Understanding the Doctrine of Patentable Subject Matter*

Jing-Yuan Chiou†

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(Preliminary and incomplete.)

Abstract

The doctrine of patentable subject matter precludes basic inventions such as abstract ideas and laws of nature from patent protection. However, current economic thinking of the patent system stresses the necessity of rewarding pioneering inventors in the cumulative innovation process. In a two-stage innovation model where the first stage invention (basic invention) has no stand-alone value and the pioneer can also participate in the second stage, I show that patent protection to the basic invention may increase rather than hamper the second stage performance. Rejecting patents on the basic invention can promote technology progress when the pioneer has high capacity, but the follower has low capacity to engage in the second stage innovation.

Keywords: Cumulative Innovation, Patentable Subject Matter, Probabilistic Patents, Search.

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*All errors are mine. Comments are welcome.

†IMT Lucca; email: jy.chiou@imtlucca.it; address: IMT Lucca, Piazza San Ponziano 6, Lucca 55100, Italy; tel: +39 0583 4326 734; fax: +39 0583 4326 565.

1 Introduction

In the past two decades, we've witnessed a paradigm shift in the economic understanding of the patent system. Economists have departed from the discrete innovation environment (Nordhaus, 1969, Gilbert and Shapiro, 1990, Klemperer, 1990), and appreciate that R&D is a continual process where each discovery opens a door for future research and each invention builds on knowledge previously acquired in the same or adjacent fields (Green and Scotchmer, 1995, Scotchmer, 1996, O'Donoghue, 1998, Denicolò, 2000, Bessen and Maskin, 2009). Reflecting its sequential nature, the literature of cumulative innovation emphasizes the needs to properly protect early stage inventions, and focuses on how to adjust patent rights to latter inventions in order to balance R&D incentives at different stages of the innovation process.[1]

Patent law does not always enthusiastically embrace the strong support of basic inventions, however. The Supreme Court of the United States has long held that "*[h]e who discovers a hitherto unknown phenomenon of nature has no claim to a monopoly of it which the law recognizes. If there is to be invention from such a discovery, it must come from the application of the law of nature to a new and useful end.*"[2] Established in case law, the doctrine of patentable subject matter (henceforth, the DPSM) precludes the following from the realm of patent protection:[3]

> principles, laws of nature, mental processes, intellectual concepts, ideas, natural phenomena, mathematical formulae, methods of calculation, fundamental truths, original causes, motives, [and] the Pythagorean theorem. . . .

Applications of abstract ideas and principles, instead, may be patented, provided that they also satisfy other requirements such as novelty, non-obviousness, and usefulness.

Understanding laws of nature or discovering new ideas very often entail substantial knowledge spillover as they provide new and fertile grounds for future research. These activities arguably require no less time and effort than designing applications from basic concepts. The DPSM seems at odds with the insights from cumulative

[1]See Scotchmer (2004) for a literature review. Bessen and Maskin (2009) argues that the patent system should be abolished in the cumulative innovation environment.

[2]Funk Bros. Seed Co. *vs.* Kalo Inoculand Co., 333 U.S. 127 (1948).

[3]*In re Bergy*, 596 F.2d 952, 201 U.S.P.Q. (BNA) 352 (C.C.P.A. 1979). See also Merges (1997). The European Patent Convention excludes the following from patentable inventions: (a) discoveries, scientific theories and mathematical methods; (b) aesthetic creations; (c) schemes, rules and methods for performing mental acts, playing games or doing business, and programs for computers; and (d) presentations of information (http://www.epo.org/patents/law/legal-texts/html/epc/1973/e/ar52.html).

innovation theory (Eisenberg, 2000). For sure, one may find other justifications for the DPSM, such as the difficulty to enforce patent rights based on abstract ideas or mental process, or the somewhat ambiguous difference between "discovery" and "invention."[4] Since the patent system is designed to "promote the Progress of Science and useful Arts,"[5] in this paper I address the discrepancy between economic theory and paten law practice from the point of view of innovation incentives. I ask: In order to promote technological progress, when is it optimal to enable the DPSM and deny patent protection to basic inventions?

Consider a two-stage innovation process, where the completion of the first stage is a pre-requisite to start the second stage (Green and Scotchmer, 1995). At the first stage, a pioneering inventor aims to create, or discover, an abstract idea. The abstract idea has no stand-alone value; further efforts are required to find practical applications of the idea. At the second stage, the pioneer and a following inventor sequentially search for the same application. I let the pioneer search first, and the follower search only when the pioneer does not come up with the application. To focus on the DPSM, I assume that the application is always patentable, and will always infringe on the abstract idea should the latter become patentable. The only policy instrument is the degree of patent protection conferred to the abstract idea.

To raise the overall innovation rate, i.e., the probability that the application will be discovered,[6] I find that the DPSM is more likely to be optimal when, at the *second* stage, the pioneer has better search capacity, while the follower is less likely to make the discovery, i.e., when his search cost is more likely to be large and so the probability to incur the cost and find the application is small. Consistent with the literature, granting a patent on the abstract idea boosts the pioneer's first stage innovation incentives and alters the follower's search decision. This policy also reduces the pioneer's search incentives at the second stage: if she does not invent the application, she can still use the patent rights on the abstract idea to get a share of the follower's expected surplus. Therefore, besides considering incentives of different generations of inven-

[4]In Gottschalk *vs.* Benson, 409 U.S. 63 (1972), the Supreme Court states that: "*It is conceded that one may not patent an idea.... The mathematical formula involved here has no substantial practical application except in connection with a digital computer, which means that if the judgment below is affirmed, the patent would wholly preempt the mathematical formula and in practical effect would be a patent on the algorithm itself*." This argument could be analyzed as one with patent scope, i.e., whether to allow a patent with a very broad scope such that it covers all inventions using the algorithm.

[5]U.S. Constitution, Art I, sect. 8, cl. 8.

[6]In Section 4, I show that the DPSM is not optimal when using the two inventors' joint surplus as the policy objective.

tors, there is a need to balance the *same* inventor's incentives at different innovation stages. This concern dominates as the pioneer's second stage capacity improves, but the follower's capacity deteriorates. When the follower has a rather small probability to find the application (even without the threat from the patent on the abstract idea), there is not much surplus to transfer from the follower to the pioneer. Patenting the abstract idea has limited benefit on the first stage innovation, and the second stage discovery probability is dominated by the pioneer's performance. When the pioneer can find the application with a significant probability, *provided that* she is willing to do so, the negative effect of such an "early reward" on her search decision can be non-negligible. The DPSM then is justified as a way to preserve the pioneer's continuing efforts in research.

This result implies that abstract ideas or basic inventions should not be patentable if great first-mover advantage can be derived from engaging in fundamental research, while a new comer, lacking the experience at the earlier stage, faces a substantial obstacle to join the rank. But as the innovation process becomes more "democratic," i.e., as knowledge and research capacity disseminate and are no longer concentrated on a few "early stars," then it would be optimal to start patenting abstract ideas or early inventions. Alternatively, the pioneering inventor's and following inventor's capacity may be different in kind. The pioneer may be good at perfecting the basic invention or better understanding its fundamental properties, and follower may have advantage in identifying particular use of the basic invention and adapting it to specific contexts. The relative importance of these two capacities then depends on the phase of technological progress. To the extent that further understanding the basic scientific principles has priority in primitive technology fields, basic inventions or abstract ideas should become patentable only in mature fields.

An interesting feature of this model is that strengthening the patent protection to the basic invention does not necessarily hamper the second stage innovation. Although it weakens the pioneer's search incentives, there are two opposite effects on the follower's search incentives. A negative effect is the direct concern to share the fruits with the pioneer. But when the application may exist only with a probability, a lower search intensity from the pioneer will boost the follower's search incentives. The positive effect comes from an information channel in the sequential search structure, and the assumption that an inventor's search cost is private information. Since the follower searches only if the pioneer hasn't found the application yet, the latter's "silence" delivers a bad news about the existence of the application, and the more so the more intensively the pioneer searches. Along the equilibrium path, the pioneer's lower search effort after obtaining a patent on the basic invention will raise the following inventor's updated belief when it is his turn to search. The direct negative effect is mitigated by the boost in belief. This positive effect may be strong enough to raise the overall second-stage performance after the patenting of abstract ideas. When this is true, patenting the basic invention is beneficial to both stages of innovation. A necessary condition for the DPSM to be optimal, then, is that it has to enhance the second-stage innovation probability.

This finding provides another interpretation of the shrinking of the DPSM since the 1980s. Through a series of court decisions, particularly in computer software and biotechnology, the scope of patentable subject matters has drastically increased in the U.S. (Kuhn, 2007). Despite rapid expansions, some commentators have warned that rewarding patents to abstract ideas would do more harm than good to the long-term development in these fields. And it is an often raised hypothesis that these industries could have done better had these basic patents been denied.[7] My result nevertheless suggests a less gloomy role of basic patents. It also implies that these patents may promote the disintegration of the innovation market. The pioneer has lower search incentives after obtaining a patent on the abstract idea. When the belief-based positive effect substantially offsets the negative effect of rent transfer, the conditional probability that the application is discovered by the follower increases, and so the concentration of innovations falls after abolishing the DPSM. Furthermore, if the optimal patent policy takes into account the concerns in my analysis, then there may be a reverse causality: abstract ideas and basic inventions become patentable precisely when there is a better follower joining the development process.

