



On numerical evaluation of effective material properties for composite structures with rhombic fiber arrangements

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ABSTRACT

This paper deals with unidirectional fiber reinforced composites with rhombic fiber arrangements. It is assumed, that there is a periodic structure on micro level, which can be taken by homogenization as a representative volume element (RVE) for the composite, where the composite phases have isotropic or transversely isotropic material characterizations. A special procedure is developed to handle the primary non-rectangular periodicity with common numerical homogenization techniques based on FE-models. Due to appropriate boundary conditions applied to the RVE elastic effective macroscopic coefficients are derived. Results are listed and compared with other publications and good agreements are shown. Furthermore new results are presented, which exhibit the special orthotropic behavior of such composites caused by the rhombic fiber arrangement.

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1. Introduction

Fiber reinforced materials are widely-used in the industry. Due to weight reducing and strengthening such composites can be found for instance in naval architecture, aviation, automotive engineering and civil engineering. For designing such structures the knowledge of effective material properties is of great importance.

There has been done a lot of research in that issue during the last decades. Two of the first pioneers in that topic are Reuss (1929) and Voigt (1910), who derive average procedures for polycrystals. They assume uniform stress or strain states to the specimen. Hill (1952) shows that these methods define upper and lower bounds for elastic moduli of polycrystals. More accurate bounds are given by Hashin and Shtrikman (1962a, 1962b). These are derived from variational principles. In Hashin and Shtrikman (1962b) the consideration of “polycrystal – cubical element” is similar to a representative volume element (RVE) taken from a random or periodic heterogeneous unlimited elastic composite. In case of periodicity Suquet (1987) shows that uniform stresses or strain states at the boundaries are not an adequate choice for this type of RVE. Uniform strains overestimate the material properties and uniform stresses underestimate them. Better results are achieved by applying certain periodicity conditions to the representative volume element. Using the concept of the RVE combined with the finite element method (FEM), as for instance in Xia, Zhang, and Ellyin (2003), Berger et al. (2005) and Kari, Berger, Rodríguez-Ramos, and Gabbert (2007), is a powerful technique. A more complex microstructure given by several inclusions and different shapes is considerable. Even random distribution of inclusions can be taken into account.

In this paper unidirectional fibrous composites are considered, which consist of a rhombic fiber arrangement. Square (Rodríguez-Ramos, Sabina, Guinovart-Díaz, & Bravo-Castillero, 2001) and hexagonal fiber arrangements (Guinovart-Díaz, Bravo-Castillero, Rodríguez-Ramos, & Sabina, 2001) are special cases of such considerations, where the rhombic fiber distribution angle is defined as 90° and 60° , respectively. Considering arbitrary rhombic angles a wider spectrum of composites

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can be investigated, which have on macro scale orthotropic material properties. The investigation on such composites is done by a numerical homogenization technique related to Berger et al. (2005) and Kari et al. (2007). Here a RVE is examined by using the finite element method in connection with the software package ANSYS. The calculated effective coefficients are checked against values reported by Jiang, Xu, Cheung, and Lo (2004) and Guinovart-Díaz et al. (2011). They present effective out-of-plane shear coefficients for several rhombic fiber arrangements by using analytical methods on periodic media. In the case of Jiang et al. (2004) a method using Eshelby's equivalent inclusion concept integrated with results from doubly quasi-periodic Riemann boundary value problems is developed. The other paper is related to the asymptotic homogenization method (AHM), which is based on so-called local problems. Further comparisons presented here are done to Rodríguez-Ramos et al. (2011) and Golovchan and Nikityuk (1981), which also deal with analytical methods considering a rhomb and a parallelogram as periodic microstructure. These methods are the previously mentioned AHM, an eigenfunction expansion-variational method (EEVM) developed in Yan and Jiang (2010) and a method (Golovchan & Nikityuk, 1981) for the solution of the problem on the shear of a regular fibrous medium underlying, which is the exact solution of the Laplace equation in a strip with an infinite number of circular holes. Since the previous mentioned analytical methods only consider out-of-plane shear coefficients, further comparisons are made to Hashin (1979) in the case of hexagonal symmetry and transversely isotropic material behavior of the fibers. Hashin presents formulas and bounds for effective material constants, where the composite consists of transverse isotropic constituents on micro level. At the end engineering constants are derived for several fiber volume fractions and plotted against the angle, which characterizes the rhombic fiber arrangement. It is to say, that up to now, there are not any comparable results found with respect to rhombic fiber arrangements except the previously mentioned out-of-plane shear coefficients.

