Technological Revolutions and Debt Hangovers: Is There a Link?

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Abstract

The Great Recession, the Great Depression, and the Japanese slump of the 1990s were all preceded by periods of major technological innovation. In an attempt to understand these facts, we estimate a model with noisy news about the future. We find that beliefs about long run income adjust with an important delay to shifts in trend productivity. This delay, together with estimated shifts in the trend of productivity in the three cases, are able to tell a common and simple story for the observed dynamics of productivity and consumption on a 20 to 25 year window. Our analysis highlights the advantages of a look at this data from the point of view of the medium run.

Keywords: Aggregate productivity, consumption-saving decision, household leverage, learning, household income dynamics.

JEL codes: E21, E27, E32, N10.

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"Shifts in the economy are rarely forecast and often not fully recognized until they have been underway for some time."

Larry Summers, *Financial Times, March 25th, 2012*

1 Introduction

A medium-run look at the three most important private-debt recessions in developed economies reveals that they were all preceded by periods of great technological innovation and economic transformation. Specifically, the recent Great Recession in the United States was preceded by a technological revolution, happening in the late 1990s, related to Information Technology (henceforth IT) (Caselli 1999; Hobijn and Jovanovic 2001; Pastor and Veronesi 2009). Similarly, the Japanese slump of the 1990s was preceded by a period of unprecedented industrial innovation in the 1980s. During this period, Japanese corporations developed and exported several electronic products that were massively consumed in many parts of the world, for instance the walkman, the VHS, and the Betamax. We view this period as containing the elements of a technological revolution, which in the particular case was mostly concentrated in Japan. Finally, before the Great Depression, the United States witnessed the so-called 2nd Industrial Revolution, happening at the beginning of the 20th century. Two key general purpose technologies here were the combustion engine and electricity: The combustion engine made possible the mass production of cars for the American household by the Ford Motor Company, starting in 1909, and about 70% of household and corporate electrification happened approximately between 1910 and 1925.

Motivated by these facts, we write and estimate a model in order to understand whether – when seen from a medium-run perspective – technological revolutions are indeed capable of generating major macroeconomic downturns.[1] Our goal here is limited. We use a simple and tractable framework in which technological revolutions imply particular movements in productivity, and these shifts in productivity are capable of generating aggregate consumption dynamics that finish on slumps.

Our model has two main ingredients. The first one is the presence of both permanent and transitory shocks to productivity (Aguiar and Gopinath 2007;

[1] To the best of our knowledge, Perez (2009) was the first to suggest a potential link between the IT revolution and the credit developments of the 2000s.

Justiniano, Primiceri, and Tambalotti 2010). As in previous work, we use permanent shocks to generate shifts in trend productivity that imply large movements in consumption. Different from previous work, we derive a closed-form solution to analyze these effects, and we apply this solution to the conditional dynamics of consumption around these so-called technological revolutions. The second one is the presence of news about the future (Beaudry and Portier 2006; Jaimovich and Rebelo 2009; Schmitt-Grohe and Uribe 2012). The idea in this literature is that information about future developments of productivity is able to generate important shifts in contemporaneous variables, in our case consumption. The novelty in our framework is the presence of noisy news (Blanchard, L'Huillier, and Lorenzoni 2013), together with agents' rational reaction to this noise in the news, and our focus on the effect of permanent shocks to productivity.

In this exercise, our main object of interest are the beliefs about long-run income held by a representative consumer, or "beliefs about the long run". Our goal is to extract a reliable measure of these beliefs from the observation of aggregate time series data. We then use our model to characterize the associated behavior of consumption and household debt. This focus on the dynamics of household debt connects our work to two recent important contributions analyzing the recent leveraging and deleveraging of U.S. households (Midrigan and Philippon 2011; Justiniano, Primiceri, and Tambalotti 2012). In some sense, our goal is less ambitious because we use a simpler model and solve it using a first order loglinear approximation. Therefore, we do not intend to provide a full quantitative account of the extent of leveraging and deleveraging. At the same time, our approach has two advantages: first, it allows for a straightforward application to the cases of Japan and the Great Depression, and second, it allows for enough tractability to elicit a particular mechanism: the evolution of beliefs about the long run. Accordingly, we check the validity of our mechanism using out-of-sample information from survey evidence.

When we look at the data through the lens of the model, we find that the joint dynamics of productivity and beliefs about long-run income are characterized by a slow moving cycle that takes between 20 to 25 years to be completed. This cycle can be summarized by the following sequence of events. First, there is an increase in the growth rates of aggregate productivity, probably caused by the technological boom. Second, this pickup of productivity generates a (rational) increase in beliefs about long-run income. This increase in beliefs increases consumption. However, due to the noise in the consumer's information, this "wave of optimism" arrives with a significant delay, and is highly persistent. Therefore, the optimism tends to coincide with the slowdown of productivity brought by the end of the revolution. In our model economy, income is determined by productivity, and therefore the decline of productivity produces a decline of income. High spending combined with low income imply a gap that is translated into a large accumulation of debt. Third, eventually the consumer receives enough information to realize that long-run productivity has declined. He decreases his beliefs about long-run income, and a deleveraging process starts. The deleverage is lengthy because at this point income has declined.

In our procedure, we intend to focus on the medium-run dynamics of aggregate consumption. To this end, we first estimate our model through standard methods and then use the variance decomposition of consumption at different horizons in order to gauge which of the shocks present in the model explain its variability on the medium run. We define the "medium run" as an horizon of about 5 years or more after the impulse of a particular shock.[2] This decomposition indicates that most of the variability of consumption in the medium run is explained by shocks to the trend-growth of productivity, or permanent productivity shocks. This result holds when estimating the model for the three cases mentioned above, and is robust to different sample spans, different specifications, and different observables in the data.

Given their importance to understand the medium-run dynamics of consumption, we estimate these shocks using a Kalman smoother. We then feed the estimated permanent shocks into the model in order to simulate the associated beliefs about the long run, which we label "model-predicted beliefs about the long-run". We then perform an out-of-sample check of these model-predicted beliefs by comparing them to survey evidence for the U.S. economy, 1994–2010 (Hoffmann, Krause, and Laubach 2011). Notice that in this exercise we shut down all other shocks in the model, thereby performing a tough check of our hypothesis that medium-run dynamics matter for the observed

[2]In the literature, little attention has been given to the study of medium-term business cycle dynamics, as most of the literature has focuses on the short term (see, for instance, Baxter and King 1999, Christiano, Eichenbaum, and Evans 2005, or Smets and Wouters 2007). A noticeable exception is the study by Comin and Gertler (2006), who, using a slightly different notion of medium term, use a perfect information model to analyze unconditional facts of the data. Pintus and Suda (2013) also stress the importance of gradual learning to understand the recent recession in the U.S.

evolution of beliefs about the long-run.[3] We find that according to both the model-predicted beliefs about the long-run and the survey, the U.S. consumer was most optimistic about his long-run income between 2000 and 2005.

In order to shed light on the properties of the data that deliver the shape of the model-predicted beliefs about the long run, we also present some reduced-form evidence by focusing on the observed dynamics of the ratio of productivity-to-consumption. We argue that – within our estimated model – this ratio is particularly informative for the estimation of the permanent productivity shocks. Indeed, in the model, productivity determines income, and beliefs about the long run determine consumption. Therefore, given the variance decomposition of consumption, the joint medium-run evolution of these two variables should be determined by permanent technology shocks. Accordingly, we find that this ratio has a similar medium-run shape in the three cases.[4]

Altogether, the exercises we perform deliver three main substantive results. First, there is a significant delay in the adjustment of beliefs about the long run. The reason is the estimated amount of noise in the information consumers receive about future income, which is quite large. We quantify this delay by computing the half-life of beliefs after an impulse to the trend-growth of productivity in our estimated model. For instance, in the case of the U.S., 1990–2013, this half-life is 5.25 years, and this figure is of a similar order of magnitude in the other two cases. Second, a simulation of household debt using the estimated permanent shocks indicates that the accumulation of debt was large, and the deleverage slow. The accumulation is large because households fail to immediately recognize the slowdown of productivity. Thus, income declines while households are still optimistic, and this implies a large build-up of debt. The deleverage is slow because income is low when households have decreased their expectations and have started to reduce their debt. Third, the trend of the productivity-to-consumption ratio computed using an HP-filter ($\lambda = 800$) has the shape of an "up-and-down wave": first increases, then decreases, and then again increases, reverting back to its value at the start of the cycle. Although the whole length of this cycle varies from case to case, it seems to be of 20 to

[3]Our empirical exercise is related to the theoretical contribution by Boz (2009), in which optimism following a "miracle" performance can lead to a downturn. Independently, Piazza (forthcoming) and Pintus and Wen (2013) model a similar interaction between development, demand, and credit.