There is a long and well established literature of the doctrine of patentable subject matter in the legal profession.[8] In economics, however, most studies either assume that early inventions always receive patent protection (Green and Scotchmer, 1995, Scotchmer, 1996, Denicolò, 2000), or give equal treatments to innovations at different stages (O'Donoghue, 1998). Matutes *et al.* (1996) and Kultti and Mittunen (2008) allow various levels of protection to the basic invention, including no protection, but conclude that some protection is always better. To the best of my knowledge,

[7] See, e.g., Merges (2007) for a discussion of these "unfulfilling" critics in the software industry.

[8] See Merges (1997) for a general discussion. A partial list of recent articles includes Gruner (2007), Kuhn (2007) and Risch (2008).

Harhoff *et al.* (2001) and Aoki and Nagaoka (2004) are the two exceptions that obtain no patent protection to the basic invention as the optimal policy. Assuming that firms have fixed research capacity, Harhoff *et al.* (2001) cautions that patenting basic inventions (gene in their model) may induce socially wasteful stockpile of basic inventions and delay applications. Aoki and Nagaoka (2004) allow firms to vary R&D efforts and is the most relevant paper to my analysis.[9]

Aoki and Nagaoka (2004) considers the same issue as here, namely, whether to grant patent protection to an intermediate invention that serves only as an input for future research, and obtains a pretty intuitive result that patent protection is desirable when conducting basic research is very costly. Aoki and Nagaoka (2004) adopts a two-stage paten race model as in Denicolò (2000), and assume that players have the same Poisson-type innovation technology. In this paper, I stress the asymmetry between inventors of different generations. I will also show that, when the first stage innovation cost has uniform distribution, the optimality of the DPSM does not depend on the cost parameter (the support of the distribution) at this stage. In this regard, my analysis is complementary to the insight derived in Aoki and Nagaoka (2004).

To proceed, section 2 introduces the basic setting; section 3 considers how the patent policy affects the innovation performance at each stage, whose results are applied in Section 4 to determine when it is optimal to enable the DPSM; section 5 (to be completed) considers some variations of the basic model; and section 6 concludes the paper. Proofs are collected in Appendix A.

2 Model

A pioneering inventor (pioneer, she) and a following inventor (follower, he) engage in a two-stage innovation process. The goal of the first stage is to create a basic in-

[9] Aoki and Nagaoka (2004) considers the same issue as here but in the name of utility requirement. Arguably there is some over-lapping between the utility requirement and the DPSM: an abstract idea is not patentable because it lacks "specific and substantial utility," i.e., it is not "useful for any particular practical purpose." (See USPTO, *Utility Examination Guidelines*, http://www.uspto.gov/web/menu/utility.pdf.) Indeed, in Brenner *vs.* Manson, 383 U.S. 519 (1966), the Supreme Court ruled that the Manson patent is at a too preliminary stage to be protected by a patent, and stated that *"a patent is not a hunting license. It is not a reward for the search, but compensation for its successful conclusion."* The Court's reasoning, however, contains some flavor of patent scope: *"Unless and until a process is refined and developed to this point–where specific benefit exists in current available form–there is insufficient justification for permitting an applicant to engross what may prove to be a broad field."* Risch (2008) suggests to abolish the DPSM but reinvigorate the utility requirement to assess the patentability of each invention. In practice, the utility requirement is not strictly applied. Few patent applications are rejected under this requirement.

vention or scientific knowledge whose application is to be discovered at the second stage. As in Matutes *et al.* (1996), I assume that only the pioneer participates in the first stage, but both players may search for the application at the second-stage. To compare my results with the cumulative innovation literature, I also assume that, at the second stage, players are looking for the same application, or applications with high substitutability in terms of payoffs.

An inventor decides whether to spend an exogenous (but *ex ante* random) innovation cost. After incurring the cost, the invention arrives with some probability. At the first stage, I assume that the basic invention will be created for sure when the pioneering inventor spends the cost c_0, which is distributed over $[0, \infty)$ with CDF $F_0(\cdot)$ and pdf $f_0(\cdot)$. The basic invention has no stand-alone value, and the game ends when the pioneer decides not to spend c_0.

After the pioneer incurs c_0, the game proceeds to the stage of application search. The application has a private value $\pi > 0$ and exists with a probability $\alpha \in (0, 1]$. The expected value is $v \equiv \alpha\pi$. These parameters are common knowledge between two inventors. Given existence, the pioneer (the follower) finds the application after incurring a cost $c_1 \in [0, \infty)$ ($c_2 \in [0, \infty)$, respectively). Denote the CDF and pdf of cost c_i as $F_i(\cdot)$ and $f_i(\cdot)$, respectively, $i \in \{1, 2\}$. Suppose that pioneer searches first, and the true cost c_i is the player's private information. An inventor cannot commit to her/his own nor observe the other's search strategy.

The distribution of search cost captures an inventor's innovation capacity. I assume that F_i as well as f_i are continuous and differentiable as necessary, $i \in \{0, 1, 2\}$. In addition, for all $i \in \{0, 1, 2\}$, $f_i(c) > 0$ for $0 \leq c < C_i$, with $C_i > v$. This guarantees that $0 < F_i(v) < 1$, and so even if an inventor can grab the whole expected surplus, from the *ex ante* point of view there is some probability that the inventor is not willing to engage in innovation.

As in the literature of cumulative innovation, the patent policy affects the division of surplus π between inventors. To focus on the doctrine of patentable subject matter, I assume that the application is patentable but always infringes on the basic invention when the latter is protected by the patent rights. The only policy instrument is the level of patent rights rewarded to the basic invention.

If the pioneer discovers the application, then she obtains a patent on the application (and maybe also one on the basic invention); she enjoys the whole surplus π. If the follower makes the discovery, then patent policy determines that the pioneer receives $\theta\pi$ and the follower receives $(1-\theta)\pi$, where $\theta \in [0, \overline{\theta}]$ and $\overline{\theta} < 1$. A higher θ then implies stronger patent rights endowed to the basic invention, and the doctrine of the patentable subject matter corresponds to the case of $\theta = 0$. The upper bound $\overline{\theta}$ is assumed to be strictly less than one because generally, in case of mutual blocking patents, each patent-holder would receive a share of surplus. In other words, I exclude the extreme case where the pioneering inventor has the full bargaining power.

Note that, if the pioneer exhausts her search opportunity but does not come up with the application, i.e, if she decides not to spend c_1 or if c_1 is incurred but she doesn't find the application, then the pioneer is (weakly) better off to disclose the basic invention, for all values of patent policy θ. When the DPSM is enabled and the basic invention is not patentable ($\theta = 0$), then whether the pioneer discloses the basic invention has no impact on her payoff. She won't get a share of π whatever happens after.[10] When the basic invention is patentable with $\theta > 0$, by disclosing the basic invention and so allow the follower to engage in application search, the pioneer may receive a surplus $\theta\pi$ with some probability. Since there is no harm of disclosing the basic invention, I assume that the pioneer will always publish the basic invention.

The optimal policy θ is derived to induce technological progress, as measured by the overall probability to complete the two-stage innovation process.[11] This objective can also be justified from the concern of the social surplus. When the *application* has significant positive externality, private parties always under-invest. It is socially desirable to raise private innovation efforts in order to achieve the application.

Figure 1 illustrates the timing of the game.

- At time 0, the patent policy θ is announced;
- at time 1, the pioneer learns the patent policy and the cost c_0 of conducting the first stage innovation. The game continues only if the inventor spends c_0 and creates the basic invention;
- at time 2, the pioneer inventor learns c_1 and decides whether to search the application;
- at time 2.5, the pioneer applies patents for the basic invention (if allowed), and for the application (if she finds it); and
- at time 3, if the pioneer doesn't find the application, then the follower learns his

[10]But after disclosure of the basic invention the pioneer may receive, say, a Nobel Prize or other reputation-based reward from the scientific community for the recognition as the inventor of important scientific knowledge or breakthrough.

[11]In Section 4, I show that the DPSM cannot maximize the joint surplus of the two inventors.

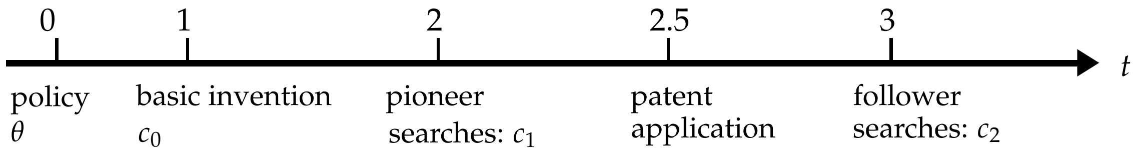

Figure 1: Timing

search cost c_2 and decides whether to search.

When the pioneering inventor holds a patent on the basic invention but doesn't find the application, the following inventor may want to negotiate a license before searching for the application. I postpone the discussion of licensing between time 2.5 and time 3 to Section 5.

3 The DPSM and Stage-wise Innovation Performance

This section evaluates the impact of patent policy θ on the innovation process. Solving the game in the backward manner, suppose that the pioneer has created the basic invention, and consider the subgame of application search.

