C2W2.B	371.5	120.7	301.4	114.7	350.0	103.3
N3C3W1.A	312.8	136.9	358.7	136.2	299.5	77.0
N3C3W1.B	351.7	146.2	400.7	187.4	411.4	101.3
N4C1W1.A	1501.1	304.7	1499.0	608.9	1482.4	499.9
N4C1W1.B	1452.8	531.5	1577.3	519.0	1490.2	503.1
N4C1W1.C	1612.7	671.4	1579.0	676.3	1535.8	555.3
N4C2W1.A	1315.8	500.4	1399.4	741.2	1584.4	463.9
Instance	GA_{v5}		GA_{v6}		EA_{v3}	
N2C1W1.A	114.0	73.4	100.4	57.1	142.7	90.4
N2C1W1.B	81.4	21.1	71.8	27.4	95.7	43.8
N3C2W2.A	300.2	112.4	327.1	99.7	350.2	198.7
N3C2W2.B	376.4	132.4	355.4	140.5	299.4	134.5
N3C3W1.A	280.7	139.5	273.0	113.6	350.7	102.7
N3C3W1.B	481.8	241.5	451.9	223.4	371.4	188.0
N4C1W1.A	1427.0	299.9	1500.2	531.5	1286.7	499.7
N4C1W1.B	1701.8	513.8	1759.0	642.3	1612.0	500.1
N4C1W1.C	1310.8	524.3	1210.4	571.8	1571.0	611.4
N4C2W1.A	1274.0	497.9	1379.6	573.4	1527.1	511.7

Table 13: Convergence behaviour of the nine techniques applied to the BPP.

7. Conclusions and further work

In this paper a study on the influence of using blind crossover operators in genetic algorithms applied to combinatorial optimization problem has been conducted. For this purpose, four different well-known combinatorial optimization problems have been used, the Traveling Salesman Problem (TSP), the Capacitated Vehicle Routing Problem (CVRP), the N-Queens Problems (NQP), and the one-dimensional Bin Packing Problem (BPP). For each problem 10 different instances have been selected, making a total set of 40 cases. In the experimentation done, the performance of six classic genetic algorithms, each with a different crossover function, has been compared with the one of three evolutionary algorithms, in three separate experimentations.

In general terms, regarding the results, the EAs obtain better results in 94.16% of the cases (113 out of 120). In addition, comparing with the GA variants, these improvements

BPP		EA_{v1} vs. GA_{v1}			EA_{v1} vs. GA_{v2}		
Instance	Results	Convergence	Time	Results	Convergence	Time	
N2C1W1_A	+	*	+	+	*	+	
N2C1W1_B	+	-	+	+	+	+	
N3C2W2_A	+	*	+	+	+	+	
N3C2W2_B	+	*	+	+	*	+	
N3C3W1_A	+	*	+	+	*	+	
N3C3W1_B	+	*	+	+	*	+	
N4C1W1_A	+	+	+	+	+	+	
N4C1W1_B	+	*	+	+	*	+	
N4C1W1_C	+	*	+	+	*	+	
N4C2W1_A	+	-	+	+	*	+	
Instance	EA_{v2} vs. GA_{v3}			EA_{v2} vs. GA_{v4}			
N2C1W1_A	+	+	+	+	*	+	
N2C1W1_B	+	*	+	+	*	+	
N3C2W2_A	+	-	+	+	*	+	
N3C2W2_B	+	*	+	+	-	+	
N3C3W1_A	+	*	+	+	+	+	
N3C3W1_B	+	-	+	+	*	+	
N4C1W1_A	+	*	+	+	*	+	
N4C1W1_B	+	*	+	+	*	+	
N4C1W1_C	+	*	+	+	*	+	
N4C2W1_A	+	-	+	+	-	+	
Instance	EA_{v3} vs. GA_{v5}			EA_{v3} vs. GA_{v6}			
N2C1W1_A	*	*	+	*	-	+	
N2C1W1_B	+	-	+	+	-	+	
N3C2W2_A	*	*	+	+	*	+	
N3C2W2_B	+	+	+	+	+	+	
N3C3W1_A	+	-	+	*	-	+	
N3C3W1_B	+	+	+	*	*	+	
N4C1W1_A	+	*	+	+	+	+	
N4C1W1_B	+	*	+	+	*	+	
N4C1W1_C	+	-	+	+	-	+	
N4C2W1_A	*	-	+	+	*	+	