2. Numerical homogenization

2.1. Basics

As mentioned before we want to consider composites with fibers, which are unidirectional embedded in a matrix phase (see Fig. 1). In addition a periodic microstructure underlies the composite. The constitutive equation according to each material phase is given by HOOKE's law using the Einstein summation convention

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}, \quad i, j, k, l = 1, 2, 3. \quad (1)$$

The quantities σ_{ij} , ε_{kl} and C_{ijkl} are the coefficients of the stress tensor, the linear strain tensor and the stiffness tensor, respectively, according to the chosen Cartesian axes x_1 , x_2 and x_3 , which are denoted as global coordinate system. These coefficients fulfill the usual symmetry conditions according to linear elasticity (Torquato, 2002). The stiffness coefficients are assumed to be constant in each phase and can be characterized by isotropic or transversely isotropic material behavior. The connectivity between the constituents is considered as perfect. This means, that the stresses and displacements are continuous on the phase interface.

By homogenization techniques it is assumed, that linear (homogeneous) displacement loads on macroscopic scale are applied. It is sufficient to consider a periodic microstructure V_{rve} , which forms the RVE, where the following problem

$$\begin{cases} \frac{\partial}{\partial x_j} \sigma_{ij}(\mathbf{x}) = 0, & \mathbf{x} \in V_{rve}, \\ u_i(\mathbf{x}) - \varepsilon_{ij}^0 x_j, & \text{periodic on } \partial V_{rve} \end{cases} \quad (2)$$

has to be solved. Here ε_{ij}^0 are the given coefficients of the macroscopic strain tensor, u is the displacement field in the microstructure and ∂V_{rve} is the boundary of the RVE. The periodicity condition in formulation (2) ensures on one hand, that extensions of displacements on adjoining cells keep being continuous. On the other hand applied loads on the microstructure realized by constraint equations can be derived. They have the form

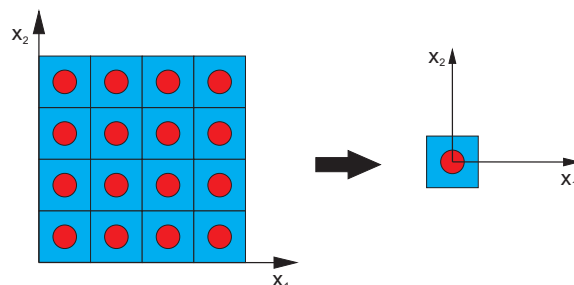


Fig. 1. Square cell arrangement and picked RVE.

$$u_i^{X_j^+} - u_i^{X_j^-} = \varepsilon_{ij}^0 (x_j^{X_j^+} - x_j^{X_j^-}). \tag{3}$$

The values $u_i^{X_j^+}$ and $u_i^{X_j^-}$ are the i th displacement components on the boundary surfaces of the cell, which are perpendicular to the x_j -axis ('+' for a positive normal direction, '-' for a negative normal direction). The locations, in which the values are calculated, are characterized by an offset in x_j -direction. The effective homogenized coefficients of the stiffness tensor, which represent the macroscopic behavior of the composite material, are calculated by

$$\langle \sigma_{ij} \rangle = C_{ijkl}^{eff} \langle \varepsilon_{kl} \rangle, \tag{4}$$

where the quantities

$$\langle \sigma_{ij} \rangle = \frac{1}{|V_{rve}|} \int_{V_{rve}} \sigma_{ij}(x) dx, \tag{5}$$

$$\langle \varepsilon_{kl} \rangle = \frac{1}{|V_{rve}|} \int_{V_{rve}} \varepsilon_{kl}(x) dx \tag{6}$$

are the related stress and strain coefficients on macro scale. Since ε_{kl}^0 is the given macro strain tensor the following equation holds

$$\langle \varepsilon_{kl} \rangle = \frac{1}{|V_{rve}|} \int_{V_{rve}} \varepsilon_{kl}(x) dx = \varepsilon_{kl}^0. \tag{7}$$

