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A COMPARISON OF HOMOGENIZATION AND DIRECT TECHNIQUES FOR THE TREATMENT OF ROUGHNESS IN INCOMPRESSIBLE LUBRICATION

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ABSTRACT

Homogenization is a formal mathematical two-scale averaging process that can be applied to roughness problems and can replace previous heuristic averaging procedures which have sometimes led to ambiguous results. This procedure was previously mathematically developed and applied to compressible flow problems. The purpose of this paper is the development of a special form of Reynolds equation for such homogenized conditions applied to the incompressible Newtonian case. The equation allows the calculation of the operating characteristics of a contact by taking into account the local geometry of surfaces, while making a substantial improvement in computing time. The method allows for the study of rough surfaces, but requires considerably fewer calculated points than for traditional deterministic discretization methods.

INTRODUCTION

An understanding of the influence of roughness on the surface of machine elements during lubrication can contribute to an improvement of the performance of the device and an increase in the lifespan of the mechanism. In turn, proper prediction of the performance of a lubricated contact depends on a rigorous characterization of the involved surfaces and on a sufficiently accurate representation of the lubricant flow behavior. When the operating conditions are severe, i.e., the fluid films are very thin, the effect of roughness is all the more significant.

As is well-known, the development of the theory of lubrication for thin films first appeared in 1886 with the mathematical model established by Reynolds. The governing equation, which can be written in various forms, is a second order elliptic partial differential equation for the pressure, with the surface film shape entering as part of known variable coefficients. Reynolds equation does not usually admit to analytical solutions, and the complexity of surfaces due to roughness is one of many complications which require numerical approaches.

In this study, our contribution is to set up a new model which takes into account the surface roughness phenomena by using a new technique of calculation known as *homogenization*, which will be explained shortly. Taking into account the roughness of surfaces in the study of lubrication can be done considering the statistical parameters of roughness. There are a number of papers in the literature which could be characterized as using stochastic analysis [1-14] (considering some sort of averaged surface properties), and another group which uses deterministic analysis of a specific surface description [15-20]. For our concerns, we introduce a third methodology called homogenized analysis [21-23] which amounts to dividing the problem into two parts: a local problem (i.e., the roughness) and a homogenized problem for the global properties. This approach will be discussed to illustrate its advantages as well as its disadvantages compared to the others.

HOMOGENIZATION ANALYSIS

The approach of homogenization amounts to rewriting the problem as two others: a *local problem* and a *homogenized problem*. The coefficients of the homogenized problem depend on the solution of the local problem. The difficulty of this technique lies in the decoupling of the two problems, starting from the homogenized problem because of the presence of nonlinearities. The coefficients of the homogenized problem can be calculated only after treatment of the former.

In 2000, this technique was developed and was applied to the compressible Reynolds equation by Buscaglia and Jaï [21]. It was further revealed that this method is well adapted to the problems with an anisotropic roughness [22-23]. It is also a technique which does not require a very fine grid account for the effect of roughness. Roughness is taken into account during the calculation of the local problem, where less computing time is required compared to deterministic techniques. To see the advantages of this approach, Fig.1 is shown, based on the compressible flow analysis of Jaï and Bou-Saïd [23].