If the pioneer incurs c_1 and finds the application, then she can patent the application (and maybe the basic invention) and gets the whole surplus π; the game ends. Suppose that the pioneering inventor does not come up with the application, either because she doesn't spend c_1 to search, or because she incurs c_1 but the application does not exist. By assumption, the follower cannot distinguish between the two events.[12] After learning his search cost c_2, the follower decides whether to search with some updated belief $\hat{\alpha}$ that the application exists. Below I will show that, for both players, the optimal search strategy takes a cut-off form. That is, an inventor will incur the search cost if and only if it is lower than a threshold value. Given this rule, when the follower believes that the pioneer's cut-off is $\tilde{c}_1$, his updated belief is

$$\hat{\alpha}(\tilde{c}_1) = \frac{\alpha[1 - F_1(\tilde{c}_1)]}{1 - \alpha + \alpha[1 - F_1(\tilde{c}_1)]} = \frac{\alpha[1 - F_1(\tilde{c}_1)]}{1 - \alpha F_1(\tilde{c}_1)}. \tag{1}$$

With probability α, the application exists, and the pioneer finds it only if incurring the cost c_1; and with probability $1 - \alpha$, the application does not exist and the pioneer-

[12] If the pioneer has incurred search cost but failed to find the application, by assumption it is a clear indication that the application does not exist. This "negative information" is valuable to the follower as well as the society for it prevents further wasteful search effort. See 5 for a discussion of "licensing" this information.

ing inventor won't be able to find it whether spending c_1 or not. The follower then updates his belief according to Bayes' rule as expressed in condition (1).

Given the patent policy $\theta \in [0, \overline{\theta}]$, the follower receives a payoff $(1-\theta)\pi$ for his discovery. He incurs c_2 and searches if and only if

$$\hat{\alpha}\pi(1-\theta) - c_2 \geq 0 \quad \Rightarrow \quad c_2 \leq \hat{c}_2 \equiv \hat{\alpha}\pi(1-\theta). \tag{2}$$

The follower adopts a cut-off rule, with an expected payoff (given that the pioneering inventor doesn't find the application)

$$\hat{U}_2 = \int_0^{\hat{c}_2} [\hat{\alpha}\pi(1-\theta) - c_2] dF_2. \tag{3}$$

For the pioneer, if she spends the cost c_1, then with probability α she will find the application and enjoy the whole surplus π; and with probability $1-\alpha$ the application does not exist and the follower will not find it either. If the pioneer does not spend c_1, then she will receive a surplus $\theta\pi$ when the application exists and the follower searches. Suppose that the pioneer believes that the follower adopts a cut-off $\tilde{c}_2$ and so will search with probability $F_2(\tilde{c}_2)$. The pioneer searches if and only if

$$\alpha\pi - c_1 \geq F_2(\tilde{c}_2)\alpha\pi\theta \quad \Rightarrow \quad c_1 \leq \hat{c}_1 \equiv \alpha\pi[1 - F_2(\tilde{c}_2)\theta] = v[1 - F_2(\tilde{c}_2)\theta]. \tag{4}$$

The pioneer also adopts a cut-off rule, and her expected payoff at the second-stage is

$$\hat{U}_1 = \int_0^{\hat{c}_1} (\alpha\pi - c_1) dF_1 + [1 - F_1(\hat{c}_1)] F_2(\tilde{c}_2)\alpha\pi\theta. \tag{5}$$

By assumption, players cannot commit to their search strategies, i.e., the cut-off values. Since the true search cost and the decision to incur it are not observable to the other party, the proper equilibrium concept at the search subgame is rational expectation equilibrium. Slightly abusing the notation, a rational expectation equilibrium is a pair of cut-offs $(\hat{c}_1, \hat{c}_2)$ such that they are determined according to conditions (2) and (4), with the belief $\hat{\alpha}$ in condition (2) evaluated at $\tilde{c}_1 = \hat{c}_1$ according to the expression (1), and the belief $\tilde{c}_2 = \hat{c}_2$ in condition (4). Given a search equilibrium $(\hat{c}_1, \hat{c}_2)$, denote the corresponding probabilities $\hat{F}_i \equiv F_i(\hat{c}_i)$, $i \in \{1, 2\}$. For the interest of this paper, I denote (c_1^*, c_2^*) as the search equilibrium under the DPSM, i.e., under the policy $\theta = 0$, with corresponding $F_i^* = F_i(c_i^*)$, $i = 1, 2$.

The DPSM guarantees a unique search equilibrium. Setting $\theta = 0$ in condition (4), the pioneer's search decision is independent of the follower's search strategy. The optimal cut-off is uniquely determined by

$$c_1^* \equiv \alpha\pi \equiv v. \tag{6}$$

This unique cut-off then pins down the follower's updated belief at search, $\hat{\alpha}(c_1^*) \equiv \alpha^*$, and the follower's optimal cut-off c_2^*:

$$c_2^* \equiv \alpha^* \pi. \tag{7}$$

When $\theta > 0$, the two inventors' search decisions become strategic substitutes. In equilibrium, a higher cut-off $\hat{c}_1$ will reduce $\hat{c}_2$, and *vice versa*. The pioneer, with $\theta > 0$, benefits from the follower's search. More intensive search by the follower, i.e., a higher cut-off $\hat{c}_2$ and so a larger probability $\hat{F}_2$, lowers the pioneer's search incentive. The pioneer's cut-off $\hat{c}_1$ is decreasing in $\hat{c}_2$ for $\theta > 0$.

The negative impact of $\hat{c}_1$ on $\hat{c}_2$ works through the belief $\hat{\alpha}$. One event leading to the follower's search is the pioneer's failure to find the application, which implies that the application does not exist. The follower's search opportunity, in other words, conveys a bad news that the application is less likely to exist. This pessimistic message becomes worse as the pioneer searches more intensively. For all $\alpha \in (0, 1)$, a higher $\hat{c}_1$ by the pioneer reduces the follower's belief at search:

$$\frac{\partial \hat{\alpha}}{\partial \hat{c}_1} = -\frac{\alpha(1-\alpha) f_1(\hat{c}_1)}{[1-\alpha F_1(\hat{c}_1)]^2} < 0. \tag{8}$$

A negative effect on belief depresses the follower's search incentives: $\hat{c}_2$ is decreasing in $\hat{c}_1$. As long as $\theta < 1$, the patent policy only changes the magnitude of this effect, but does not affect its presence.

The mutual dependence of search decisions may lead to multiple search equilibria. Consider an increase in $\hat{c}_1$. Along the equilibrium path, a more intensive search from the pioneer lowers the follower's belief, and so the follower's equilibrium cut-off $\hat{c}_2$. A lower search intensity from the follower in turn justifies the initial increase in $\hat{c}_1$. By the same token, expecting an increase of the follower's cut-off, the pioneer will search over a smaller range of search cost. The follower, along the equilibrium path, will a have a higher updated belief, and so is willing to raise the cut-off.

Despite the possibility of multiple equilibria under $\theta \in (0, \overline{\theta}]$, granting patent rights to the basic invention always reduces the pioneer's search incentive, $c_1^* > \hat{c}_1$ for all θ such that $0 < \theta \leq \overline{\theta} < 1$. By $\overline{\theta} < 1$, in any search equilibrium $\hat{c}_2 > 0$ and so $\hat{F}_2 > 0$. It follows that $c_1^* = v > \hat{c}_1 = v(1 - \hat{F}_2\theta)$, for all $\theta \in (0, \overline{\theta}]$. For the follower, a lower cut-off adopted by the pioneer boosts his belief at search: $\hat{\alpha}(\hat{c}_1) > \alpha^*$, for all $\hat{c}_1 < c_1^*$. Whether $\hat{c}_2 \gtrless c_2^*$ then depends on whether $\hat{\alpha}(1-\theta) \gtrless \alpha^*$. In this model, the negative effect of transferring surplus $\theta\pi$ from the follower to the pioneer on the former's search incentive is mitigated by the opposite effect on the belief.

Patent protection to the basic invention does not necessarily weaken the follower's search incentives.

Given the parameter α and search equilibrium $(\hat{c}_1, \hat{c}_2)$, the probability to discover the application is $\alpha[\hat{F}_1 + (1-\hat{F}_1)\hat{F}_2]$. Define $\hat{E} \equiv \hat{F}_1 + (1-\hat{F}_1)\hat{F}_2$, which measures the overall search effort, or the innovation performance at the second stage. Define the corresponding measure under the DPSM as $E^* \equiv F_1^* + (1-F_1^*)F_2^*$. Since $c_1^* > \hat{c}_1$, the comparison between E^* and $\hat{E}$ depends on the relative size of c_2^* and $\hat{c}_2$. If $c_2^* \geq \hat{c}_2$, then the DPSM surely boosts the second-stage innovation performance, $E^* > \hat{E}$. If $\hat{c}_2 >> c_2^*$, however, we may have the opposite outcome, $\hat{E} > E^*$. Different from Aoki and Nagaoka (2004), in my model patent protection to the basic invention is not necessarily detrimental to the second stage innovation performance. The following example use two-point search technology to illustrate this point, as well as the possibility of multiple search equilibria.