In order to prevent rigid body motions one structural point of the cell is fixed to a constant value. The fixed value and the location in the cell can be arbitrarily chosen. They do not influence the effective stiffness coefficients. In order to calculate the stiffness coefficients C_{ijkl}^{eff} different states of macro strains are considered. They are classified into six states, three pure normal strain states

$$\begin{pmatrix} \varepsilon_{11}^0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_{22}^0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{33}^0 \end{pmatrix} \tag{8}$$

and three pure sliding states

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \varepsilon_{23}^0 \\ 0 & \varepsilon_{23}^0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & \varepsilon_{13}^0 \\ 0 & 0 & 0 \\ \varepsilon_{13}^0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \varepsilon_{12}^0 & 0 \\ \varepsilon_{12}^0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

2.2. RVE for rhombic fiber arrangements

In this section the derivation of an appropriate RVE for the rhombic fiber arrangement is explained. Normally the rhombic pattern of the structure leads to a RVE, where the periodicity is characterized by an oblique coordinate system ω_1, ω_2 shown in Fig. 2. But this makes the handling of the cell in order to apply appropriate boundary conditions very difficult. That's why a new periodic micro cell is created by rotating the global coordinates into coordinates, whose axes coincide with the diagonals of the rhombic cell (see Fig. 3). The new rotated system given by x'_1, x'_2 , and x'_3 , is denoted as local. This coordinate system coincides with the principle axes of an orthotropic material.

Assuming unit edge length of the rhomb the rectangular RVE shown in Fig. 3 has edge lengths

$$\begin{aligned} l_1 &= 2 \cos \frac{\alpha}{2}, \\ l_2 &= 2 \sin \frac{\alpha}{2}, \end{aligned} \tag{10}$$

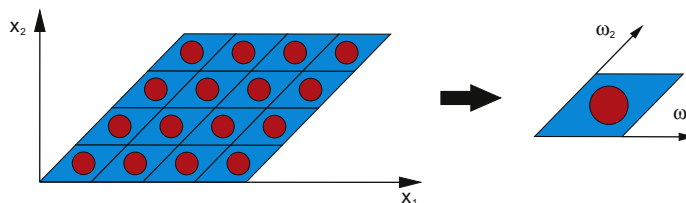


Fig. 2. Rhombic cell arrangement and rhombic RVE.

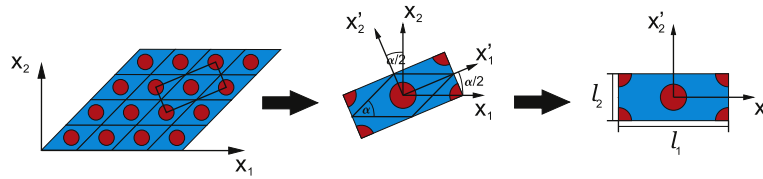


Fig. 3. New RVE with rectangle shape by rotation.

where l_1 and l_2 are the width and the height of the cell. The quantity α is the smaller angle of the periodic rhomb. For varying the fiber volume fraction an appropriate abortion for the restriction of the fiber radius

$$r_f > \min \left\{ \frac{l_2}{2}, 0.5 \right\} \tag{11}$$

has to be formulated for the geometric generation. This guarantees non-overlapping of the fibers.

2.3. Numerical model and algorithm

In order to evaluate the effective coefficients the software package ANSYS is used. Here algorithms are written in APDL (ANSYS Programming Design Language), which is delivered by the software and it makes the handling much more comfortable. The model of the micro cell (see Fig. 4) consists of three-dimensional SOLID226-elements. These elements are characterized by twenty nodes and quadratic shape functions.

Symmetry planes are used in order to generate and mesh the model in such a way, that the boundary conditions from Eq. (3) can be applied, which are realized by defining constraint equations in ANSYS. Because of finite element discretization the integrals in Eqs. (5) and (6) are changed to

$$\langle \sigma_{ij} \rangle = \frac{1}{|V_{rve}|} \sum_e \sigma_{ij}^e |V_e|, \tag{12}$$

$$\langle \varepsilon_{kl} \rangle = \frac{1}{|V_{rve}|} \sum_e \varepsilon_{kl}^e |V_e|. \tag{13}$$

The quantities σ_{ij}^e , ε_{kl}^e are averaged element values for the corresponding coefficients of the stress and strain tensor and $|V_e|$ is the volume of the finite element. Since the symmetry conditions of linear elasticity are fulfilled and a special boundary condition is applied (fixed k and l), the coefficients C_{ijkl}^{eff} from Eq. (4) are calculated with respect to the local coordinate system by

$$C_{ijkl}^{eff} = \frac{\langle \sigma_{ij} \rangle}{\langle \varepsilon_{kl} \rangle} \quad \begin{matrix} i, j, k, l = 1, 2, 3 \\ k = l \end{matrix}, \quad C_{ijkl}^{eff} = \frac{\langle \sigma_{ij} \rangle}{2 \langle \varepsilon_{kl} \rangle} \quad \begin{matrix} i, j, k, l = 1, 2, 3 \\ k \neq l \end{matrix}, \tag{14}$$

where only six of nine calculated coefficients are independent in the case of total anisotropy of the homogenized material.