Table 14: z -test for BPP. '+' indicates that EA is better. '-' depicts that it is worse. '*' indicates that the difference between the two algorithms is not significant (at 95% confidence level)

are significant in the 81.25% of the cases (195 out of 240). In 17.91% of the cases (43 out of 240) these differences are insignificant, and in the remaining 0.84% (2 out of 240) one GA obtains substantially better results than its corresponding EA. For these reasons:

- Conclusion 8: Regarding the results and applicable to the experimentation performed, it is concluded that the use of blind crossover operators in genetic algorithms for solving combinatorial optimization problems provides no significant improvement in the results.

In relation to the runtime, the EAs needs less time than their corresponding GAs in all of the instances. In addition, these improvements are substantial in 92.91% of the cases (223 out of 240). These data suggest the following finding:

- Conclusion 9: In relation to runtime and according to the experimentation performed, the use of blind crossover operators in genetic algorithms substantially

increases the execution time of the technique, without providing an improvement in results.

Regarding the convergence behaviour, the GAs show better performance than the EAs. This means that they need less generations/iterations to find their final solution. Anyway, this fact does not entail better results, or less runtime, as has been mentioned in Conclusion 8 and Conclusion 9. What it really involves, is a greater exploration capacity of the technique. Additionally, this fact is subject to the problem that is being treated, being more effective if the problem has an easy formulation. For the experimentation conducted, the EAs show better convergence behaviour in 45.83% of the cases (55 out of 120). Moreover, the statistical test conducted shows that for simple formulation problems (TSP, NQP, and BPP), the EAs have a significantly better convergence in 12.77% (23 of 180) of the cases. On the other hand, in 41.66% (75 out of 180) of the comparisons, the GAs are substantially better. In the remaining 45.57% the differences are not remarkable. For the CVRP, as has been seen in Section 4, the EAs show a significantly better convergence in the 71.66% (43 out of 60) of the cases. As a result of this, the following finding can be deduced:

- Conclusion 10: Finally, regarding the convergence behaviour and according to the experimentation performed, the study concludes that the use of blind crossover operators in genetic algorithms for solving combinatorial optimization problems with simple formulation entails a better convergence behaviour of the technique, needing less generations to obtain the final solution. Anyway, this fact does not mean better results. On the other hand, for more complex problems, the use of blind crossover operators does not imply a better convergence behavior.

Finally, as a final conclusion of this work, and based on the findings that have been proposed along the paper, the following assertion can be concluded:

- Conclusion 11: Based on the experimentation performed, an evolutionary algorithm (based only on mutation and survivor selection functions) is more efficient than a classic genetic algorithm to solve combinatorial optimization problems.

As a final comment, the authors of this study want to clarify that they are aware that there is a large amount of combinatorial optimization problems in the literature. In the same way, there are a lot of blind crossover operators. For these reasons, it could be pretentious to generalize the conclusions of this study to all the combinatorial optimization problems. In this work, to perform the tests, four well-known and widely used problems have been used. The goal of this selection is to choose problems of different type, and to obtain conclusions as objective as possible. Following the same philosophy, all the crossover operators that have been used in this study have been previously utilized in many studies in the literature. Thereby, the authors of this study are aware that the conclusions drawn are objective and rigorous, but just for the conducted experimentation.

As future work, and in order to verify the conclusions of this study, it could be interesting to extend this work to some other combinatorial optimization problems, such as the minimum spanning tree problem [80], or the job-shop scheduling problem [81]. Furthermore, it may be worthwhile to investigate whether these same findings are also applicable to other types of optimization problems, such as continuous optimization.

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