Example 1. (Two-point search technology). Suppose that both the pioneer's and follower's search cost have two-point distributions, $c_i \in \{C_i, K\}$, with $K > v \geq C_i \geq 0$, and the probability of low search cost is $\Pr(c_i = C_i) = p_i \in (0,1)$, $i = 1,2$. An inventor will not incur the high search cost $K > v$. In any search equilibrium, the pioneer's (follower's) search probability is at most $\hat{F}_1 = p_1$ ($\hat{F}_2 = p_2$, respectively).

Fixing $\theta > 0$, I first show that both $(\hat{F}_1, \hat{F}_2) = (0, p_2)$ and $(p_1, 0)$ can be search equilibria. To have $(0, p_2)$ as the equilibrium, the pioneer must find it too costly to incur C_1, given that the following inventor will incur C_2. We need $C_1 > v(1 - p_2\theta)$. And for the follower to be willing to incur C_2, given that the pioneering inventor does not search at all, we need $C_2 \leq v(1-\theta)$. In this search equilibrium, the follower's belief maintains at the *ex ante* level. For $(p_1, 0)$ to be the search equilibrium, the pioneer incurs C_1 but the follower will not search. We need $C_1 \leq v$ and $C_2 > \hat{\alpha}\pi(1-\theta)$, where $\hat{\alpha} = \alpha(1-p_1)/(1-\alpha p_1) < \alpha$. We have multiple equilibria when

$$v(1 - p_2\theta) < C_1 \leq v \text{ and } \hat{\alpha}\pi(1-\theta) < C_2 \leq v(1-\theta). \quad (9)$$

An implication of multiple equilibria is mis-allocation of search activity. Even though the overall search performance is the same, different equilibria may entail different levels of total search cost. To see this, suppose $p_1 = p_2 = p \in (0,1)$ and condition (9) holds. Both search equilibria have the same probability to find the application (given existence), $\hat{E} = p$, but different search costs depending on which inventor searches. When $C_1 > C_2$, then the equilibrium where only the follower searches is more cost-efficient. In fact, if $p_2 > p_1$, then this equilibrium also has a higher probability to find the application.

Lastly, suppose that $p_2 > p_1$. Under the DPSM ($\theta = 0$), the search equilibrium is unique, $(p_1, 0)$, with $E^* = p_1$. But if we let the basic invention be patentable with $\theta > 0$ such that condition (9) holds, then in the search equilibrium $(0, p_2)$, we have $\hat{E} = p_2 > E^*$. Patenting the basic invention boosts the second stage innovation performance when the "good" search equilibrium prevails. $\|$

The following proposition summarizes the results at the second stage.

Proposition 1. *(Search equilibrium). When $\theta \in (0, \overline{\theta}]$ and $\alpha \in (0, 1)$, there may be multiple search equilibria at the second innovation stage. But the search equilibrium is unique under the DPSM.*

The pioneer has a higher search incentive under the DPSM than under other policy $\theta \in (0, \overline{\theta}]$, $c_1^* > \hat{c}_1$. The impact of the patent policy on the follower's search incentive is ambiguous, $c_2^* \gtrless \hat{c}_2$. Consider stable search equilibria. When evaluated at $\theta = 0$, $d\hat{c}_1/d\theta < 0$. For $\theta > 0$, $d\hat{E}/d\theta \gtrless 0$, but not both $d\hat{c}_1/d\theta$ and $d\hat{c}_2/d\theta > 0$.

Remark 1. (Market structure). An important difference between my model and the analytical framework adopted by Aoki and Nagaoka (2004) is the effect of the patent policy θ on the "market structure" of the innovation market. Aoki and Nagaoka (2004) uses a two-stage patent race model from Denicolò (2000) and also allows the first inventor to engage in the second stage innovation.[13] Due to the assumptions of a homogeneous Poisson race and identical research capability, if the basic invention is patentable, the patent-holder has no incentive to let other inventors pursue the second stage innovation. The only meaningful policy space is a binary set, namely, whether the basic invention is patentable or not. The pioneer does not benefit from other inventor's innovation capacity. Patenting the basic invention generates a monopoly at the second stage, and increases the concentration of the innovation activity, i.e., the extent to which different inventions are created by different inventors.

By contrast, I have a "hybrid" structure where the pioneer enjoys head-start advantage at the second stage and at the same time could extract some surplus from the follower when the basic invention is patentable and her search fails. My model has a richer policy space $\theta \in [0, \overline{\theta}]$, and can easily incorporate asymmetric innovation capacities by different inventors, as captured by different distributions F_1 and

[13] They assume free entry at the first stage. The "pioneer," therefore, refers to the first inventor to finish the race and create the basic invention (or the intermediate technology as they called it).

F_2. In addition, granting the patent protection to the basic invention may reduce the concentration of the innovation market. Given that the second stage innovation is completed, the probability that it is finished by the follower is $[(1-F_1^*)F_2^*]/E^*$ when the basic invention is not patentable, and $[(1-\hat{F}_1)\hat{F}_2]/\hat{E}$ when it is patentable with $\theta > 0$. Compare the two levels,

$$\frac{(1-\hat{F}_1)\hat{F}_2}{\hat{E}} > \frac{(1-F_1^*)F_2^*}{E^*} \quad \Leftrightarrow \quad F_1^*(1-\hat{F}_1)\hat{F}_2 > \hat{F}_1(1-F_1^*)F_2^*. \tag{10}$$

Since $F_1^* > \hat{F}_1$, as long as $\hat{F}_2$ is not too small relative to F_2^*, patenting the basic invention helps the decentralization of innovation activities. ||

Remark 2. (Impact of α). Fixing the expected value v, the level of the parameter α captures how "abstract" the basic invention is, or how far it is from commercial applications. A lower α means that it is more difficult to find or develop the application, although the expected value is not affected. In the proof of Proposition 1, I show that a higher α does not necessarily raise the overall second stage performance. Given the pioneer's search strategy, a higher α will raise the follower's belief $\hat{\alpha}$ and increase his incentive to search, $d\hat{c}_2/d\alpha > 0$. When $\theta > 0$, this boost in the follower's search intensity provides a negative feedback to the pioneer's search decision, $d\hat{c}_1/d\alpha < 0$, for she can free ride on the follower's search result. The overall impact on the second stage performance $\hat{E}$ is ambiguous, and may be negative when the pioneer has better search capacity than the follower. For instance, when c_1 and c_2 have uniform distributions over $[0, \gamma_1 v]$ and $[0, \gamma_2 v]$, respectively, with γ_1 and $\gamma_2 > 1$, then $d\hat{E}/d\alpha < 0$ for $\gamma_1 < 1 + \theta$ and γ_2 large enough. ||

Turn to the first stage. Expecting a payoff U_1 from the search subgame, the pioneer will incur a cost c_0 to create the basic invention as long as $c_0 \leq U_1$. The basic invention will be produced with a probability $F_0(U_1)$, and a higher U_1 raises the pioneer's incentive to engage in basic research. The pioneer's expected payoff at the first stage is

$$\hat{U}_0 = \int_0^{U_1} (U_1 - c_0) dF_0. \tag{11}$$

Denote $\hat{U}_1$ and U_1^* as the pioneer's payoffs in the search equilibrium when $\theta \in (0, \overline{\theta}]$ and $\theta = 0$, respectively. By previous discussion, $c_1^* > \hat{c}_1$ and $\hat{c}_2 > 0$. Together with the definition of $\hat{c}_1$,

$$\begin{aligned} U_1^* = & \int_0^{c_1^*}(v-c_1)dF_1 = \int_0^{\hat{c}_1}(v-c_1)dF_1 + \int_{\hat{c}_1}^{c_1^*}(v-c_1)dF_1 \\ < & \int_0^{\hat{c}_1}(v-c_1)dF_1 + \int_{\hat{c}_1}^{c_1^*}\hat{F}_2 v\theta dF_1 < \hat{U}_1 = \int_0^{\hat{c}_1}(v-c_1)dF_1 + (1-\hat{F}_1)\hat{F}_2 v\theta. \end{aligned} \tag{12}$$

As in the standard cumulative innovation literature, denying patent protection to the basic invention reduces the pioneer's first stage incentive. The DPSM imposes a cost of harming the basic innovation.

Proposition 2. *(First stage innovation incentives). Granting patent protection to the basic invention increases the pioneer's incentive to engage in basic research.*

4 When to Impose the DPSM?

Considering the impact on both innovation stages, when is it optimal to impose the DPSM? Using the overall technology progress rate, $F_0(\hat{U}_1)\alpha\hat{E}$, as the policy criterion, I am interested in situations where $\theta = 0$ is the solution to the program $\max_\theta \alpha F_0(\hat{U}_1)\hat{E}$. Fixing α, it is equivalent to finding conditions such that $F_0(U_1^*)E^* \geq F_0(\hat{U}_1)\hat{E}$ for all $\theta \in (0, \overline{\theta}]$.

By previous analysis, $U_1^* < \hat{U}_1$ and so the DPSM is detrimental to the first stage innovation incentive. If, at the second stage, $E^* < \hat{E}$ for some $\theta \in (0, \overline{\theta}]$, then the DPSM is dominated at both stages of the innovation process. A necessary condition to reject patent protection to the basic invention therefore is $\hat{E} < E^*$ for all $\theta \in (0, \overline{\theta}]$. Suppose that this necessary condition holds. That is, the indirect effect of $\theta > 0$ on the follower's belief, $\hat{\alpha}(\hat{c}_1) > \alpha^*$, is not large enough to dominate the sum of the effect on the pioneer's search decision ($c_1^* > \hat{c}_1$) and the direct effect on the follower's search incentive due to surplus transfer.