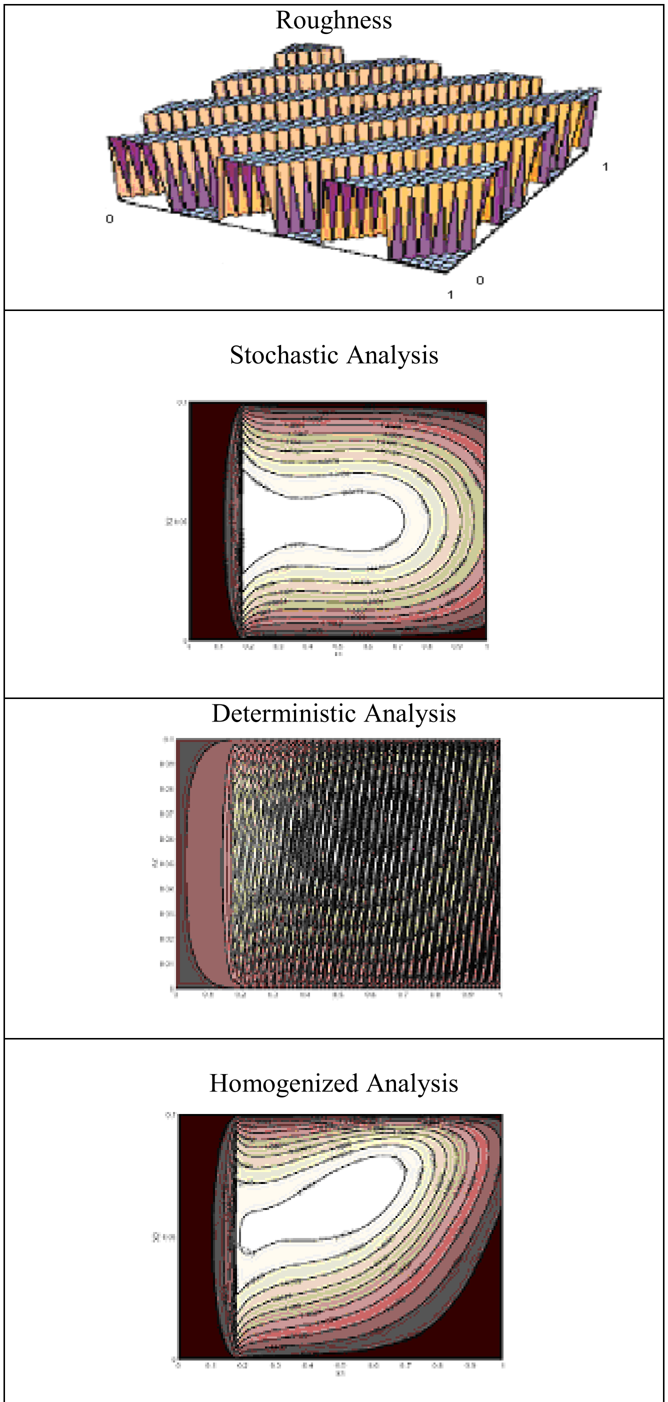


Figure 1. Comparison of roughness methodologies – dimensionless pressure

From this figure we can observe that the stochastic analysis does not capture the directional aspect of the roughness, i.e., the pressure is symmetric about the mid-plane. As for the deterministic analysis, it accurately portrays the pressure shape, but requires a very high number of discretization points to do so. On the other hand, the results obtained from homogenized analysis accurately capture the shape and magnitude of the pressure field but with considerably less numerical effort.

In the following section, this technique will be applied to a Newtonian incompressible flow analysis to obtain the homogenized Reynolds equation.

HOMOGENIZED REYNOLDS EQUATION

We begin with the incompressible Reynolds equation:

$$\nabla \cdot \left(h^3 \, \nabla p \right) = \Lambda \frac{\partial h}{\partial x_1} \tag{1}$$

where $p(x, y)$ is the pressure, $h(x, y)$ the film thickness, the viscosity is μ, sliding occurs only in the x- direction at speed V, and $\Lambda = 6\mu V$. The coordinates of the Cartesian reference system are $x = x_1$, $y = x_2$, $z = x_3$, and the domain is $\Omega = \left(x_1, x_2 \right)$.