[4]This historical stylized fact is akin to the well-known work by Reinhart and Rogoff (2008,2011). However, we do not seek to address the abrupt financial meltdowns emphasized there. Instead, we are interested in the medium-run dynamics of consumption and productivity, and their association with movements of debt.

30 years. As argued below, in either the "no-news" or perfect foresight benchmarks, the ratio would have a different shape. Therefore, our model is a way of accommodating this particular feature of the data. To sum up, we find that a medium-run look at the dynamics of productivity and consumption is useful for understanding the build-up of debt and the deleveraging in the three cases.[5]

The rest of the paper proceeds as follows. We first present the model (Section 2). We then discuss its estimation and present these results (Section 3). Here, we generate the model-predicted beliefs about the long run, and perform the out-of-sample check. We then turn to the properties of the productivity-to-consumption ratio in the data (Section 4). Afterwards we use our model to analyze the implication of our results of household debt (Section 5). We then conclude (Section 6). The Appendix contains several proofs and a detailed description of our data. The Online Appendix presents the results of a number of empirical robustness exercises.

2 The Model

2.1 Productivity Process and Information Structure

We model an open economy similar to Aguiar and Gopinath (2007), adding a "news and noise" information structure (Blanchard, L'Huillier, and Lorenzoni 2013, henceforth BLL).[6] Specifically, productivity a_t (in logs) is the sum of two components, permanent, x_t, and transitory z_t:

$$a_t = x_t + z_t \quad . \tag{1}$$

Consumers do not observe these components separately. The permanent component follows the unit root process

$$\Delta x_t = \rho \Delta x_{t-1} + \varepsilon_t \quad . \tag{2}$$

[5]Reinhart and Reinhart (2010) also take a medium-run look at a number of aggregate indicators as unemployment, housing prices, inflation and credit. We differ in combining both structural and reduced form approaches, and in our focus on productivity, consumption, and private debt.

[6]Boz, Daude, and Durdu (2011) use a similar framework. We simplify it further by removing labor supply and capital. Those extra ingredients do not change anything to our analysis, as we explain below (p. 12, footnote 14).

The transitory component follows the stationary process

$$z_t = \rho z_{t-1} + \eta_t \quad . \tag{3}$$

The coefficient ρ is in $[0, 1)$, and ε_t and η_t are i.i.d. normal shocks with variances σ_ε^2 and σ_η^2. Similar to BLL, we assume that these variances satisfy

$$\rho\sigma_\varepsilon^2 = (1-\rho)^2 \sigma_\eta^2 \quad , \tag{4}$$

which implies that the univariate process for a_t is a random walk, that is

$$\mathbb{E}[a_{t+1}|a_t, a_{t-1}, ...] = a_t \quad . \tag{5}$$

This assumption is analytically convenient and broadly in line with productivity data. To see why this property holds, note first that the implication is immediate when $\rho = \sigma_\eta = 0$. Consider next the case in which ρ is positive and both variances are positive. An agent who observes a productivity increase at time t can attribute it to an ε_t shock and forecast future productivity growth or to an η_t shock and forecast mean reversion. When (4) is satisfied, these two considerations exactly balance out and expected future productivity is equal to current productivity.[7]

Consumers have access to an additional source of information, as they observe a noisy signal about the permanent component of productivity. The signal is given by

$$s_t = x_t + \nu_t \quad , \tag{6}$$

where ν_t is i.i.d. normal with variance σ_ν^2.

We think of ε_t as the "news" shock, because it builds up gradually, has large, permanent effects on productivity, and delivers (noisy) information about the future through the signal. We think of ν_t as the "noise" shock. Our focus throughout the paper is on the dynamics implied by ε_t.[8]

[7] See BLL for the proof.

[8] Related and important contributions on the impact of noise, or more broadly, changes in expectations are by Angeletos and La'O (2009,2013). As it will become clear, our noisy news approach is different, especially because it captures, once the model is estimated, medium-term fluctuations. Forni, Gambetti, Lippi, and Sala (2013b) also use the term "noisy news", but they use a different specification of the information structure. See also Forni et al. (2013a).

2.1.1 Slow Adjustment of Beliefs and Technological Revolutions

Here we focus on an important property of the signal extraction problem for our purposes.

First, we borrow the idea from Greenwood and Jovanovic (1999) (among others) that "technological revolutions come in waves". According to this idea, the start of a technological revolution should create an increase in the growth rate of permanent productivity Δx_t – away from the old, deterministic trend – and the end of a technological revolution should create a decrease in the growth rate of permanent productivity Δx_t – away from the new trend.

In this setup agents optimally form beliefs about the permanent component x_t using a Kalman filter. Then, they form beliefs about the future path of x_t. The following definition is useful to make these ideas precise.

Definition 1 (BLR) *Given information at time t, the agent's best estimate of the productivity in the future is*

$$\lim_{\tau \longrightarrow \infty} \mathbb{E}_t \left[a_{t+\tau} \right] = \frac{\mathbb{E}_t \left[x_t - \rho x_{t-1} \right]}{1-\rho} = \frac{x_{t|t} - \rho x_{t-1|t}}{1-\rho}, \tag{7}$$

*where $x_{\tau|t}$ denotes the conditional expectation $\mathbb{E}_t[x_\tau]$ of x_τ on information available at time t. We call the estimate of long-run productivity, **beliefs about the long run (BLR)** and denote it by $x_{t+\infty|t}$.*

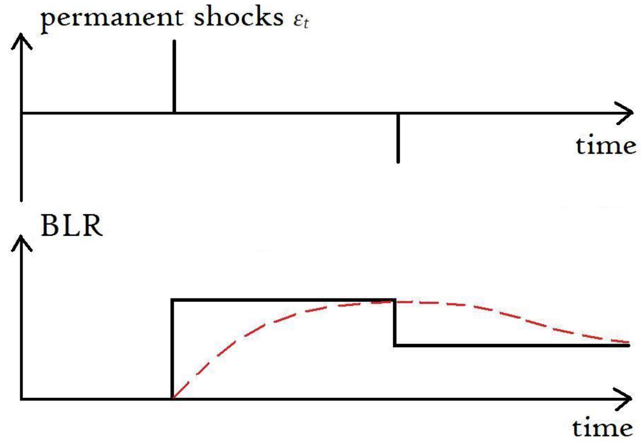
The first equality is proved in the Appendix and the second equality comes directly from the definition of $x_{\tau|t}$. In Proposition 1 below we show that these BLR will determine consumption.

Because of noisy information, agents will be slow to adjust their beliefs $x_{t+\infty|t}$. In particular, they will be slow to adjust their beliefs following an impulse to Δx_t.

Definition 2 (Delayed adjustment of beliefs) *After a permanent shock, $\epsilon_t = 1$, under perfect information, BLR jumps immediately to the long-run level $1/(1-\rho)$ and stays at that level in the absence of future shocks. However, under imperfect information, it takes time for the BLR to reach the long-run level. We define the **delay** by the time it takes BLR to reach half of the long-run level.*

For illustrative purposes, consider the example of a technological revolution given by Figure 1. The upper panel of the figure plots permanent shocks, and

Figure 1: An Example of a Technological Revolution and Beliefs



the lower panel plots the implied long run levels of the permanent component. The technological revolution initially increases the long run level of the permanent component, and then decreases it in off-trend terms.[9] As we will show below, our structural estimations, which allows to estimate permanent shocks, will give support to this view of technological revolutions for the three cases we consider.[10]

The figure also sketches the evolution of beliefs around this technological revolution in the lower panel. Beliefs under perfect information ($\sigma = 0$) are plotted with a solid line, beliefs under noisy information ($\sigma > 0$ but finite) are plotted with a dashed line. In the later case, agents slowly learn about the increase in the long-run level of the component, and about the subsequent decrease. Overall, the adjustment of beliefs lags the actual changes in the permanent component, which is the key property needed for our results.

One may wonder why agents in this economy do not anticipate the second (negative) shock when learning about the first one. We think about these technological revolutions as happening rarely, for instance once every century. Accordingly, in our simple specification of the evolution of technology agents are "surprised" by the slowdown of aggregate productivity implied by the second

[9] In this example, the technological revolution has a total positive effect on the off-trend level of the long-run permanent component.