With the tensor transformation rule (Torquato, 2002)

$$C_{pqrs}^{eff, global} = a_{pi} a_{qj} a_{kr} a_{sl} C_{ijkl}^{eff, local}, \tag{15}$$

where a_{ij} , $i, j = 1, 2, 3$ are the direction cosines, the stiffness tensor related to the global coordinates can be derived, which is implemented in a MATLAB-routine.

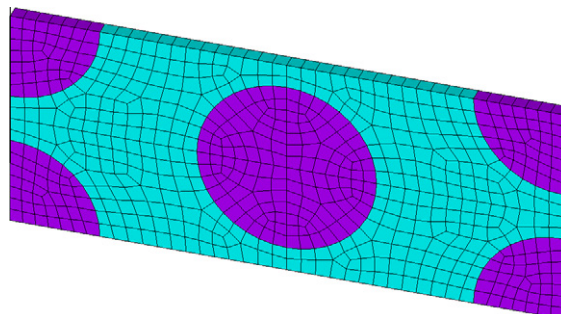


Fig. 4. Meshed RVE for 0.4 fiber volume fraction and 45° fiber arrangement.

3. Results and comparisons

Here an overview about the calculated effective coefficients of the stiffness tensor for rhombic fiber arrangements is given. At first results are compared with values from Jiang et al. (2004), Guinovart-Díaz et al. (2011), Golovchan and Nikityuk (1981) and Rodríguez-Ramos et al. (2011) to evaluate the numerical algorithm. Only a few papers can be found, which present results for effective material values of composites with rhombic fiber arrangements. Furthermore they are limited in calculating only out-of-plane shear coefficients C_{2323}^{eff} and C_{1313}^{eff} . That's why in the next step a comparison to Hashin (1979) is presented, where in the case of hexagonal fiber arrangement effective material constants are verified. In addition a more realistic material behavior of the fibers is assumed here (transversely isotropic). At the end effective material constants in dependence of the rhombic angle and the fiber volume fraction are plotted.

As in Jiang et al. (2004) and Guinovart-Díaz et al. (2011) a RVE is considered, where the rhombic structure is characterized by an angle of 45°, 60° and 90°. The volume fraction of the fiber is varying from 0.1 to 0.6. The material properties are taken from Jiang, where he has only given the ratio of the shear moduli

$$\frac{G^f}{G^m} = 120. \quad (16)$$

The quantities are the shear modulus of the fiber (index 'f') and the matrix (index 'm') phase. For an analysis in ANSYS a full material description is necessary. Thus in addition a Poisson's ratio of 0.3, which characterizes both material phases, is chosen and the shear modulus of one phase is fixed to an appropriate value. It is mentioned here, that the Poisson's ratio can be arbitrarily chosen, since shear deformation states are considered. For each out-of-plane shear state six independent effective coefficients can be calculated, where because of the local coordinate system only C_{2323}^{eff} and C_{1313}^{eff} are not equal to zero. Related results, which are normalized by the shear modulus of the matrix phase, are listed in Tables 1 and 2 for 60°, 90° and 45°, respectively. From literature (Jiang et al., 2004) it is known, that in Table 1 the considered cases provide equality of the values C_{2323}^{eff} and C_{1313}^{eff} . There is a good agreement with the compared values from Jiang et al. (2004). In Tables 2 and 3 values for a rhombic fiber arrangement of 45° are listed. As written before the effective stiffness coefficients are calculated related to local coordinates, which can be seen in Table 2. Hence C_{2323}^{eff} and C_{1313}^{eff} are the only non-zero values. Due to rotation into global axes the coefficients change to the values listed in Table 3 and C_{2313}^{eff} becomes non-zero. In this table also result given by AHM (Guinovart-Díaz et al., 2011) are presented. These values seems to be coincident with values received by Jiang et al. (2004), which implies also coincidence of values with respect to the local coordinates. At all values (FEM, Jiang, AHM) there is a good agreement in all coefficients.