Consider the overall impact of the patent policy on the technological progress:

$$\begin{aligned} \frac{dF_0(\hat{U}_1)\hat{E}}{d\theta} &= \hat{E} f_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta} + F_0(\hat{U}_1)\frac{d\hat{E}}{d\theta} \\ &= \hat{E} f_0(1-\hat{F}_1)v\left(\hat{F}_2 + \theta\hat{f}_2\frac{d\hat{c}_2}{d\theta}\right) + F_0(1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right) \\ &= (1-\hat{F}_1)\left[\hat{E} f_0 v\left(\hat{F}_2 + \theta\hat{f}_2\frac{d\hat{c}_2}{d\theta}\right) + F_0(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right)\right]. \end{aligned} \tag{13}$$

In my model, the patent policy affects three decisions: besides the pioneer's innovation decision at the first stage and the follower's decision at the second stage, it also affects the pioneer's incentive at the second stage. The cumulative innovation literature, such as Green and Scotchmer (1995), emphasizes the trade-off between different generations of inventors at different innovation stages, but overlooks the *same* inventor's innovation incentives across stages. As shown in Section 3, a patent on the basic invention encourages the first stage innovation ($F_0(\hat{U}_1) > F_0(U_1^*)$) and changes the follower's innovation performance ($d\hat{c}_2/d\theta \gtreqless 0$). This early reward also *discourages* the pioneer from continuing her research activity ($\hat{F}_1 < F_1^*$). When this effect is strong enough, we may find a reason to reject patenting the basic invention.

To better illustrate the key result, I make a few simplifications on the cost distributions. Suppose that c_0 follows the uniform distribution over the support $[0, \gamma_0 v]$, with $\gamma_0 > 1$, and c_2 follows the two-point distribution, $c_2 \in \{0, K\}$, with $K > v$ and $\Pr(c_2 = 0) = p_2 \in (0, 1)$. The first simplification brings about an interesting case where the optimal policy θ is independent of the cost parameter at the first stage, namely, γ_0. Different from Aoki and Nagaoka (2004), within the class of uniform distributions, I can derive the optimality of the DPSM without referring to the difficulty of obtaining the basic invention. The two-point search technology, $c_2 \in \{0, K\}$, implies that the follower has a fixed probability $\hat{F}_2 = F_2^* = p_2$ to find the application. The second simplification is introduced to point out how the follower's capacity p_2 affects the trade-off between the pioneer's innovation incentives at different stages.[14]

Under these specifications, $d\hat{c}_2/d\theta = 0$ and $d\hat{c}_1/d\theta = -vp_2$. Fixing the follower's search capacity, stronger patent protection to the basic invention θ always raises the pioneer's incentive to engage in basic invention. By integration by parts and the opti-

[14]Suppose that, instead, the pioneer has a fixed search capacity, $c_1 \in \{0, K\}$ with $\Pr(c_1 = 0) = p_1 \in [0, 1)$. The optimal θ then is determined according to the classical trade-off between different inventor's incentives at different stages. (When $p_1 = 0$, it corresponds to the standard model where different generations of innovations are conducted by different players.) By $d\hat{c}_1/d\theta = 0$ and so $d\hat{c}_2/d\theta = -v\phi$, where $\phi = (1 - \hat{F}_1)/(1 - \alpha \hat{F}_1)$,

$$sign\left(\frac{dF_0(\hat{U}_1)\hat{E}}{d\theta}\right) = sign\left(\frac{f_0}{F_0}[p_1 + (1-p_1)\hat{F}_2](\hat{F}_2 - \theta\phi\hat{f}_2) - \hat{f}_2\phi\right). \tag{14}$$

If both f_0 and f_2 take uniform distributions, the sign of $dF_0(\hat{U}_1)\hat{E}/d\theta$, when evaluated at $\theta = 0$, is the same as

$$\frac{1}{p_1 v}[p_1 + (1-p_1)F_2^*] - \frac{\phi}{\phi v} = \frac{1}{v}\left(1 + \frac{1-p_1}{p_1}F_2^* - 1\right) > 0. \tag{15}$$

The DPSM is never optimal.

mal cut-off $\hat{c}_1 = v(1-\hat{F}_2\theta) = v(1-p_2\theta)$,

$$\begin{aligned}\hat{U}_1 &= \int_0^{\hat{c}_1}(v-c_1)dF_1 + (1-\hat{F}_1)p_2v\theta = (v-c_1)F(c_1)|_0^{\hat{c}_1} + \int_0^{\hat{c}_1}F_1dc_1 + (1-\hat{F}_1)p_2v\theta \\ &= \int_0^{\hat{c}_1}F_1dc_1 + p_2v\theta,\end{aligned} \tag{16}$$

and so

$$\frac{d\hat{U}_1}{d\theta} = \hat{F}_1\frac{d\hat{c}_1}{d\theta} + p_2v = (1-\hat{F}_1)p_2v > 0, \text{ when } \hat{F}_1 < 1. \tag{17}$$

This positive effect, however, comes at a cost of lower second stage innovation performance,

$$\frac{d\hat{E}}{d\theta} = (1-p_2)\hat{f}_1\frac{d\hat{c}_1}{d\theta} = -(1-p_2)\hat{f}_1p_2v < 0. \tag{18}$$

The term p_2v appear in both $d\hat{U}_1/d\theta$ and $d\hat{E}/d\theta$. Raising patent protection to the basic invention directly affects the surplus transfer from the follower to the pioneer. Its impact is proportional to the follower's expected return from search, which is p_2v in this case. Beyond this common factor, we can see that the positive effect on the first stage innovation is also proportional to $1-\hat{F}_1$, the probability that the pioneer finds the application (given existence). For the pioneer will use the patent on the basic invention to get a share of the follower's search surplus only when her search fails. On the other hand, the negative impact on the second stage innovation is also proportional to $1-p_2$, namely, a lower search effort from the pioneer becomes a more serious problem when the follower is less likely to make the discovery. Combining the two factors, it might be optimal to maintain the DPSM when the pioneer has significant search capacity (and so $\hat{F}_1$ is high for all $\theta \in [0,\overline{\theta}]$), but not the follower (and so p_2 is small).

Under the specified distributions of c_0 and c_2, the sign of the first-order condition, $dF_0(\hat{U}_1)\hat{E}/d\theta$, is the same as

$$\hat{E}(1-\hat{F}_1) - \hat{U}_1(1-p_2)\hat{f}_1. \tag{19}$$

The first term, $\hat{E}(1-\hat{F}_1)$, captures the incentive effect of an increase in θ on the pioneer's willingness to engage in the first stage research, $\hat{U}_1$. The second term, $\hat{U}_1(1-p_2)\hat{f}_1$, is associated with the effect of θ on the pioneer's second stage decision $\hat{c}_1$. If the expression (19) is negative for all $\theta \in [0,\overline{\theta}]$, then the DPSM is the optimal policy.

To illustrate how the optimality of the DPSM is determined by the pioneer's search capacity, let's assume that c_1 also has uniform distribution over the support $[0, \gamma_1 v]$, with $\gamma_1 > 1$. When γ_1 is smaller, the support of c_1 shrinks, the pioneer is more likely to have smaller search cost. Fixing $\hat{c}_1$, the pioneer is more likely to make the discovery, $\hat{F}_1$ is decreasing in γ_1. This parameter also affects the density function $f_1(c_1) = 1/\gamma_1 v$. A reduction in γ_1 increases f_1, provided that $c_1 \in [0, \gamma_1 v]$, which in turn magnifies the negative impact of θ on $\hat{F}_1$ and thus $\hat{E}$. A lower γ_1, then, implies a better search capacity by the pioneer, and so the more likely to find the DPSM optimal.

After some calculation,

$$\hat{U}_1 = \frac{v}{2\gamma_1}\left[1 + 2(\gamma_1 - 1)\theta p_2 + (\theta p_2)^2\right]. \tag{20}$$

The sign of the first-order condition is the same as

$$\begin{aligned} &2(\gamma_1 - 1)[(\gamma_1 - 1)p_2 + 1] - (1 - p_2) - 3(1 - p_2)(p_2\theta)^2 \\ &\qquad + 2p_2\theta[(\gamma_1 - 1)p_2 + 1 - 2(\gamma_1 - 1)(1 - p_2)]. \end{aligned} \tag{21}$$

When $\gamma_1 \to 1^+$ and $p_2 \to 0$, the first-order condition is strictly negative for all $\theta \in [0, \overline{\theta}]$. The DPSM is the optimal policy. In the proof of the following proposition, I consider the case where c_2 also has a uniform distribution, $f_2 = 1/(\gamma_2 v)$, with $\gamma_2 > 1$. In this case, p_2 is no more fixed, but the DPSM is optimal when γ_2 is large enough and γ_1 small enough. As the pioneer's search capacity expands, but the follower's capacity shrinks, denying patent protection to the basic invention is more likely to be optimal.

Proposition 3. *To promote the technology progress, a necessary condition for the DPSM to be the optimal policy is that it encourages the overall efforts to search the application, $E^* > \hat{E}$, for all $\theta > 0$.*

Suppose that c_i following uniform distribution, $f_i = 1/(\gamma_i v)$, with $\gamma_i > 1$, $i \in \{0, 1, 2\}$. The DPSM is an optimal policy when γ_1 is small enough and γ_2 is large enough.