[10] Closely related is an interesting type of stochastic processes considered by a number of papers (Barbarino and Jovanovic 2007; Zeira 1987; Zeira 1999; Boldrin and Levine 2001; Li 2007). The approach adopted here is similar to the one used by Christiano, Ilut, Motto, and Rostagno (2008), and has the advantage of being compatible with linear, Kalman filter, learning, and therefore it is more suited for statistical inference.

shock. To justify this assumption, in the Appendix we build a model of an economy with two regimes. In the first regime, the behavior of productivity (including the permanent and temporary components) behaves exactly in the same way as we assumed in this section. In the second regime - the technological revolution regime - first there is an increase in permanent component, i.e. , $\epsilon_t = 1$, and then the technological revolution ends after a random (or deterministic) time $T > t$, i.e. $\epsilon_T = -\frac{1}{2}$, and the economy switches back to the first regime. Starting from the first regime, in each period, there is a probability γ that the economy moves to the second regime. We show that as long as the regime is unknown to agents in the economy and they only observe a_t as well as the signal s_t, their estimate of the permanent component remains arbitrarily close to their estimate presented in this section as long as γ is small. To sum up, the Kalman filter used in our baseline model is a good approximation of learning in the regime switching model as long as technological revolutions are rare.[11]

2.2 Consumption, Production and Net Exports

We now describe the rest of the model. A representative consumer maximizes

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \log C_t\right]$$

where $\mathbb{E}[\,\cdot\,]$ is the expectation operator conditional on information available contemporaneously. The maximization is subject to

$$C_t + B_{t-1} = Y_t + Q_t B_t \quad , \tag{8}$$

where B_t is the external debt of the country, Q_t is the price of this debt, and Y_t is the output of the country.

Output is produced using only labor through the linear production function:

$$Y_t = A_t N \quad , \tag{9}$$

We abstract from fluctuations on employment, i.e. the consumer supplies labor N inelastically.[12] The price of debt is sensitive to the level of outstanding

[11] Bianchi and Melosi (2013) offers a more comprehensive treatment of learning under Markov switching.

[12] This approach is, to some extent, justified by our focus on the medium-run. However, we have used labor supply in previous versions of this model and obtained very similar results. We comment more on this feature of the model below (p. 12, footnote 14).

debt, taking the form used by Schmitt-Grohe and Uribe (2003), and Aguiar and Gopinath (2007), among others:

$$\frac{1}{Q_t} = R_t = R^* + \psi \left\{ e^{\frac{B_t}{Y_t} - b} - 1 \right\} \quad , \tag{10}$$

where b represents the steady state level of the debt-to-output ratio.[13]

The only first-order conditions from the optimization problem of the consumer is:

$$\frac{1}{C_t} = \beta R_t \mathbb{E}_t \left[\frac{1}{C_{t+1}} \right] \quad , \tag{11}$$

We define four endogenous variables c_t, nx_t, r_t, and b_t as follows:

$$c_t \equiv \log\left(C_t/A_t\right) - \log\left(C/A\right) \quad ,$$

$$r_t \equiv \log R_t \quad ,$$

and

$$b_t \equiv \frac{B_t}{Y_t} - b \quad ,$$

$$nx_t \equiv \frac{NX_t}{Y_t} - \frac{NX}{Y} \quad .$$

In the definition of c_t, we need to use the ratio of C_t over A_t to ensure stationarity. Moreover, exogenous productivity is

$$a_t = \log\left(A_t/A\right) \quad .$$

In order to examine the dynamics of consumption, we also define another variable which is the logdeviation of consumption:

$$\widehat{c}_t = c_t + a_t.$$

In the Online Appendix, we derive the loglinearization of the equilibrium. This equilibrium is given by the equations for the shock processes (1), (2), and (3), and other four equations:

[13] It is straightforward to generalize our model to a two-country economy, and our main results do not change in that case. See the discussion in Appendix D.

$$c_t = -r_t + \mathbb{E}_t[c_{t+1} + \Delta a_{t+1}] \quad , \tag{12}$$

$$r_t = \psi \cdot b_t, \tag{13}$$

$$c_t + \frac{1}{C/Y} nx_t = 0, \tag{14}$$

$$nx_t = b_{t-1} - \beta b_t + \frac{1 - C/Y}{1 - \beta} \left(-\Delta a_t + \beta r_t \right). \tag{15}$$

This simple model admits a closed-form solution. It is presented in Appendix C.

To illustrate the effect of a permanent shock on the endogenous variables of this system, we parameterize the model as follows. The discount factor β is set at 0.99. The elasticity of the interest rate, ψ, is set to low value, 0.0010, following previous literature (Neumeyer and Perri 2004; Schmitt-Grohe and Uribe 2003; Aguiar and Gopinath 2007). Under this common parametrization, BLR is the main driver of consumption, as established by the following proposition.

Proposition 1 *As $\beta \longrightarrow 1$, and $\psi/(1-\beta) \longrightarrow 0$, consumption is only a function of BLR. Specifically,*

$$\widehat{c}_t = \frac{1}{C/Y} x_{t+\infty|t} \quad .$$

The proof is in the Appendix.[14]

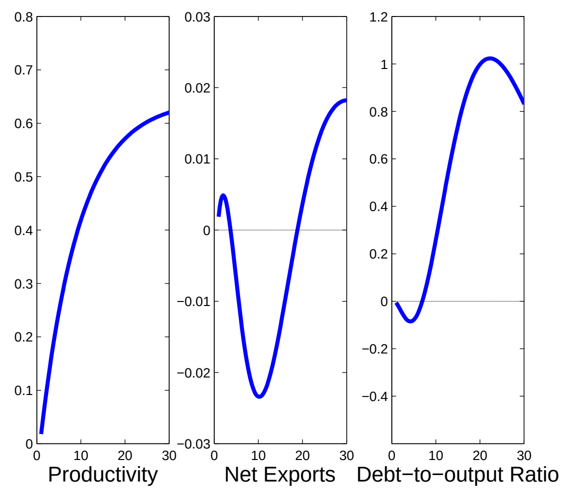
The rest of the parameters is taken from the estimation of the model for the United States (1990–2013) below. The parameter ρ is set at 0.97, implying slowly building permanent shocks and slowly decaying transitory shocks. The standard deviation of productivity growth, σ_a, is set at 0.64. These values for ρ and σ_a yield standard deviations of the two technology shocks, σ_ε and σ_η, equal to 0.02% and 0.63%, respectively. The standard deviation of the noise shock, σ_ν, is set to 7.39%, implying a fairly noisy signal.

Figure 2 shows a simulation of the model for these parameter values. The

[14] In Cao and L'Huillier (2014) (available on our webpages) we prove a version of this theorem for a more general model that includes labor supply and capital. Therefore, for the standard parametrization in the literature, including those ingredients in our framework does not change our results. This simplification of our model and our focus on consumption finds empirical support in the work by Mian and Sufi (2012) and Mian, Rao, and Sufi (2013).

figure shows the responses of productivity a_t, net exports nx_t (or equivalently in this model, the current account), and external debt b_t, to a one-standard deviation increase in ε_t (the permanent technology or "news" shock). The time unit on the x-axis is four quarters (one year). The scale of productivity is relative percentage deviations from steady state. The scale of both net exports and the debt-to-output ratio are absolute percentage deviation from the steady state value of net exports-to-output, NX/Y, and debt-to-output, b.

Figure 2: Impulse Response Functions to Permanent Technology Shock



Notes: Units in the vertical axis are percentage relative deviations from steady state in the case of productivity, and absolute percentage deviations from steady state in the case of net exports and the debt-to-output ratio. The time unit on the x-axis is four quarters (one year).

In response to a one-standard-deviation increase in ε_t, the permanent technology shock, productivity increases slightly on impact, and then gradually continues to increase until it reaches a new long-run level. This sustained increase is slow; in fact, half of the productivity increases are reached only after 6 years. Initially, net exports rise, mainly because productivity increases faster than beliefs about long run productivity. This is a reflection of the high amount of noise in this simulation. After 3 years net exports fall, because agents have received enough "news" and a standard income effect kicks in. This is translated into a sharp accumulation of external debt. In the long run, productivity reaches a new level (at 0.63) and net exports and the debt-to-output ratio go back to zero.

3 Estimation

In this section we first explain how we estimate the model. We then show the results for the Great Recession, and we perform an out-of-sample check of this estimation by comparing the estimated model-predicted BLR to survey evidence. We then show the results for Japan and the Great Depression. The Online Appendix presents several other estimations in order to assess the robustness of our results.

3.1 Data Sets and Estimation Procedure

Data. Our data set includes series on productivity, TFP, consumption, and net exports. We used quarterly data. The series for the Great Recession were obtained from the Bureau of Economic Analysis and the Bureau of Labor Statistics. The series for Japan were obtained from the OECD.

In the case of the Great Depression, we have data for the components of GDP from the Gordon-Krenn data set.[15] Gordon and Krenn (2010) used the Chow and Lin (1971) method for interpolating annual national accounts series and obtain cyclical variation at quarterly frequency, thereby obtaining an estimated series for GDP components. In order to obtain a series for labor productivity, we obtained an estimate for GDP from the Gordon-Krenn data set, and we used the Kendrick (1961, Appendix A, Table XXIII, 2nd column) data set for employment, using a linear interpolation out of the annual series.