Also in Rodríguez-Ramos et al. (2011) and Golovchan and Nikityuk (1981) results for effective out-of-plane shear coefficients are presented. These coefficients are derived by analytical methods. In these papers rhombs and parallelograms are considered as periodic microstructure. Here only rhombs are of interest, more precisely a rhomb with an angle $\alpha = \arccos(1/4)$, which is about 75°. For the microscopic material behavior shear ratios (see Eq. (16)) of 20 and 120 are used. The fiber volume fraction varies from 0.3 to 0.7 with an increment of 0.2. The calculated coefficients are rounded to the second decimal place. The obtained results with respect to the global axes and the results of the methods given by Rodríguez-Ramos et al. (2011) (AHM and EEM) and Golovchan and Nikityuk (1981) (G&N) can be seen in Table 4. As in the previous tables a good coincidence with the compared values can be achieved.

In the next step results for a full set of effective material constants using a more realistic composite structure consisting of epoxy with embedded graphite fibers are presented. The material parameters for each phase are taken from Hashin (1979), listed below in Table 5, where results are published for the special case of hexagonal fiber arrangement. The matrix phase has

Table 1
Normalized effective coefficients for 60° and 90°.

Vol. fraction		0.1	0.2	0.3	0.4	0.5	0.6
$\frac{C_{2323}^{eff}}{G_m} = \frac{C_{1313}^{eff}}{G_m}$	FEM	1.21833	1.49016	1.83776	2.29902	2.94139	3.90591
	Jiang	1.21815	1.48971	1.83711	2.29764	2.93931	3.90319
$\frac{C_{2323}^{eff}}{G_m} = \frac{C_{1313}^{eff}}{G_m}$	FEM	1.21835	1.49047	1.84098	2.31489	3.01174	4.18919
	Jiang	1.21816	1.49000	1.83990	2.31343	3.00863	4.18637

Table 2
Normalized effective coefficients for 45° in local coordinates.

Vol. fraction		0.1	0.2	0.3	0.4	0.5
$\frac{C_{2323}^{eff}}{G_m}$	FEM	1.22532	1.52645	1.95185	2.60760	3.81020
	Jiang	1.22501	1.52595	1.95099	2.60663	3.80354
$\frac{C_{1313}^{eff}}{G_m}$	FEM	1.21200	1.45898	1.75184	2.10768	2.55697
	Jiang	1.21170	1.45854	1.75116	2.10682	2.55497

Table 3
Normalized effective coefficients for 45° in global coordinates.

Vol. fraction		0.1	0.2	0.3	0.4	0.5
$\frac{C_{232}^{eff}}{G_m}$	FEM	1.22337	1.51657	1.92256	2.53439	3.62667
	Jiang	1.22306	1.51608	1.92172	2.53344	3.62069
	AHM	1.22306	1.51608	1.92172	2.53343	3.62069
$\frac{C_{233}^{eff}}{G_m}$	FEM	-0.00471	-0.02385	-0.07071	-0.17675	-0.44308
	Jiang	-0.00471	-0.02383	-0.07065	-0.17671	-0.44144
	AHM	-0.00471	-0.02383	-0.07065	-0.17671	-0.44144
$\frac{C_{1313}^{eff}}{G_m}$	FEM	1.21395	1.46886	1.78113	2.18089	2.74050
	Jiang	1.21365	1.46841	1.78042	2.18002	2.73782
	AHM	1.21365	1.46841	1.78042	2.18002	2.73782

Table 4
Normalized effective coefficients with respect to $\alpha = \arccos(1/4)$.