In light of this result, the DPSM should be applied, and the basic invention should not be patentable when the pioneer has superior technology at the subsequent research stage, but not the follower. But where does this persistence of innovation dominance come from? One source of this advantage would be some knowledge the pioneer acquired during the first stage. The follower cannot benefit from this knowledge either because of its tacit nature and so the intrinsic difficulty to transfer among different inventors, or the pioneer's unwillingness to disclose and help the follower to understand this knowledge. The former in turn may relate to the landscape of the research environment, for instance, how easily it would be for a late-comer to digest the knowledge required to effectively participation in the innovation process. To the extent that, at its nascent phase, the background information of a field may not be widely distributed, but rather concentrated on very few key players, there may not be many capable followers who can readily pursue the pioneer's research line. The insight of Proposition 3 suggests that patents shouldn't be granted to basic inventions in order to maintain the pioneer's continuation effort. The latter, on the other hand, may depend on the disclosure requirement of the patent law. That is, when weak disclosure or enablement requirements significantly hampers other parties' ability to exploit the patented technology, the patent should not be granted. Although it is a common argument that patent system should be designed to diffuse technology, the reasoning here is based on a somewhat reason, namely, the pioneer's incentive to continue doing research.

Another factor that would affect the pioneer's and follower's chance to develop the application are their commercialization capacity. Although the model is developed as a two-stage innovation process, the second stage can be equivalently interpreted as one that involves not research, but commercialization activity. A party is more likely to successfully commercialize the basic invention if, say, she controls more key physical assets used to develop useful and marketable application. The assumption that the second stage result is patentable then corresponds to the protection to (tangible) property rights. And the condition identified in Proposition 3 implies that the patentability of basic invention hinges on the degree of vertical integration. It should not be patentable when the upstream pioneer extends her dominance to the downstream stage of commercialization.

Remark 3. (Research grants). Basic research is often funded by research grants. The main advantage of monetary rewards, it is often argued, is to avoid monopolization of fundamental knowledge. Our analysis, however, indicates that monopoly rights over basic innovation do not necessarily hinder subsequent innovation. By the same logic, monetary rewards, instead of patent rights, to the pioneer's basic invention may deteriorate rather than enhance the overall performance of subsequent innovation. ||

Remark 4. (Alternative objective). This remark considers another objective function, namely, the joint surplus between the two inventors. It turns out that setting $\theta = 0$ will not maximize the joint surplus. In the search of the optimality of the DPSM, this justifies the use of technology progress as the policy objective.

Given the policy θ and the payoffs from the search subgame, $\hat{U}_1$ and $\hat{U}_2$, the joint surplus is

$$S = \int_0^{\hat{U}_1} (\hat{U}_1 - c_0) dF_0 + F_0(\hat{U}_1)(1 - \alpha \hat{F}_1)\hat{U}_2. \tag{22}$$

Since the basic invention has no stand-alone value, when the pioneer is willing to incur c_0, she expects a payoff $\hat{U}_1$ from the subsequent subgame. And the follower gets a payoff $\hat{U}_2$ only when the basic invention is created and the pioneer does not come up with the application. In the proof of Proposition 4, I show that, when evaluating at $\theta = 0$, a marginal increase in θ always raises the joint surplus S. This result does not need further restrictions on the distributions of innovation costs. Intuitively, raising θ beyond zero only exerts a negative impact on the follower's payoff $\hat{U}_2$. This negative effect, however, is canceled by a positive impact on the pioneering inventor's payoff $\hat{U}_1$. Therefore, the DPSM cannot be justified with the inventor's joint surplus as the policy objective.

Proposition 4. *Imposing the DPSM, i.e., setting $\theta = 0$, does not maximize the joint surplus of the two inventors.* ||

5 Extensions (to be completed)

☐ **Licensing:** When the pioneer holds a patent on the basic invention, the two parties may negotiate a license between time 2.5 and 3.[15] Licensing bargaining takes place around two issues.

First, by limited liability, a license only contains a revenue-sharing rule between the two parties, namely, the portion of π transferred from the follower to the pioneer. The patent protection θ may be too strong, for instance, in the extreme case of $\theta = 1$, the follower has no incentive to search; it may be mutually beneficial to a lower royalty term, namely, the portion of π transferred from the follower to the pioneer. This concern justifies our restriction on the upper bound of patent protection, $\overline{\theta} < 1$, as the range of protection that would matter along the equilibrium path, after taking into account licensing.

[15] To the extent that π reflects the maximal revenue from holding a patent on the application, there is no benefit to license this patent.

Second, the follower is interested in the pioneer's private information, i.e., when she has incurred the search cost at the second stage (time 2). By learning this information, the follower can save on the search cost c_2 if the pioneer has searched yet failed, and in the case where the pioneer didn't search, the follower can raise his belief $\hat{\alpha}$ to the *ex ante* level α.

Consider the pioneer's incentive to disclose her information.[16] Suppose that the pioneer does not incur the search cost at time 2. As long as she can get a stake from the follower's search result, e.g., when $\theta > 0$, the pioneer has a strong incentive to transmit this information to the follower in order to raise his belief and the search effort.[17] For the pioneer who has spent the search cost and learned that the application does not exist, she knows that the follower's search is doomed to fail and so loses the interests in the stake from the follower's innovation activity. The pioneer is indifferent to making (or accepting) an offer or not. By breaking this indifference in different ways, the follower may or may not learn the pioneer's private information.

The indifference, however, is not robust to some modifications of the model, e.g., if the pioneer may make mistakes in search.[18] Suppose that with some probability $\varepsilon > 0$ the pioneer fails to find the application even when it exists and the search cost is spent. In this case, the pioneer's failure is still a bad news, but not as desperate as before, and the pioneer will retain some interests in the follower's search activity. The updated belief about the existence of the application, after the pioneer's search failure, is

$$\alpha^{\varepsilon} = \frac{\varepsilon\alpha}{1 - \alpha + \varepsilon\alpha}. \tag{23}$$

[16] When the true level of c_1 is the pioneer's private information and whether she has spent this cost is non-verifiable, it is unclear which patent policy tool could be used to encourage the pioneer to disclosure this information. Since patents are public records, whenever the identity of following inventors are unknown *ex ante*, it may be difficult to enforce patent rights that are granted to knowledge that is used to *prevent* some activities from happening. A monetary reward might be useful, though. That is, the pioneering inventor brings the hard evidence of spending c_1 and receives a prize related to the follower's expected saving. But when the follower cannot be traced down to finance the monetary reward, we go beyond the scope of the patent system and public funds become necessary. I do not consider how the patent system should be designed to directly tackle this issue.

[17] When $\theta = 0$, whether the pioneer is willing to reveal this information depends on the contracting environment. For instance, if the pioneer makes the offer, then after learning the pioneer's information via license offering, the follower can simply turn down the offer and run away with the pioneer's private information. The Arrow problem applies here. But this strategy does not work when the follower makes an enforceable offer to the pioneer.

[18] An alternative way is to relax the limited liability constraint and so the follower can purchase information with cash, or if the follower's saving on the search cost c_2 is transferrable to the pioneer. The pioneer may be able to "sell" her negative information in exchange for some rent from the follower. The question, then, is whether this broader contracting space could help information transmission.

For any $\varepsilon \in (0,1)$, $0 < \alpha^{\varepsilon} < \alpha$. Denote the pioneer's belief, or "type" as $\alpha^P \in \{\alpha, \alpha^{\varepsilon}\}$. When the pioneer gets a share l from the follower's successful search, her expected payoff is

$$\alpha^P F_2(\tilde{\alpha}^l (1-l)\pi) l\pi, \tag{24}$$

where $\alpha^P \in \{\alpha, \alpha^{\varepsilon}\}$, and the follower forms his belief at search, $\tilde{\alpha}^l$, according to the contract term l (offered by the pioneer in the signaling model, or accepted by the pioneer in the screening model). Note that, apart from the first term, the pioneer's own belief α^P affects her expected payoff only through its impact on the follower's belief $\tilde{\alpha}^l$ via the contract term l. There, at the bargaining stage, the pioneer acts to maximize $F_2(\tilde{\alpha}^l(1-l)\pi)l\pi$, regardless of her type. The pioneer's behavior is not affected by her private information. It is then natural to select an equilibrium where both types of pioneer take the same action and so the follower learns no new information, i.e., a pooling equilibrium when the pioneer makes the offer, or no separation (bunching) when the follower makes the offer.[19] Once we restrict our attention to such equilibria, then our previous analysis goes through.

☐ **Endogenous search capacity:**

☐ **Endogenous order to search:** In the basic model I let the pioneer search first. An implicit assumption is that the pioneer can protect the basic invention under secrecy until her search fails, or until she decides not to search. This assumption captures some first-mover advantage and, more importantly, avoids the extreme situation where the pioneer is "forced" to disclose the basic invention even if it is *not* patentable. Here I consider whether the pioneer will exploit this advantage, or instead will want to wait until after the follower's search.