Procedure. For our baseline estimations, we fix β and ψ. The discount factor β is set at 0.99. ψ is set to low value, 0.0010, following previous literature (Neumeyer and Perri 2004; Schmitt-Grohe and Uribe 2003; Aguiar and Gopinath 2007). We estimate the remaining parameters as described below. Notice, given the random walk assumption (5) for a_t, σ_ε and σ_η are determined by ρ and σ_a. For robustness, the Online Appendix[16] presents an estimation including ψ among the parameters to estimate.

Our loglinearized model can be represented in state-space form. Our model is similar to the model in BLL, and there more details are provided on how to compute the likelihood function for a general representative-agent model with

[15] In this case our sample length is restricted by the fact that there are no quarterly data on GDP components before the end of World War I in 1918.

[16] This appendix is available on our webpages.

signal extraction.[17] The main idea is first to solve the consumer's Kalman filter to obtain the dynamics of consumer's expectations, and next to build the econometrician's Kalman filter, including in the list of unobservable state variables the consumer's expectations. The model can then be estimated through Maximum Likelihood (ML).

Following BLL, one can exploit the existence of an observationally equivalent full information model to the model with noisy information. This immensely facilitates the implementation of our estimation by allowing us to use standard computational tools even if we did not have a closed-form solution (Appendix C). BLL provides more details about this point.

In our baseline estimations we include the demeaned first differences of labor productivity Δa_t and net exports nx_t as observable variables. Using consumption instead of net exports did not change the results – the Online Appendix[18] presents supplementary robustness estimations varying the set of observables. From BLL we know that in this type of model, identification of ρ, σ_a, and σ_ν is usually obtained – in the sense of finding a unique maximum for the likelihood function – when two time series are used, one directly providing information about a_t and another being forward looking.[19] Therefore, we can limit ourselves to this approach without having to recur to Bayesian statistics.

3.2 Great Recession

Here we present our baseline results for the Great Recession.

Table 1 contains the parameter estimates. The persistence parameter ρ is estimated at 0.97, implying very persistent processes both for the permanent and the transitory components of productivity. The standard deviation of productivity is estimated at 0.64% in the case of the Great Recession. Given the random walk assumption (5) for productivity, the high values of ρ imply a standard deviation for permanent technology shocks that is fairly small, of 0.02%, and a fairly big standard deviation for the transitory technology shock, of 0.63%. The standard deviation of noise shocks is large, 7.39%.

The fact that permanent shocks in productivity are small compared to tran-

[17]See Appendix 5.1 of BLL.

[18]This appendix is available on our webpages.

[19]The use of the permanent income logic together with rational expectations to identify transitory and permanent shocks connects our approach to a large body of work on household income dynamics, e.g. Blundell and Preston (1998), or Blundell, Pistaferri, and Preston (2008). Within this literature, Guvenen (2007) also uses a learning model.

Table 1: Parameter Estimates, Great Recession

Parameter	Description	Value	s.e.
ρ	Persistence tech. shocks	0.97	0.01
σ_a	Std. dev. productivity	0.64	0.04
σ_ε	Std. dev. permanent tech. shock (implied)	0.02	–
σ_η	Std. dev. transitory tech. shock (implied)	0.63	–
σ_ν	Std. dev. noise	7.39	2.04

Notes: ML estimates of the loglinearized state-space representation of the model. The observation equation is composed of the first differences of U.S. labor productivity and net exports. The sample is from 1990:Q1 to 2013:Q1. Standard errors are reported to the right of the point estimate. The standard deviations σ_ε and σ_η are implied by the random walk assumption (5) for productivity.

sitory shocks, and that the amount of noise in the signal is large suggests that learning is slow, because both a_t and s_t are fairly uninformative signals about x_t. This illustrates the major signal extraction problem that consumers face according to our estimation. This has a direct implication for the delay in learning defined above, which is 5.25 years for this estimation.[20]

Given the simplicity of our model, there is no need to recur to Bayesian methods. In fact, in this estimation we hit a unique global maximum for the likelihood function.

Figure 3 shows the variance decomposition of BLR in the estimated model at different horizons. At short horizons, the forecast error of BLR is mostly accounted for by both transitory noise and shocks, and the opposite holds at a medium horizon (after, say, 7 years). Given our emphasis on the medium run, we focus on the effect of permanent shocks throughout the paper.

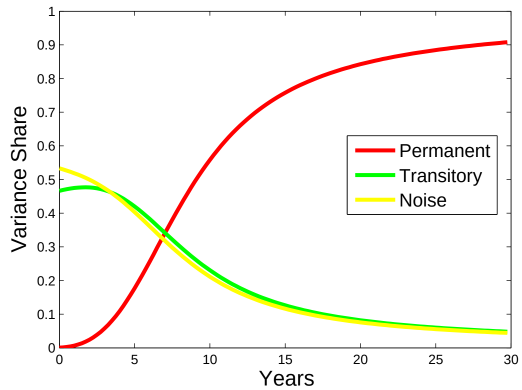
The state-space representation of the estimated model can be used in order estimate the shocks and states of the model using a Kalman smoother. Figure 4 shows our estimated permanent technology shocks for the case of the Great Recession.[21] Consistent with the idea of the effects of a technological revolution spelled out in the previous section, we estimate positive shocks in the early 1990s, up to 1998, and negative shocks in the second part of the sample.

The serial correlation of our estimated permanent shocks is not a violation

[20]Notice that the signal is about the level of the permanent component. Even though the standard deviation of permanent shocks is small, the permanent component is very persistent, and therefore its unconditional standard deviation is substantially larger. Therefore, it is misleading to directly compare the standard deviation of noise shocks to the standard deviation of permanent shocks. Instead, our definition of the delay gives a sense of the amount of uncertainty faced by agents in our model.

[21]For brevity we do not show the estimated transitory and noise shocks here, see the Online Appendix (available on our webpages).

Figure 3: Variance Decomposition of BLR at Different Horizons



Notes: Percentage of forecast error explained by each shock.

of the i.i.d. assumption in the model, but instead purely a reflection of the information available to the econometrician. Given the small size of permanent shocks, it difficult to the econometrician to pin point with precision the quarter when each particular shock hits. This introduces an estimation error that it autocorrelated, and the smoothed shocks turn out autocorrelated as well. This has implications for the interpretation of the estimated series. Indeed, there is fairly strong evidence in the data of either a large positive shock or a high proportion of positive shocks somewhere in the early 90s, although it is not possible to know exactly when. The opposite holds starting 1998.[22]

The estimated permanent shocks in our sample imply that we should have observed a productivity acceleration in the mid-90s, and a subsequent slowdown starting some time later, say around the turn of the century. This can be verified by evidence outside our exercise. In particular, Fernald (2012a) documents that the growth of both labor and total-factor productivity slowed after the early to mid 2000s, the slowdown preceding the Great Recession. Moreover, annualized productivity growth rates in our sample are on average 1.88% from the first quarter of 1990 to the first quarter of 2000 on a yearly basis, and 1.41% from the second quarter of 2000 to the first quarter of 2013.[23]

[22]We have verified that Kalman smoothed shocks out of simulated data have a similar degree of autocorrelation.

[23]On its special report on the world economy, *The Economist* also documented (Oct. 7th 2010) a slowdown of GDP per hour worked in the U.S. that started around 2001 (Figure 12).

Figure 4: Smoothed Permanent Shocks (U.S., 1990–2013)

Notes: Shocks estimated using a Kalman smoother on the U.S. 1990–2013 sample. The data is composed by the first differences of labor productivity and net exports. The unit on the y-axis is percentages. Shocks are scaled by their ML estimated standard deviation.

3.3 Model-predicted Beliefs About the Long Run and Out-of-sample Check

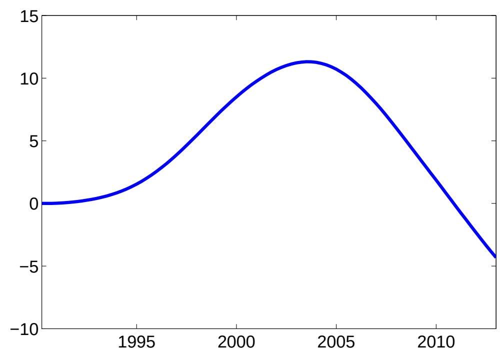
Our main goal in this paper is to obtain a reliable measure of BLR. Proposition 1 provides a clear and intuitive interpretation of BLR in terms of aggregate consumption. Here, we present model-predicted BLR and compare them to survey data. Notice that this exercise is quite challenging because the survey was not used in the estimation.

We proceed by a standard historical decomposition as follows. We feed into the model the series of estimated permanent shocks shown in Figure 4, setting the other two shocks η_t and ν_t to zero. We then simulate the associated BLR using our model. Figure 5 shows the resulting BLR.