Vol. fraction		Shear ratio					
		20			120		
		0.3	0.5	0.7	0.3	0.5	0.7
$\frac{C_{232}^{eff}}{G_m}$	FEM	1.74	2.67	4.83	1.84	2.96	6.16
	G&N	1.74	2.66	4.83	1.83	2.96	6.16
	AHM	1.74	2.66	4.83	1.83	2.96	6.16
	EEVM	1.74	2.66	4.83	1.83	2.96	6.16
$\frac{C_{233}^{eff}}{G_m}$	FEM	0.02	0.08	0.34	0.02	0.11	0.60
	G&N	0.02	0.08	0.34	0.02	0.11	0.60
	AHM	0.02	0.08	0.34	0.02	0.11	0.60
	EEVM	0.02	0.08	0.34	0.02	0.11	0.60
$\frac{C_{1313}^{eff}}{G_m}$	FEM	1.75	2.71	5.01	1.85	3.01	6.47
	G&N	1.75	2.70	5.00	1.84	3.01	6.47
	AHM	1.75	2.70	5.00	1.84	3.01	6.47
	EEVM	1.75	2.70	5.00	1.84	3.01	6.47

isotropic and the fiber transversely isotropic material behavior. E denotes the Young's modulus in GPa, G the shear modulus in GPa and ν the Poisson's ratio. In the case of transverse isotropy the indices a and t characterize axial and transverse parameters, respectively. The results for the hexagonal fiber arrangement can be seen in Figs. 5–10. They are compared to data received by analytical closed-form expressions for the effective constants E_a^{eff} , G_a^{eff} , ν_a^{eff} and bounds for E_t^{eff} , G_t^{eff} and ν_t^{eff} (Hashin, 1979). The calculated values are given in GPa and are plotted against the fiber volume fraction ν_f . In the cases of axial direction the elastic constants fit well with the analytical values. Considering the transverse case the values are close to the lower bound and the higher the fiber volume fraction the closer the results are approaching the upper bound. Altogether there is a good agreement with the compared data.

Afterwards Figs. 11–14 show effective elastic quantities in dependence of the rhombic fiber arrangement given by the angle α using same material properties as in Table 5. These are the in-plane Young's moduli E_1^{eff} , E_2^{eff} , the out-of-plane shear moduli G_{23}^{eff} , G_{13}^{eff} , the in-plane and out-of-plane Poisson's ratios ν_{12}^{eff} , ν_{21}^{eff} , ν_{32}^{eff} , ν_{31}^{eff} related to the local coordinate axes for a fiber volume fraction of 0.1, 0.3, 0.5 and 0.7. Due to avoiding geometric overlapping (see Eq. (11)), the ranges of the angle differ for the plotted cases. A good numerical coincidence of values for the cases of 60° and 90° can be observed, which is caused by certain symmetry properties of the effective stiffness tensors (Guinovart-Díaz et al., 2001; Rodríguez-Ramos et al., 2001). It is mentioned here, that there are no comparable results found except the out-of-plane coefficients listed before in the tables.

For some effective quantities, as for instance E_2 and G_{12} , there is a range of value over all considered rhombic arrangements, where the minimum and maximum differ up to 10%. A list of such discrepancy for chosen material constants can be seen in Table 6.

Table 5
Material phase properties.

Epoxy				Graphite			
E	ν	E_a	ν_a	E_t	ν_t	G_a	G_t
3.45	0.35	345	0.2	9.66	0.3	2.07	3.72

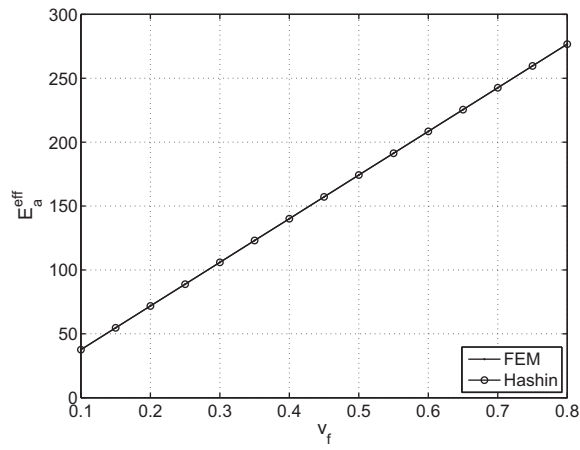


Fig. 5. Effective axial Young's modulus.

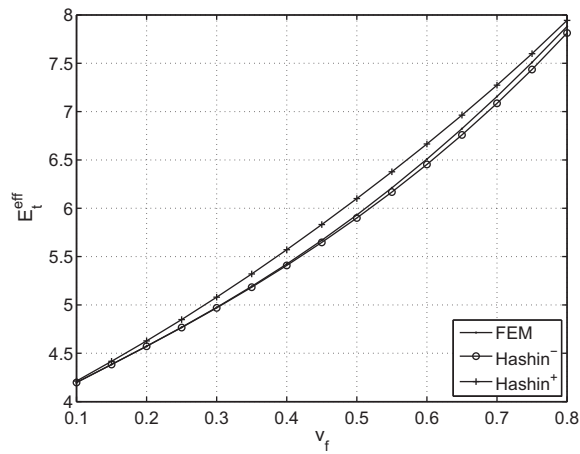


Fig. 6. Effective transverse Young's modulus.