I keep the assumption that a player cannot observe the other's true search cost nor the decision to incur the cost, and that the pioneer still learns the trust cost at time 2, but add an additional stage, time 4, where the pioneering inventor can spend her cost c_1 to search, if she hasn't done that at time 2. For a policy $\theta \in [0, \overline{\theta}]$, denote $\hat{c}_1$ and $\hat{c}_2$ as the equilibrium cut-offs without time 4. I derive conditions under which this additional timing is irrelevant.

[19]When the pioneer makes the offer, by carefully structuring the follower's off-path beliefs we may have separating equilibria. However, in any such equilibrium both types of pioneer must be indifferent to the two equilibrium offers.

When endowed with this additional timing to search, the pioneer knows that if she delays search, she will need to incur c_1 only if the follower doesn't come up with an application. Similar to the reasoning in section 3, the pioneer can update her belief about α at this event. The pioneer's updated belief at time 4 is $\alpha \cdot \hat{\phi}$, where

$$\hat{\phi} = \frac{1 - F_2(\hat{c}_2)}{1 - \alpha F_2(\hat{c}_2)}. \tag{25}$$

Since the follower holds no claim again the pioneer, the latter will incur search at time 4 as long as $c_1 \leq \alpha \hat{\phi} \pi = \hat{\phi} v$. For $c_1 > \hat{\phi} v$, this additional timing to search is irrelevant.

Suppose that the pioneer has search cost $c_1 \leq \hat{\phi} v$. If she searches at time 2, the expected payoff is $v - c_1$. If she delays to time 4, the expected payoff is

$$F_2(\hat{c}_2)\theta v + [1 - \alpha F_2(\hat{c}_2)](\hat{\phi} v - c_1) = F_2(\hat{c}_2)\theta v + [1 - F_2(\hat{c}_2)]v - [1 - \alpha F_2(\hat{c}_2)]c_1. \tag{26}$$

When $F_2(\hat{c}_2) > 0$,

$$v - c_1 \geq F_2(\hat{c}_2)\theta v + (1 - F_2(\hat{c}_2))v - [1 - \alpha F_2(\hat{c}_2)]c_1 \Leftrightarrow c_1 \leq (1 - \theta)\pi. \tag{27}$$

For c_1 smaller than $(1 - \theta)\pi$, the pioneer will search at time 2 rather than wait.

Compare the pioneer's different thresholds. If

$$1 - \theta \geq \alpha \hat{\phi} = \alpha \frac{1 - F_2(\hat{c}_2)}{1 - \alpha F_2(\hat{c}_2)} \Rightarrow \theta \leq \frac{1 - \alpha}{1 - \alpha F_2(\hat{c}_2)}, \tag{28}$$

then time 4 is irrelevant. The pioneer will even search at time 4 only for $c_1 \leq \alpha \hat{\phi} \pi$. But by $1 - \theta \geq \alpha \hat{\phi}$, and so $c_1 \leq (1 - \theta)\pi$, for this range of search cost the pioneer prefers searching at time 2 than time 4.[20] When α and θ are not too large, such that condition (28) holds, previous results are robust to the pioneer's endogenous search timing.

If condition (28) fails, then the search equilibrium is not robust to the pioneer's additional search opportunity. The pioneer will want to delay search for $c_1 \in ((1 - \theta)\pi, \hat{\phi} v]$, and only spend $c_1 \leq (1 - \theta)\pi$ at time 2. The search equilibrium is characterized by three cut-offs: $c_1' = (1 - \theta)\pi$, $c_2' = \hat{\alpha}(c_1')\pi(1 - \theta)$, and $c_1'' = \hat{\phi}(c_2')v$, where

$$\hat{\alpha}(c_1') = \frac{\alpha[1 - F_1(c_1')]}{1 - \alpha F_1(c_1')} \text{ and } \hat{\phi}(c_2') = \frac{1 - F_2(c_2')}{1 - \alpha F_2(c_2')}. \tag{29}$$

That is, the pioneer adopts the cut-off c_1' at time 2, and cut-off c_1'' at time 4, and the follower adopts cut-off c_2'. The search equilibrium is unique, but the patent policy has

[20]The same condition also guarantees $(1 - \theta)\pi \geq \hat{c}_1 = \alpha\pi[1 - \theta F_2(\hat{c}_2)]$, the cut-off obtained in section 3. That is, the additional time 4 expands the range of search cost the pioneer is willing to spend at time 2. Time 4, again, is irrelevant.

similar impact as before. An increase in θ will decrease c_1', and has a direct negative impact on c_2'. But a lower c_1' exerts a positive indirect effect on c_2' via the follower's belief $\hat{\alpha}$. The net change in c_2', then, has an opposite effect on the pioneer's second cut-off c_1'' through $\hat{\phi}$. The overall impact on the search performance, again, is ambiguous.

6 Conclusion

In this paper, I provided a simple theory about the patentability of basic inventions. I found both necessary conditions and a sufficient condition for the DPSM to be the optimal policy. Necessary conditions, i.e., the strong externality of the application and the DPSM's positive effect on the second stage innovation, are derived in a fairly general situation. The sufficient condition concerning the research capacities of the pioneer and follower at the second stage, however, is obtained under specific cost distributions. A future task is to test its robustness in more general settings.

A few other avenues for future research come to mind: multiple pioneers at the first stage innovation as in Denicolò (2000) and Aoki and Nagaoka (2004); secrecy protection to the basic invention; and the combination of the DPSM with other policy instruments, such as patent length and protection to second stage inventions, to name a few. A better understanding of the doctrine of the patentable subject matter would advance our knowledge on the optimal design of the patent system. This paper constitutes an early step.

Appendix: Proofs

A Proofs

□ Proposition 1

Proof. For the comparative static results, keep $v \equiv \alpha\pi$ constant and denote $\phi \equiv (1 - \hat{F}_1)/(1 - \alpha\hat{F}_1)$. Differentiate conditions (2) and (4):

$$d\hat{c}_1 + \theta v \hat{f}_2 d\hat{c}_2 = -v\hat{F}_2 d\theta \tag{30}$$

$$-(1-\theta)v\frac{\partial\phi}{\partial\hat{c}_1}d\hat{c}_1 + d\hat{c}_2 = -v\phi d\theta + (1-\theta)v\frac{\partial\phi}{\partial\alpha}d\alpha, \tag{31}$$

where

$$\frac{\partial \phi}{\partial \hat{c}_1} = -\frac{\hat{f}_1(1-\alpha)}{(1-\alpha \hat{F}_1)^2} \leq 0 \text{ and } \frac{\partial \phi}{\partial \alpha} = \frac{\hat{F}_1(1-\hat{F}_1)}{(1-\alpha \hat{F}_1)^2} > 0, \tag{32}$$

with $\hat{f}_i \equiv f(\hat{c}_i), i \in \{1,2\}$.

On the $c_1 - c_2$ plane, a stable equilibrium $(\hat{c}_1, \hat{c}_2)$ requires that the pioneer's reaction curve have a larger slope (in absolute value) than the follower's reaction curve. That is,

$$\left|\frac{\partial \hat{c}_2}{\partial \hat{c}_1}\right|_{\hat{c}_1} > \left|\frac{\partial \hat{c}_2}{\partial \hat{c}_1}\right|_{\hat{c}_2} \Leftrightarrow \triangle \equiv 1 + \theta(1-\theta)v^2 \hat{f}_2 \frac{\partial \phi}{\partial \hat{c}_1} = 1 - \theta(1-\theta)v^2 \frac{(1-\alpha)\hat{f}_1 \hat{f}_2}{(1-\alpha \hat{F}_1)^2} > 0. \tag{33}$$

Suppose that this is true.

By Cramer's rule, the impact of an exogenous change in θ are

$$\frac{d\hat{c}_1}{d\theta} = \frac{v}{\triangle}(v\theta\phi\hat{f}_2 - \hat{F}_2) \gtreqless 0 \text{ and } \frac{d\hat{c}_2}{d\theta} = \frac{v}{\triangle}[-(1-\theta)v\hat{F}_2 \frac{\partial \phi}{\partial \hat{c}_1} - \phi] \gtreqless 0. \tag{34}$$

When $\theta = 0$, $d\hat{c}_1/d\theta < 0$. When $\theta > 0$, if both terms are strictly positive,[21] then

$$v\theta\phi\hat{f}_2 > \hat{F}_2 > \frac{\phi}{-v(1-\theta)(\partial\phi/\partial\hat{c}_1)}, \tag{35}$$

which contradicts the requirement of $\triangle > 0$.

The overall effect of θ on $\hat{E}$ is

$$\frac{d\hat{E}}{d\theta} = (1-\hat{F}_2)\hat{f}_1 \frac{d\hat{c}_1}{d\theta} + (1-\hat{F}_1)\hat{f}_2 \frac{d\hat{c}_2}{d\theta} = (1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right). \tag{36}$$

The comparative static results with respect to α are

$$\frac{d\hat{c}_1}{d\alpha} = \frac{-(1-\theta)v}{\triangle} v\theta \hat{f}_2 \frac{\partial \phi}{\partial \alpha} < 0 \text{ and } \frac{d\hat{c}_2}{d\alpha} = \frac{(1-\theta)v}{\triangle}\frac{\partial \phi}{\partial \alpha} > 0. \tag{37}$$

The impact on the overall search performance is

$$\begin{aligned} \frac{d\hat{E}}{d\alpha} &= (1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\alpha} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\alpha}\right) \\ &= (1-\hat{F}_1)(1-\hat{F}_2)\frac{(1-\theta)v}{\triangle}\frac{\partial \phi}{\partial \alpha}\left(\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v\right). \end{aligned} \tag{38}$$

Under the DPSM, $\theta = 0$, $dE^*/d\alpha > 0$ for $\partial\phi/\partial\alpha > 0$. When $\theta > 0$, the sign of $d\hat{E}/d\alpha$ depends on

$$\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v. \tag{39}$$

[21] This excludes the case where $\alpha = 1$ and so $\partial\phi/\partial\hat{c}_1 = 0$.