Consensus Forecasts publishes a survey including a question of participants' expectations of GDP growth up to 10 years ahead. The survey is done in major industrialized economies[24], and this is the longest horizon available in the survey. Figure 6 reproduces from (Hoffmann, Krause, and Laubach 2011, p. 6) a series of GDP weighted average answers of these Forecasts of real GDP growth 6–10 years ahead (upper panel, top series, marked with a '+'), a series of U.S. answers of these Forecasts (upper panel, bottom series, marked with an 'x'), and the difference of these Forecasts between the U.S. and the Rest-of-the-World (RoW, bottom panel, unique series, marked with an 'x'). Given

[24]The countries included are the U.S., Japan, Germany, France, the U.K., Italy, Canada, China, Korea and Taiwan.

Figure 5: Model-predicted BLR (U.S., 1990–2013)



Notes: Historical effect of permanent shocks on BLR.

that on average the RoW was more optimistic about domestic growth than the U.S., the series on the bottom is negative. This is consistent with the higher average growth rate of countries like China, Korea and Taiwan. From the perspective of out model, we are interested in the relative evolution of trend-growth expectations in the U.S. versus the RoW, i.e. the evolution of the series in the bottom panel (unique series, marked with an 'x').

Figure 7 compares the evolution of trend-growth expectations according to the survey (the bottom panel of Figure 6) and BLR about the long run generated by our estimated model. In the upper panel we plot our beliefs series, aligning the time axis to the Hoffmann et al. (2011) series. A qualitative comparison between the two series suggests that according to both measures the U.S. agent seems to have been relatively most optimistic between 00 and 05.[25]

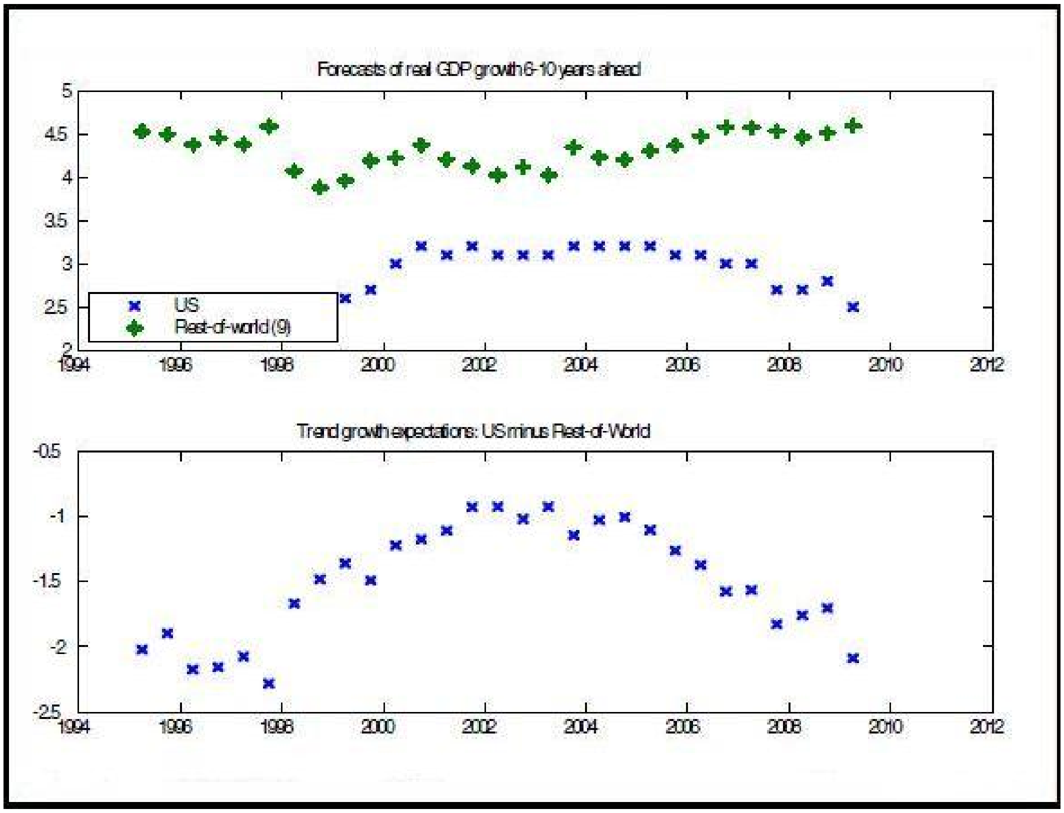
3.4 Japan and Great Depression

Here we present our baseline results for the case of Japan (1975–2005) and the Great Depression in the U.S. (1920–1935).

Table 2 contains the parameter estimates. The persistence parameter ρ is estimated at 0.94 in the case of Japan, and at 0.86 in the case of the Great Depression. Both values imply persistent processes both for the permanent and

[25] In Subsection 3.3 of their paper, Hoffmann et al. (2011) perform a similar exercise by computing a Kalman filtered trend of productivity and comparing it to the sample. Their exercise and ours complement each other. The most important difference is our use of both productivity and net exports – following the permanent income logic mentioned previously – while they use only productivity (of course not imposing the random walk Assumption 5). Another difference is our computation of the variance decomposition for BLR and focus on permanent shocks for getting at their model-implied medium-run dynamics.

Figure 6: Survey Evidence on Long-run Growth Forecasts, U.S. versus RoW (Lower Panel)

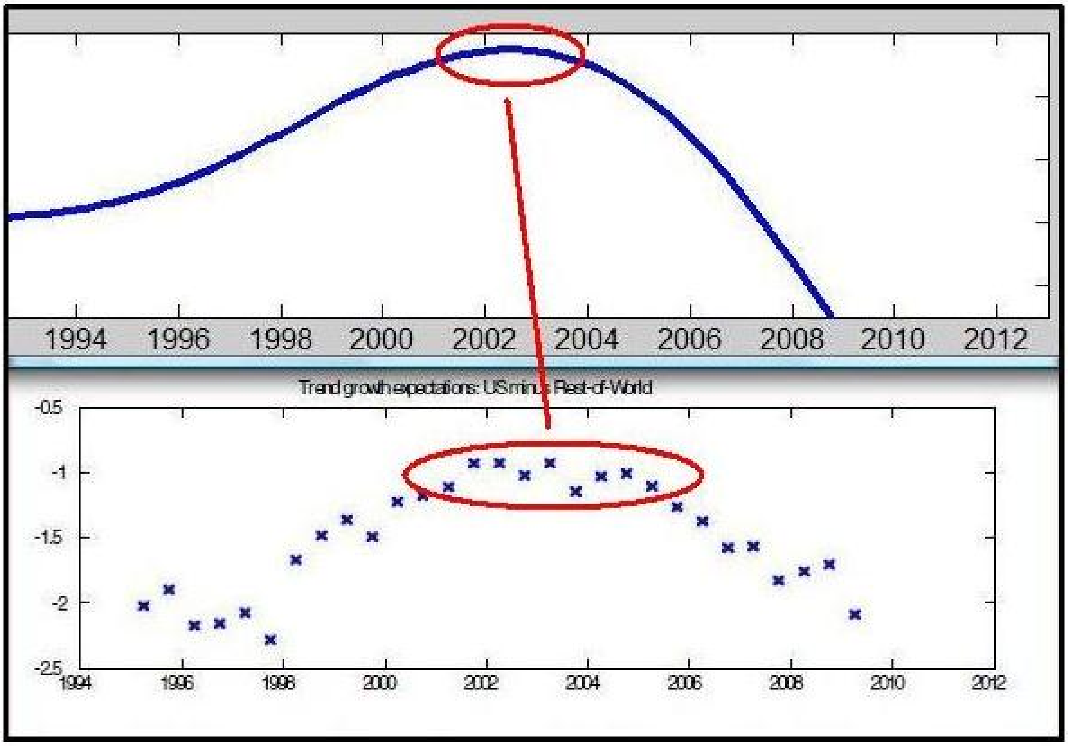


Notes: Reproduced from Hoffman et al. (2011). The upper panel plots the weighted average response in a sample of major industrialized countries countries (Japan, Germany, France, the U.K., Italy, Canada, China, Korea and Taiwan, upper panel, top series, marked with a '+'), and the average response in the U.S (upper panel, bottom series, marked with an 'x'). The lower panel plots the difference between the two (unique series, marked with an 'x').

the transitory components of productivity. The standard deviation of productivity is estimated at 1.00% in the case of Japan, and at 1.66% in the case of the Great Depression. These values are considerably larger than the ones obtained for the Great Recession. Given the random walk assumption (5) for productivity, these values imply a standard deviation for permanent technology shocks of 0.06% in the case of Japan, and of 0.24% in the case of the Great Depression, and a standard deviation for the transitory technology shock of 0.97% in the case of Japan, and of 1.53% in the case of the Great Depression. The standard deviation of noise shocks is large, 14.49% and 20.05% respectively.