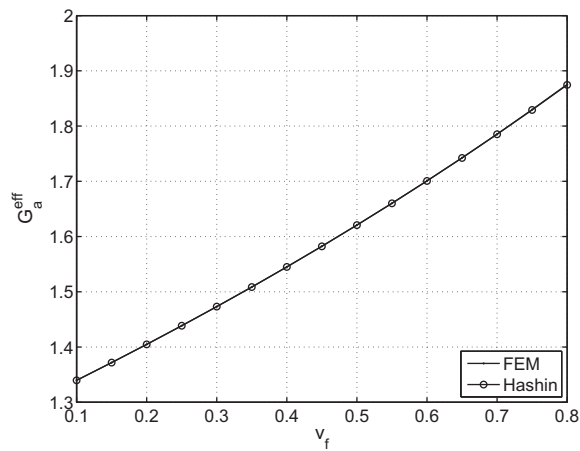


Fig. 7. Effective axial shear modulus.

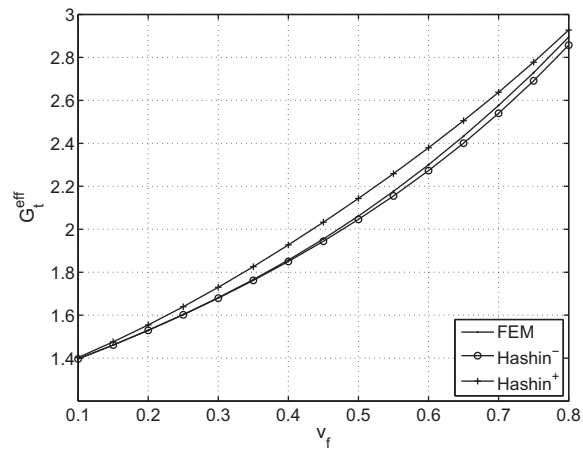


Fig. 8. Effective transverse shear modulus.

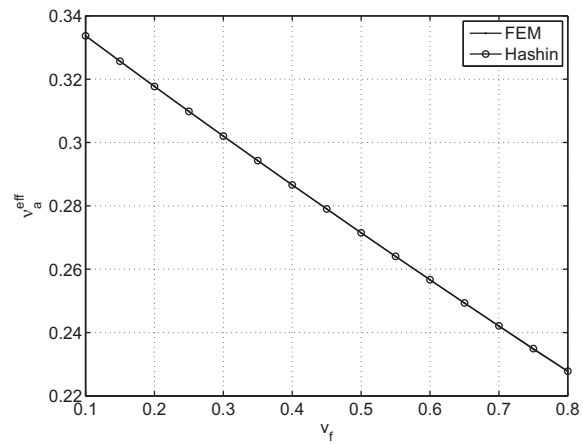


Fig. 9. Effective axial Poisson's ratio.

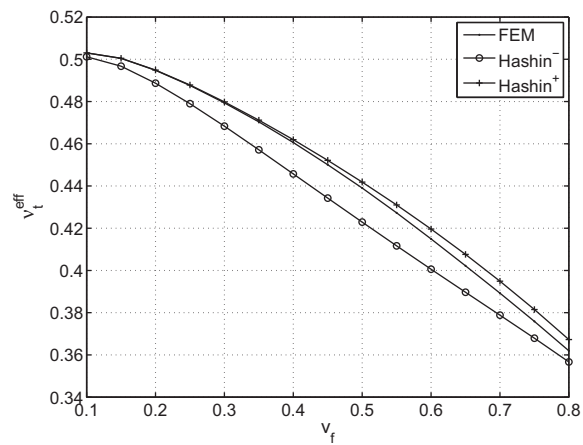


Fig. 10. Effective transverse Poisson's ratio.