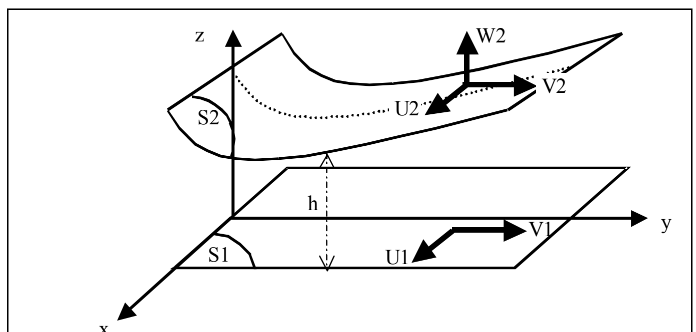


Figure 2. Schematic of film coordinates

We introduce the concept of local coordinates by writing:

$$h(x_1, x_2) = h_0(x_1, x_2) + \delta\left(\frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon}\right) \tag{2}$$

The symbol h_0 denotes the global film thickness and δ is the roughness contribution. The latter is a periodic function of period $\varepsilon = 1/n_r$, where n_r is the “roughness number” or the number of roughness cycles across the contact (in an order-of-magnitude sense) .

Let us introduce the concept of local variables setting $\left(\xi_1, \xi_2 \right) = \left(x_1/\varepsilon, x_2/\varepsilon \right)$ and make an asymptotic development of the pressure by writing:

$$p(x_1, x_2) = p_0(x_1, x_2) + \varepsilon p_1(x_1, x_2, \xi_1, \xi_2) + \varepsilon^2 p_2(x_1, x_2, \xi_1, \xi_2) + \ldots \tag{3}$$

where $p_1, p_2, \ldots$ are periodic functions of the variables $\left(\xi_1, \xi_2 \right)$.

We use the following rule of differentiation:

$$\frac{\partial}{\partial x_i}=\frac{\partial}{\partial x_i}+\frac{1}{\varepsilon}\frac{\partial}{\partial \xi_i} \qquad (4)$$

and substitute Eq. (3) in Eq. (1). We then gather terms of power of ε and obtain the following equations for powers ε^0, ε^1, and ε^2, respectively.

$$\frac{\partial}{\partial x_1}\left(h^3\left(\frac{\partial p_0}{\partial x_1}+\frac{\partial p_1}{\partial \xi_1}\right)\right)+\frac{\partial}{\partial x_2}\left(h^3\left(\frac{\partial p_0}{\partial x_2}+\frac{\partial p_1}{\partial \xi_2}\right)\right)=\Lambda\frac{\partial h}{\partial x_1}-\nabla_{\xi}\cdot\left(h^3\nabla_x p_1\right) \qquad (5)$$

$$\nabla_{\xi}\cdot\left(h^3\nabla_{\xi}p_1\right)=\Lambda\frac{\partial h}{\partial \xi_1}-\nabla_{\xi}h^3.\nabla_x p_0 \qquad (6)$$

$$\nabla_x\cdot\left(h^3\nabla_x p_1\right)=0 \qquad (7)$$

With ∇_x and ∇_{ξ}, respectively, equivalent to $\left(\partial/\partial x_1,\partial/\partial x_2\right)$ and $\left(\partial/\partial \xi_1,\partial/\partial \xi_2\right)$

To uncouple p_0 and p_1 the following *local* problems are considered in terms of the roughness coordinates and the auxiliary variables w_1, w_2, and w_3.

$$-\nabla_{\xi}\cdot\left(h^3\nabla_{\xi}w_1\right)=\frac{\partial h^3}{\partial \xi_1}$$
$$-\nabla_{\xi}\cdot\left(h^3\nabla_{\xi}w_2\right)=\frac{\partial h^3}{\partial \xi_2}$$
$$\nabla_{\xi}.\left(h^3\nabla_{\xi}w_3\right)=\Lambda\frac{\partial h}{\partial \xi_1} \qquad (8)$$

The boundary conditions for the local problems are that w_1, w_2, and w_3 equal zero on the boundary and are periodic functions. The following relation is postulated to exist between p_1 w_1, w_2, and w_3 and the partial derivatives of p_0:

$$p_1=w_1\frac{\partial p_0}{\partial x_1}+w_2\frac{\partial p_0}{\partial x_2}+w_3+C(x_1,x_2) \qquad (9)$$

By substituting this expression in Eq. (5) and integrating with respect to ξ, we obtain the homogenized Reynolds equation:

$-\nabla\cdot\left(\left[A\right]\nabla p_0\right)=\nabla\cdot\left[\theta\right]$ in the domain Ω, $p_0 = 0$ on $\partial\Omega$ (10)

with

$$\left[A\right]=\begin{pmatrix}\int_{\Xi}h^3\left(1+\frac{\partial w_1}{\partial \xi_1}\right)d\xi & \int_{\Xi}h^3\left(\frac{\partial w_2}{\partial \xi_1}\right)d\xi \\ \int_{\Xi}h^3\left(\frac{\partial w_1}{\partial \xi_2}\right)d\xi & \int_{\Xi}h^3\left(1+\frac{\partial w_2}{\partial \xi_2}\right)d\xi\end{pmatrix}$$

$$\left[\theta\right]=\begin{pmatrix}\int_{\Xi}\left(h^3\frac{\partial w_3}{\partial \xi_1}-\Lambda h\right)d\xi \\ \int_{\Xi}\left(h^3\frac{\partial w_3}{\partial \xi_2}\right)d\xi\end{pmatrix} \qquad (11)$$

The notation of Eq. (11) is that we integrate the roughness variables $\left(\xi_1,\xi_2\right)$ over their domain Ξ. Thus, one obtains the local problems [Eqs. (8)-(9)] and the homogenized problem [Eqs. (10)-(11)].

These problems do not have analytical solutions, thus it is necessary to use numerical techniques.

GEOMETRY OF THE CONTACT

Before carrying out the calculation of the homogenized pressure, it is necessary to define the geometry of the contact. The height of film can be written as :

$$h=h_0\left(x_1\right)+\delta\left(\xi_1,\xi_2\right) \qquad (12)$$

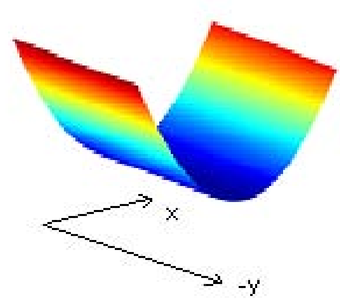
In turn, for demonstration, we use a parabolic cylinder contact

$$h_0=h_{\min}+\frac{x^2}{2R} \qquad (13)$$

This film shape is the parabolic cylinder approximation to a highly loaded journal bearing contact. All the characteristics of the contact have been non-dimensionalized and we use the following parameter values: $h_{\min}=1$, $\mu=1$, $V=1$, $R=1$, $\alpha=0.3\,h_{\min}$ roughness amplitude, $L_x/L_y = 1$ (the "unwrapped" contact is rectangular). The roughness configurations have been obtained using the following roughness description:

$$\delta=\alpha\sin\left(2\pi\frac{L_x\xi_1+L_y\xi_2}{L_x+L_y}\right)=\alpha\sin\left(2\pi n_r\frac{L_x x+L_y y}{L_x+L_y}\right) \qquad (14)$$

In Eq. (14) above, $L_x = 1$ and $L_y = 0$ represents transverse roughness, $L_x = 0$ and $L_y = 1$ represents longitudinal roughness, with anisotropic roughness being characterized by intermediate values.