The standard deviations of all shocks and, in particular, the noise shock are larger in both cases than in the Great Recession. However, the overall amount of noise in the news agents receive – quantified by the delayed learning – is actually smaller. This delay is of 2.5 years in the case of Japan, and of 1 year in

Figure 7: Out-of-sample Check: Comparison of Model-predicted BLR (Upper Panel) and Survey Evidence on Long-run Growth Forecasts (Lower Panel)



Notes: The survey data is reproduced from Hoffman et al. (2011), see Figure 6.

the case of the Great Depression. It is difficult to analyze the mapping between the parameter values to the delay, but from numerical exercises it seems that the main reason the delay is smaller here is that in both cases, and especially in the case of the Great Depression, the value of ρ is smaller, implying a less persistent permanent technology process and, through Assumption (5), larger and easier to detect permanent shocks.[26]

Figure 8 plots the estimated permanent shocks. As for the Great Recession, we estimate a high number of positive shocks in the first part of the two samples, and a high number of negative shocks later on. In the case of Japan, the positive shocks hit roughly between 1980 and 1987. This estimated permanent shocks imply that we should have observed a productivity acceleration and

[26]We find the delay in the case of the Great Depression a bit too low compared to the other two. For data availability reasons it is unfortunately not possible to see if, having a sample spanning more or less the same amount of years as in the other two cases, we would have obtained a larger delay. It is apparent from the smoothed shocks (Figure 8) that the Great Depression sample has less information about the ups and downs in the productivity trend (see below.) In the Online Appendix (available on our webpages), we estimate the limit model according to Proposition 1 and we obtain a larger delay with the Great Depression data.

Table 2: Parameter Estimates, Japan and Great Depression

Parameter	Description	Japan Value	Japan s.e.	Great Dep. Value	Great Dep. s.e.
ρ	Persistence tech. shocks	0.94	0.02	0.86	0.05
σ_a	Std. dev. productivity	1.00	0.06	1.66	0.15
σ_ε	Std. dev. permanent tech. shock (implied)	0.06	–	0.24	–
σ_η	Std. dev. transitory tech. shock (implied)	0.97	–	1.53	–
σ_ν	Std. dev. noise	14.49	3.50	20.05	8.06

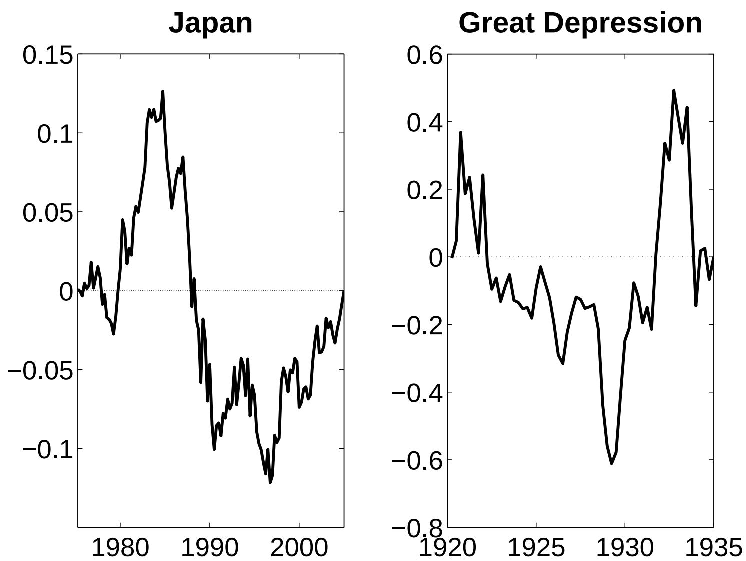
Notes: ML estimates of the loglinearized state-space representation of the model. The observation equation is composed of the first differences of labor productivity and net exports. In the case of Japan, the sample spans 1975–2005. In the case of the Great Depression, the sample spans 1920–1935 (due to data availability it does not start earlier). Standard errors are reported to the right of the point estimate. The standard deviation of the permanent and transitory shocks are implied by the random walk assumption (5) for productivity.

deceleration. Consistently, Japanese annualized growth rates of productivity averaged to 3.22% between 1975 and 1990, and 1.14% from then on. In the case of the Great Depression, the positive shocks hit roughly between 1920 and 1922, the negative shocks roughly between 1926 and 1932, and then again positive shocks hit starting 1932, probably related to the strong economic recovery that started around 1933. In the later case our sample does not seem to start early enough (due to data availability) to appreciate the full extent of the productivity pickup related to the 2nd Industrial Revolution, because the range that mostly contains positive shocks is rather short. Looking at the dates in which some of the technological innovations were implemented – for instance the Ford Model-T was introduced in 1908 – suggest that one would like to have a reliable sample for quarterly consumption and productivity starting at least 10 years before 1920. Still, starting in 1920 captures some of the trend productivity increases of the period. Consistently, annualized productivity growth rates for the U.S. economy average 2.75% between 1920 and 1925, and drop to -.48% between 1925 and 1933. Productivity growth recovers later, between 1933 and 1935, to 4.59%.[27]

Figure 9 plots the model-predicted BLR for Japan and the Great Depression. Even though the positive shocks seem to have hit the Japanese economy mostly in the mid-1980s, consumers there seem to have been most optimistic around 1990. In the case of the Great Depression, the consumer is most optimistic around 1923, which implies a shorter delay with respect to the positive

[27] Productivity growth rates seem to have been high for a number of years during the recovery from the Great Recession, a fact noted by Field (2003,2006), among others.

Figure 8: Smoothed Permanent Shocks (Japan 1975–2005, and U.S. 1920–1935)



Notes: Shocks estimated using a Kalman smoother on the Japanese 1975–2005 sample, and on the U.S. 1920–1935 sample. The latter is restricted by data availability. The data is composed by the first differences of labor productivity and net exports. The time unit on the x-axis is percentages. Shocks are scaled by their ML estimated standard deviation.

permanent shocks. The reason is the smaller delay in learning.

4 The Productivity-to-Consumption Ratio in the Data

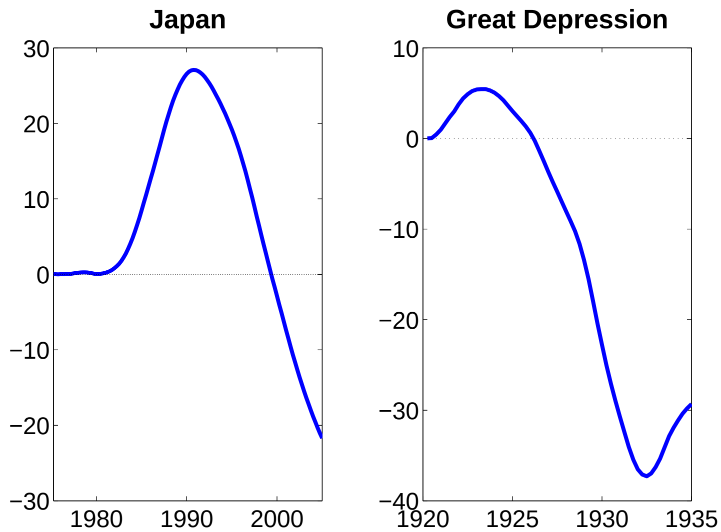
In order to understand which feature of the data deliver the results above, here we focus on the shape of the productivity-to-consumption ratio in the three cases.

Productivity here (and throughout the paper) is real GDP divided by employment. Consumption is consumption from the National Income and Product Accounts (NIPA) divided by population. Notice that we do *not* detrend any of these series for these plots.

Great Recession. Figure 10 plots the logarithm of the ratio of productivity-to-consumption around the Great Recession (U.S. 1990–2013). The vertical axis is centered around the average of the ratio over the period considered. The trend of this series computed using an HP-Filter ($\lambda = 800$) is also plotted.

As the figure shows, the ratio has relatively high values at the start of this

Figure 9: Model-predicted BLR (Japan 1975–2005, and U.S. 1920–1935)



Notes: Historical effect of permanent shocks on BLR.

time window, with a slight increasing portion between 1990 and 1992. This is because during this period productivity is growing at a higher rate than consumption. The ratio starts declining around 1992, and this decline becomes more dramatic starting in 1997, where consumption grows at a considerably stronger rate than productivity. The ratio reaches its lowest point around 2007, after which a reversal starts in which the ratio quickly goes back to its level from 20 years earlier. The reversal is quite sharp and coincides with the start of the Great Recession in 2007. Overall, the ratio appears to follow a slow moving "up-and-down" wave.