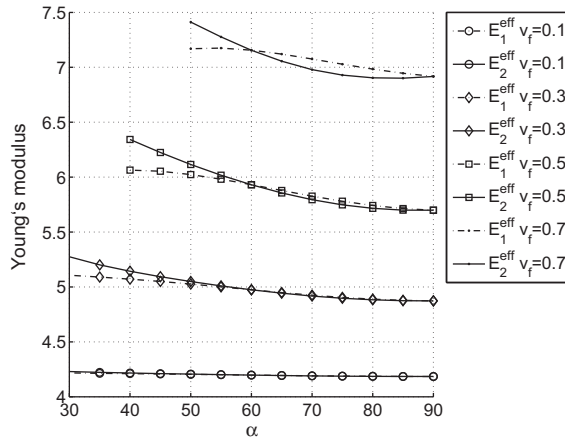


Fig. 11. Effective in-plane Young's moduli in local coordinates.

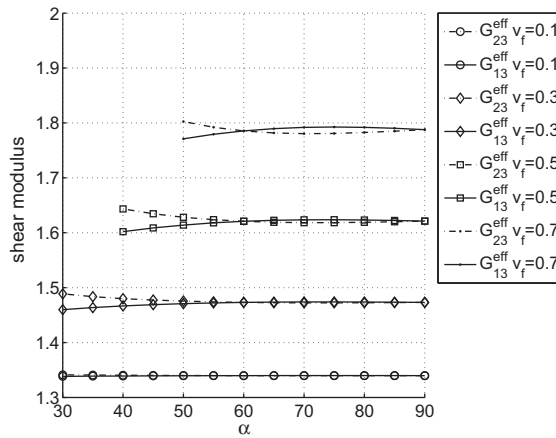


Fig. 12. Effective out-of-plane shear moduli in local coordinates.

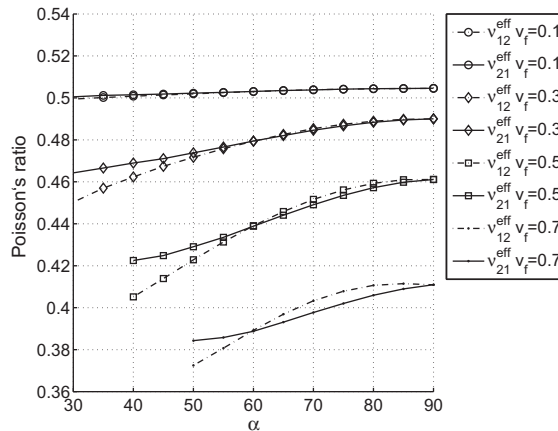


Fig. 13. Effective in-plane Poisson's ratios in local coordinates.

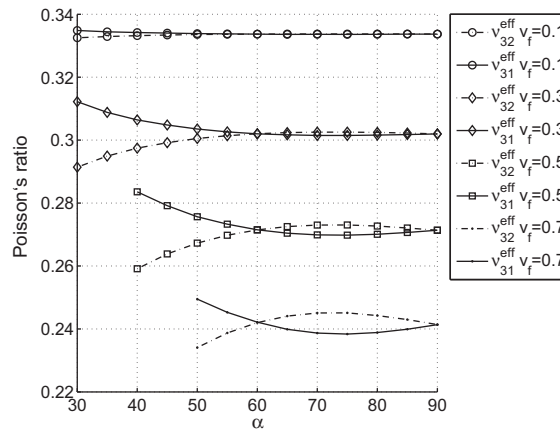


Fig. 14. Effective out-of-plane Poisson's ratios in local coordinates.

Table 6

Difference between maximum and minimum in % with respect to local coordinates.

Vol. fraction	E_1	E_2	G_{12}
0.4	5.7	11.2	9.3
0.7	3.6	6.9	7.2

4. Conclusion

A technique to evaluate effective material properties related to unidirectional fiber reinforced composites having rhombic and periodic microstructures has been presented. One primary aspect of it is the development of an appropriate micro cell model for a FE-analysis in ANSYS with periodic boundary conditions. The constituents of the considered composites have isotropic and transversely isotropic material behavior.

The calculated effective values presented in Tables 1–4 are in a good agreement compared to Jiang et al. (2004), AHM, EVM and G&N. Since these comparisons are restricted to out-of-plane shear coefficients, further results (Figs. 5–10) are shown for a complete material description in case of hexagonal fiber arrangements and compared to values given by Hashin (1979). These comparisons to literature demonstrate the validity of the technique at least for the considered cases. Finally results (Figs. 11–14) plotted against the rhombic angle are presented, where for some of them an appreciable change of value can be observed.

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