Suppose that c_1 and c_2 are distributed uniformly over $[0, \gamma_1 v]$ and $[0, \gamma_2 v]$, respectively. In this case, given that $\hat{c}_1 = v(1 - \hat{F}_2\theta)$ and $\hat{c}_2 = v\phi(1-\theta)$,

$$\frac{\hat{f}_2}{1-\hat{F}_2} = \frac{1/(\gamma_2 v)}{1 - [\hat{c}_2/(\gamma_2 v)]} = \frac{1}{v[\gamma_2 - \phi(1-\theta)]}, \quad \frac{\hat{f}_1}{1-\hat{F}_1} = \frac{1}{v[\gamma_1 - 1 + \hat{F}_2\theta]}, \tag{40}$$

and so

$$\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v = \frac{1}{v}\left[\frac{1}{\gamma_2 - (1-\theta)\phi} - \frac{\theta}{\gamma_2(\gamma_1 - 1 + \hat{F}_2\theta)}\right]. \tag{41}$$

The sign of $d\hat{E}/d\alpha$ at $\theta > 0$ is determined by

$$\begin{aligned}\gamma_2(\gamma_1 - 1 + \hat{F}_2\theta) - \theta[\gamma_2 - (1-\theta)\phi] &= \gamma_2[\gamma_1 - (1+\theta)] + \gamma_2\theta\frac{\phi(1-\theta)v}{\gamma_2 v} + \phi\theta(1-\theta) \\ = \gamma_2[\gamma_1 - (1+\theta)] + 2\phi\theta(1-\theta) &\le \gamma_2[\gamma_1 - (1+\theta)] + \frac{1}{2},\end{aligned} \tag{42}$$

for $\phi \le 1$ and $\theta(1-\theta) \le 1/4$. Therefore, given any $\theta > 0$, $d\hat{E}/d\alpha < 0$ for $\gamma_1 < 1 + \theta$ and γ_2 large enough. *Q.E.D.*

□ Proposition 3

Proof. When all three cost components have uniform distributions, but different supports, the objective function is $F_0(\hat{U}_1)\alpha\hat{E} = [\alpha/(\gamma_0 v)]\hat{U}_1\hat{E}$. Finding the analytical solutions of $\hat{U}_1$ and $\hat{E}$, the relevant part of the objective function is

$$\hat{U}_1\hat{E} = \frac{v}{2\gamma_1^2}\left[1 + 2\theta\hat{F}_2(\gamma_1 - 1) + \theta^2\hat{F}_2^2\right]\left[1 + (\gamma_1 - 1)\hat{F}_2 - \theta\hat{F}_2(1-\hat{F}_2)\right], \tag{43}$$

where $\hat{F}_2 = \hat{c}_2/(\gamma_2 v) = \phi(1-\theta)/\gamma_2$. Ignoring $v/(2\gamma_1^2)$, the objective function is proportional to

$$\begin{aligned}1 + (\gamma_1 - 1)\hat{F}_2 + \theta\hat{F}_2\Big\{&[1 + (\gamma_1 - 1)\hat{F}_2][2(\gamma_1 - 1) + \theta\hat{F}_2] - \\ &(1-\hat{F}_2)[1 + 2(\gamma_1 - 1)\theta\hat{F}_2] - \theta^2\hat{F}_2^2(1-\hat{F}_2)\Big\}.\end{aligned} \tag{44}$$

The DPSM is optimal if the whole term is decreasing in θ, for all $\theta > 0$. According to the comparative static results in Proposition 1, under uniform distribution,

$$\frac{d\hat{c}_2}{d\theta} = -\frac{v}{\triangle}\left[\phi + (1-\theta)v\hat{F}_2\frac{\partial\phi}{\partial\hat{c}_1}\right] = -\phi\frac{v}{\triangle}\left[1 + \frac{v}{\gamma_2}(1-\theta)^2\frac{\partial\phi}{\partial\hat{c}_1}\right], \tag{45}$$

which is negative when γ_2 large enough, for $|\partial\phi/\partial\hat{c}_1| \leq \hat{f}_1/(1-\alpha) < \infty$ as long as $\alpha < 1$. (If $\alpha = 1$, then $\phi = 1$, a constant.) By $\hat{f}_1 = 1/(\gamma_1 v)$,

$$1 + \frac{v}{\gamma_2}(1-\theta)^2 \frac{\partial\phi}{\partial\theta} \geq 1 - \frac{v}{\gamma_2} \frac{(1-\theta)^2}{(1-\alpha)\gamma_1 v}. \tag{46}$$

When γ_2 is large enough such that $\gamma_2 > 1/[(1-\alpha)\gamma_1] \geq (1-\theta)^2/[(1-\alpha)\gamma_1]$, an increase in θ will reduce $\hat{c}_2$ and so $(\gamma_1 - 1)\hat{F}_2$.

Consider the whole term associated with $\theta\hat{F}_2$. It is negative for all θ as long as both $\gamma_1 - 1$ and $\hat{F}_2$ are small enough. For instance, if $\gamma_1 - 1$ is close to zero, it becomes $\theta\hat{F}_2 - (1-\hat{F}_2) - \theta^2\hat{F}_2^2(1-\hat{F}_2) < 2\hat{F}_2 - 1 - \theta^2\hat{F}_2^2(1-\hat{F}_2)$, by $\theta \leq \overline{\theta} < 1$. When γ_2 is large enough such that $\hat{F}_2 \leq v/(\gamma_2 v) \leq 1/2$, it is strictly negative. Or, if $\gamma_1 - 1 = 1/4$, then, since $\theta \leq \overline{\theta} < 1$,

$$\begin{aligned} &[1 + (\gamma_1 - 1)\hat{F}_2][2(\gamma_1 - 1) + \theta\hat{F}_2] - (1-\hat{F}_2)[1 + 2(\gamma_1 - 1)\theta\hat{F}_2] \\ = &-\frac{1}{2} + \frac{3\theta}{4}\hat{F}_2^2 + (\frac{9}{8} + \frac{\theta}{2})\hat{F}_2 < -\frac{1}{2} + \frac{3}{4}\hat{F}_2^2 + \frac{13}{8}\hat{F}_2, \end{aligned} \tag{47}$$

which is strictly negative if γ_2 is large enough such that $\hat{F}_2$ is smaller than, say, 1/8.

Q.E.D.

□ Proposition 4

Proof. The impact of the policy θ on the total surplus is

$$\frac{dS}{d\theta} = F_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta} + (1-\alpha\hat{F}_1)f_0(\hat{U}_1)\hat{U}_2\frac{d\hat{U}_1}{d\theta} - F_0(\hat{U}_1)\left[\alpha\hat{f}_1\hat{U}_2\frac{d\hat{c}_1}{d\theta} - (1-\alpha\hat{F}_1)\frac{d\hat{U}_2}{d\theta}\right]. \tag{48}$$

By the envelope theorem, the direct effect of θ on an inventor's choice variable can be ignored:

$$\frac{d\hat{U}_1}{d\theta} = \frac{\partial\hat{U}_1}{\partial\hat{c}_2}\frac{d\hat{c}_2}{d\theta} + \frac{\partial\hat{U}_1}{\partial\theta} = (1-\hat{F}_1)v\left[\hat{F}_2 + \theta\hat{f}_2\frac{d\hat{c}_2}{d\theta}\right], \tag{49}$$

$$\frac{d\hat{U}_2}{d\theta} = \frac{\partial\hat{U}_2}{\partial\hat{c}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\partial\hat{U}_2}{\partial\theta} = \hat{F}_2 v\left[(1-\theta)\frac{\partial\phi}{\partial\hat{c}_1}\frac{\partial\hat{c}_1}{\partial\theta} - \phi\right]. \tag{50}$$

At $\theta = 0$, $d\hat{U}_1/d\theta = (1-\hat{F}_1)\hat{F}_2 v > 0$. By the comparative static results in the proof of Proposition 1, $d\hat{c}_1/d\theta = -\hat{F}_2 v < 0$. When $\theta = 0$, the only negative term in $dS/d\theta$ is the one associated with ϕ in $d\hat{U}_2/d\theta$, i.e.,

$$F_0(\hat{U}_1)(1-\alpha\hat{F}_1)(-\hat{F}_2 v\phi) = -F_0(\hat{U}_1)(1-\hat{F}_1)\hat{F}_2 v, \tag{51}$$

which is exactly canceled by the first term in $dS/d\theta$, for

$$F_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta}|_{\theta=0} = F_0(\hat{U}_1)(1-\hat{F}_1)\hat{F}_2 v. \tag{52}$$

Therefore, $dS/d\theta > 0$ at $\theta = 0$. *Q.E.D.*

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