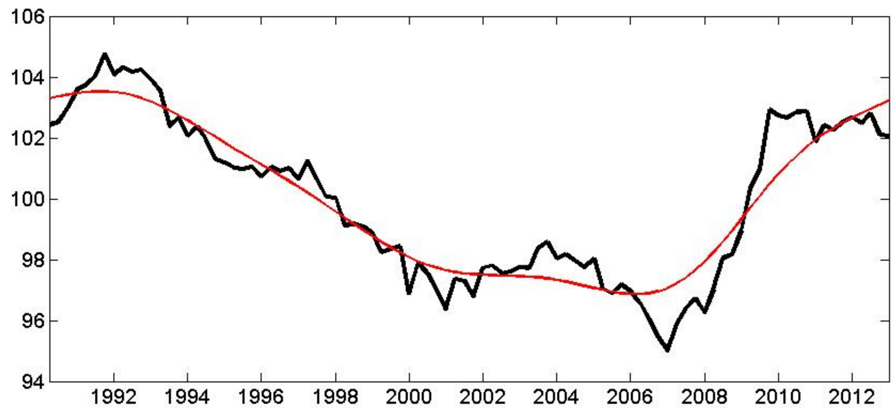
To shed light on these dynamics, it is useful to considered two theoretical benchmarks.

Benchmark (a): "No-news". In this case, σ_ν tends to infinity and thus the signal is completely uninformative. Given the random walk assumption (5), BLR are

$$x_{t+\infty|t} = a_t \quad ,$$

and so, under the conditions of Proposition 1, consumption is equal to productivity:

Figure 10: Productivity-to-consumption ratio, in logs (U.S., 1990–2013), and Trend

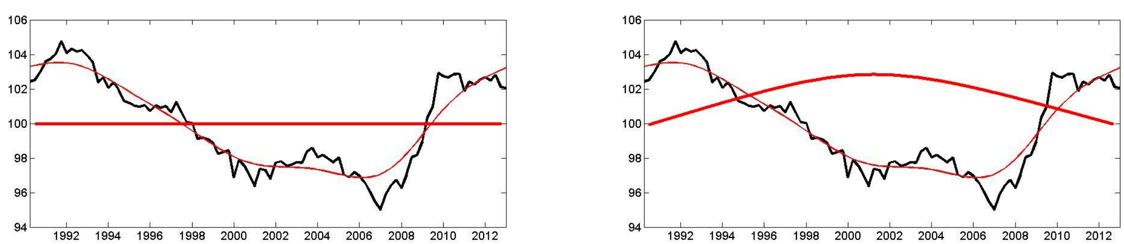


Notes: Productivity is real GDP divided by employment. Consumption is NIPA consumption divided by population. Neither series is detrended. The trend is computed with an HP-Filter ($\lambda = 800$).

$$c_t = a_t, \quad \forall t \quad .$$

Thus, the ratio of productivity-to-consumption is flat. As illustrated by Figure 11 (left panel), this clearly fails to fit the data.

Figure 11: Benchmarks for the Productivity-to-consumption Ratio



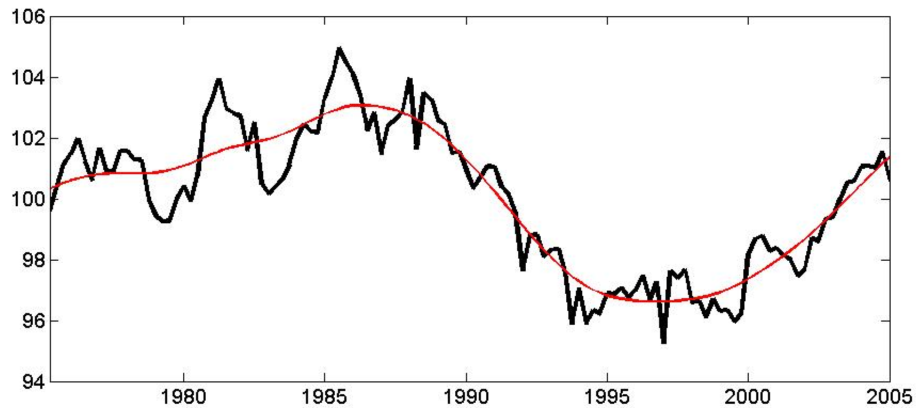
(a) No-news

(b) Perfect Foresight

Notes: Ratio for the U.S. 1990–2013, HP-filter trend ($\lambda = 800$), and theoretical benchmarks.

Benchmark (b): Perfect Foresight. Under perfect foresight, agents have knowledge of all future shocks right from 1990:Q1. Under the conditions of Proposition 1, consumption jumps immediately to the long-run level of productivity, say $x_{t+\infty}$, and remains there. As a result of the positive and then negative permanent shocks, productivity first increases and then decreases, and then stays there. The ratio of productivity and consumption, thus, has the same

Figure 12: Productivity-to-consumption ratio, in logs (Japan, 1975–2005), and Trend



Notes: Productivity is real GDP divided by employment. Consumption is NIPA consumption divided by population. Neither series is detrended. The trend is computed with an HP-Filter ($\lambda = 800$).

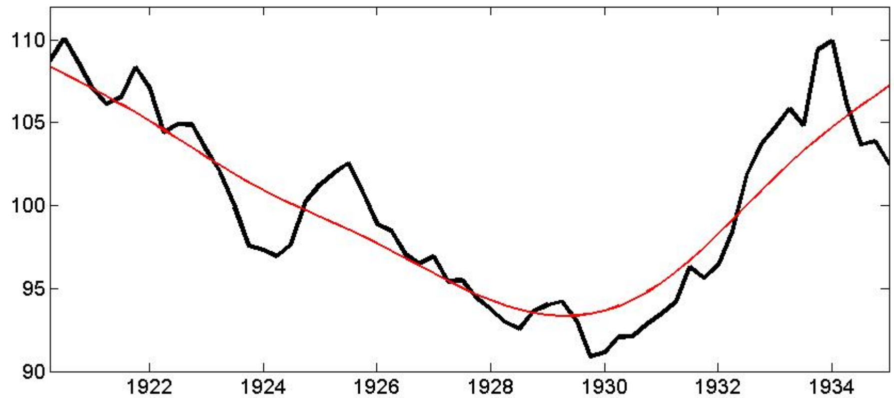
dynamics: it increases, then decreases, and then stay there. As illustrated by Figure 11 (right panel), this, again, fails to fit the data.

To conclude, in both the "no-news" and the perfect foresight benchmarks, the model has a strongly counterfactual prediction for the behavior of the productivity-to-consumption ratio. Indeed, in the data, the ratio finishes in a U-shaped motion. As explained above, noisy signals ($\sigma_\nu > 0$ but finite) imply a delay in learning that helps accommodate this behavior of the ratio, i.e. its decline as consumption catches up with the productivity increase, and rise when consumption growth slows down.

Japan. Figure 12 plots the same ratio for Japan. In this case we can see a more gradual increase in the ratio from its average over the period considered, reaching a peak in 1985. From this point on, the average growth rate of consumption is higher than the growth rate of productivity, and therefore the ratio decreases up to 1994. The lowest point of the ratio is reached in 1997, after which an upward movement brings the ratio back to its level in 1975, suggesting that similar to the previous case, the ratio followed a slow moving up-and-down wave.

Great Depression. Figure 13 plots the ratio for the Great Depression. Due to data availability, we look at this data starting 1920. However, the ratio in this case seems to follow a similar "wavy" pattern as in the two previous figures. It starts at high values, then decreases, reaches a lowest point at the onset of

Figure 13: Productivity-to-consumption ratio, in logs (U.S., 1920–1935), and Trend



Notes: Productivity is real GDP divided by employment. Consumption is NIPA consumption divided by population. Neither series is detrended. The trend is computed with an HP-Filter ($\lambda = 800$). The sample starts in 1920 due to data availability.

the Great Depression in 1929, and then reverts back to its level of 14 years before.

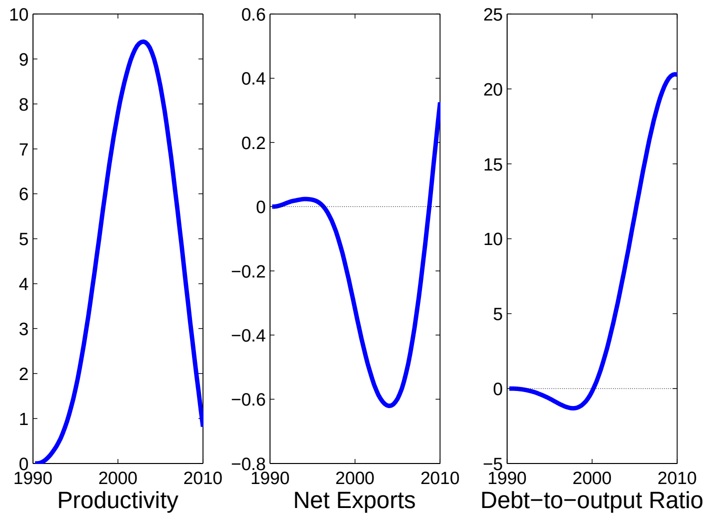
To summarize, this reduced-form analysis improves our understanding of the results obtained through structural estimation. In the three cases considered, the productivity-to-consumption ratio appears to follow similar medium-term dynamics. Together with the evidence on productivity growth rates presented in Section 3, the overall conclusion is that in the three cases there was a slow-moving boom of aggregate productivity, followed by a slowdown. Furthermore, consumption features similar dynamics, but adjusts with a significant lag. We would like to highlight that this way of looking at the data has the advantage of not requiring any particular detrending – a sensitive issue in medium frequency analysis, and more generally, in macroeconomic time series analysis.