global film thickness

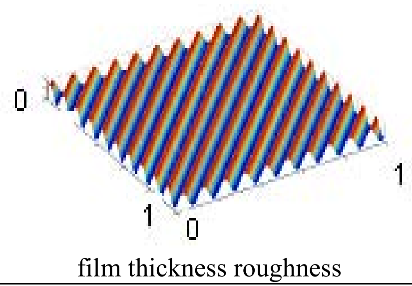


Figure 3. Geometry of global film thickness and roughness

RESULTS

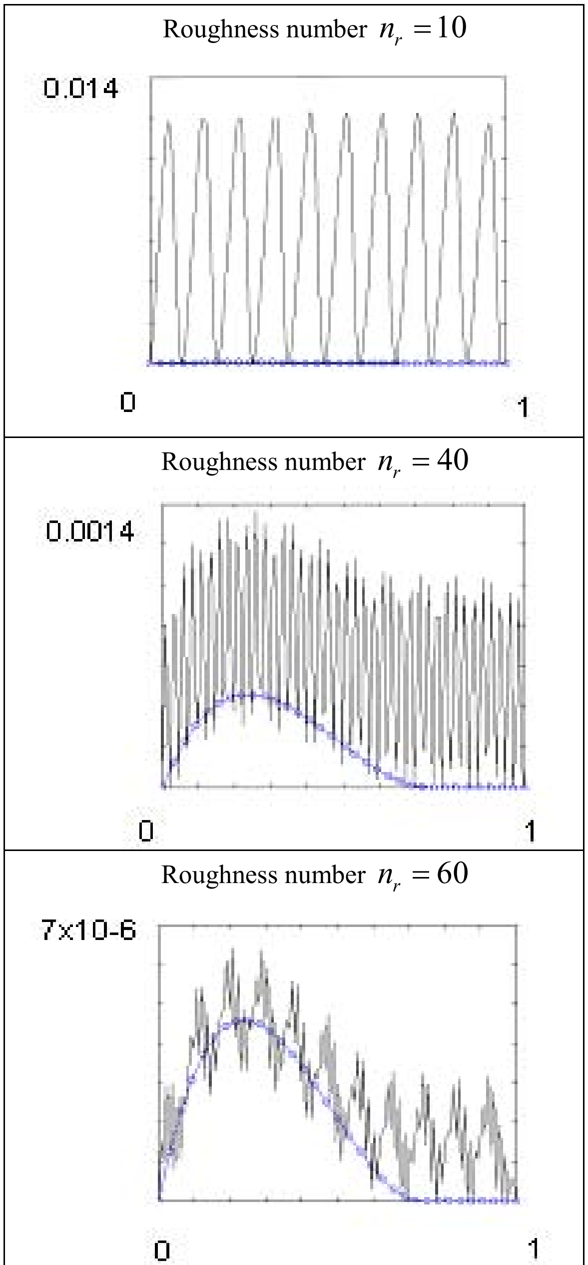


Figure 4. Dimensionless pressure profile variation with roughness number

We can see in Fig. 4 that the deterministic solution becomes erratic when the number of roughness cycles becomes significant. This is due to numerical error. Thus to have a well behaved deterministic solution, it is necessary to increase the number of discretization points. However, we notice that the homogenized solution remains insensitive to this variation in roughness number.

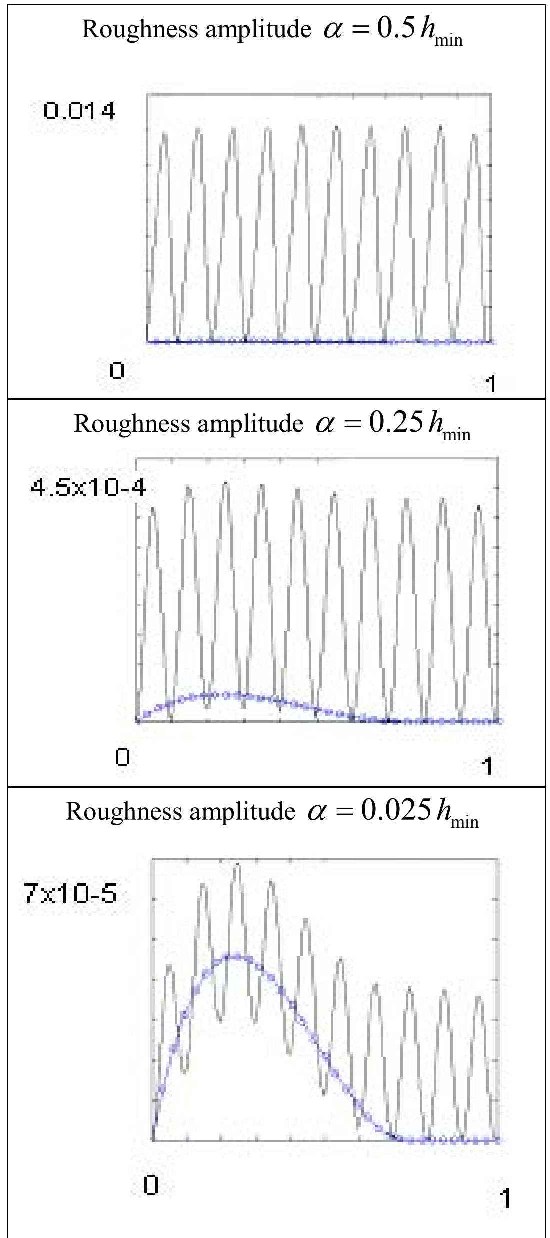


Figure 5. Dimensionless pressure profile variation with roughness amplitude

We notice that for low roughness number, the deterministic solution remains far away from the homogenized solution, see Fig. 5. A reduction in the height of roughness peaks allows the two solutions to converge. Thus, this makes it possible to validate the model in the case of low amplitudes of roughness.

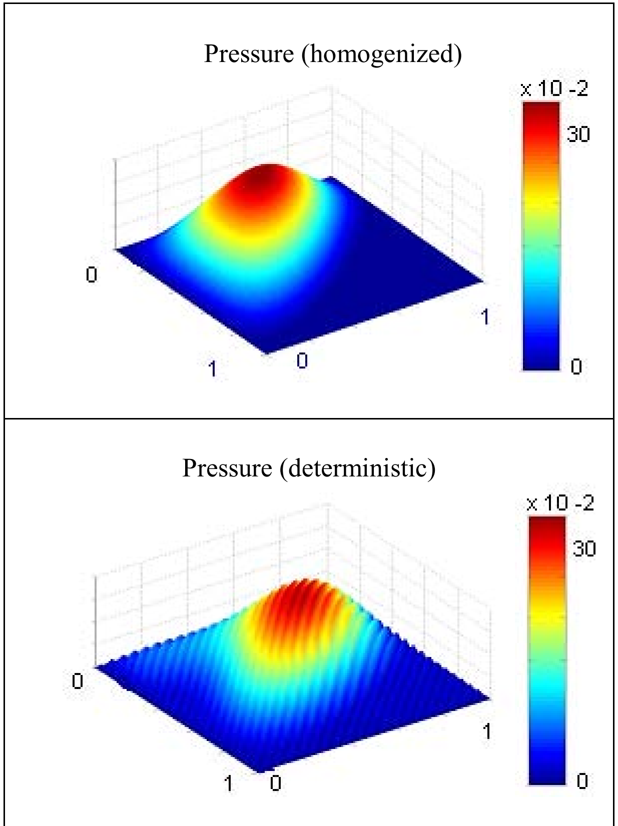


Figure 6. Dimensionless pressure field – homogenized and deterministic calculations

We observe the three-dimensional pressure field for $n_r = 40$ and $\alpha = 0.3\,h_{\min}$ in the homogenized and deterministic cases. We note that the direction of roughness is evident in the homogenized pressure field, i.e., the pressure field is skewed due to the roughness orientation, which is expected in a physical sense.

CONCLUSIONS

After analysis of the various results, we can draw several conclusions on this method of calculation. Homogenization makes it possible to obtain pressure fields for rough surfaces, using far fewer calculation points than for deterministic methods, while capturing important non-symmetric features oftern missed by heuristic averaging. We note that the higher the period of roughness, the more the deterministic solution approaches the homogenized solution, which makes it possible to validate the model under the conditions where the number of roughness cycles is large.

For small roughness amplitude (compared to the minimum height of the film) and a significant number of roughness peaks, the homogenized analysis remains effective regardless of the type of roughness. Under these conditions this technique gives a good physical insight as to the distribution of pressure on the surface of contact. For anisotropic roughness, the technique of the homogenization is essential to give realistic information on the amplitude and the direction of roughness, contrary to the traditional (stochastic) approaches which prove to be defective in this case.

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