5 Characterization of the Dynamics of Debt

In this section we study the model-predicted dynamics of debt, that is, the dynamics implied by the estimated permanent shocks shown in Figure 4. For brevity, we do this only for the case of the Great Recession.

Figure 14 plot these dynamics. The left panel shows productivity, the center panel shows net exports and the right panel shows the debt-to-output ratio. Vertical axes are percentage deviations for steady state. Productivity increases and then decreases, the peak happening in the early 2000s. Net exports are first

Figure 14: Model-predicted Productivity, Net Exports, and Debt-to-output Ratio



Notes: Historical effect of permanent shocks on productivity, net exports, and the debt-to-output ratio. On the left panel, vertical axis' units are relative percentage deviations from the steady state. On the mid- and right panels, vertical axes' units are absolute percentage deviations from the steady state.

slightly positive, then turn negative, and turn positive around 2008. When net exports are negative the economy accumulates debt, with the debt-to-output ratio reaching its highest point around 2008.[28]

The dynamics of debt are determined by three elements. First, they depend on the persistence of the technology process ρ, because it governs the size of the income effect and the persistence of beliefs. The higher ρ, the larger the long-run effect of a shock ε_t, and the larger the income effect. The larger the income effect, the larger the accumulation of debt. Furthermore, the higher ρ, the more persistent the process for Δx_t, and the more persistent BLR.[29] The more persistent BLR, the more time they take to realize the end of the technological revolution. Second, the dynamics of debt depend on the relative size of the standard deviations σ_ε, σ_η, and σ_ν, because these determine the

[28]A close inspection of equation (15) reveals that changes of debt away from the steady state are slightly persistent, which is why the ratio starts declining a bit after net exports turn positive.

[29]The mentioned persistence of BLR is subtle. Indeed, because of the law of iterated expectations, BLR are a random walk, i.e.

$$E[x_{t+1+\infty|t+1}] = x_{t+\infty|t} \quad .$$

However, suppose that the signals agents receive at $t+1$ are equal to those received at t ($a_{t+1} = a_t$ and $s_{t+1} = s_t$). Then, if $x_{t+\infty|t} - x_{t-1+\infty|t-1} > 0$, we have that $x_{t+1+\infty|t+1} - x_{t+\infty|t} > 0$. In this sense, BLR are persistent.

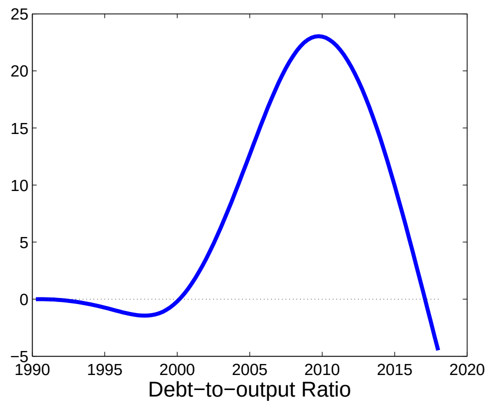
informativeness of a_t and s_t as signals about the permanent component x_t, and thus the speed of learning. The smaller σ_ε with respect to the other two, the less informative a_t and s_t, the slower learning, and the longer it takes for beliefs and consumption to adjust. Third, the dynamics are also determined by the timing of the positive and negative shocks. Suppose there is only one positive and only one negative shock, of same size, and that they hit one after the other in two consecutive quarters. In this case, the effect of the shocks in the economy would be virtually nil. As shocks spread out, they can have an effect in the economy, in particular, agents can be optimistic when the negative shock hits. In the opposite extreme, if the negative shock never hits, agents are never "surprised".

We stop this simulation in 2010 to make the following qualitative point. At this point the state of the U.S. economy has two adverse features: high debt, and low productivity growth. Debt is high because it took some time for agents to recognize the productivity slowdown of the early 2000s. Productivity growth is low because of the negative permanent shocks that hit the economy after 1998. It is obvious that both ingredients imply, at least qualitatively, a slow deleverage. One can quantify the length of the deleverage by simulating the model forward from 2010, assuming all shocks are equal to zero after the end of our sample (2013:Q1). Figure 15 shows the results. According to this simulation, it would take 7 years after 2010 for the debt-to-output ratio to return to steady state. Of course, the model is too simple to take this quantitative prediction seriously, for instance, the aggressive monetary easing after 2008 is not taken into account, among other factors. However, the qualitative prediction of the model is clear: because the amount of debt in 2010 is large, and productivity low, the deleveraging process should be notoriously slow.

6 Conclusion

We have explored the joint medium-run dynamics of productivity and consumption associated with the Great Recession, the Japanese crisis of the 1990s, and the Great Depression. We found that it is useful to look at these dynamics over a 20 to 25 year window. In the three cases, we find evidence of an initial acceleration of productivity, followed by a subsequent slowdown. We also find evidence of a consumption boom, which lags considerably the developments of productivity, peaking when productivity has already started to decelerate. The

Figure 15: Simulation of the Debt-to-output Ratio After 2010



Notes: Model-produced forecast of b_t assuming all shocks after 2013 (the end of our sample) are zero.

dynamics end with a consumption decline. These joint dynamics of productivity and consumption are visible by looking at the ratio of these two variables, which have the shape of an up-and-down wave, commonly across episodes.

In an effort to understand these dynamics, we estimate a model with noisy news about the future. In the model, the productivity process is exogenous and is therefore estimated using time series data. Consumption is determined by beliefs about long-run income. We find that consumers seem to form these beliefs with a delay. The delay is generated by the noise in their information, and it helps explain the lagged behavior of consumption. Altogether, this is a useful exercise to understand the build-up of debt and the deleveraging process in the three cases.

We find the predictions of the model intuitive, and capable to provide a simple account of the behavior of consumption in these episodes. In this exercise, we have intentionally abstracted away from other factors that certainly affected the most salient of these episodes, i.e. the Great Recession, and also probably the other two. Among the most important ones, it is relevant to mention the role of financial deregulation, housing, and financial innovation. However, this abstraction has allowed us to pinpoint to simple medium-run dynamics common to the three episodes. These dynamics find a precise meaning in the context of our model.

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A Beliefs About the Long Run

Proof. To prove (7), we make use of equations (1), (2), and (3):

$$\mathbb{E}_t\left[a_{t+\tau}\right] = \mathbb{E}_t\left[x_{t+\tau} + z_{t+\tau}\right] \quad .$$

It follows that

$$\begin{aligned}
\mathbb{E}_t\left[x_{t+\tau}\right] &= \mathbb{E}_t\left[x_t + \sum_{\tau'=1}^{\tau-1}\left(x_{t+\tau'+1} - x_{t+\tau'}\right)\right] \\
&= \mathbb{E}_t\left[x_t + \sum_{\tau'=1}^{\tau-1}\left\{\rho^{\tau'+1}\left(x_t - x_{t-1}\right) + \sum_{\tau''=1}^{\tau'}\rho^{\tau''}\varepsilon_{t+\tau''}\right\}\right] \\
&= \mathbb{E}_t\left[x_t + \left(x_t - x_{t-1}\right)\sum_{\tau'=1}^{\tau-1}\rho^{\tau'+1}\right]
\end{aligned}$$

where the last inequality comes from the fact that $\mathbb{E}_t\left[\varepsilon_{t+\tau''}\right] = 0$ for all $\tau'' \geq 1$. The geometric sum $\sum_{\tau'=1}^{\tau-1}\rho^{\tau'+1}$ simplifies to $\rho\frac{1-\rho^\tau}{1-\rho}$. So

$$\mathbb{E}_t\left[x_{t+\tau}\right] = \mathbb{E}_t\left[x_t + \rho\frac{1-\rho^\tau}{1-\rho}\left(x_t - x_{t-1}\right)\right] \quad .$$

Now taking the limit of τ to infinity and noticing that $\lim_{\tau\longrightarrow\infty}\rho^\tau = 0$ we obtain

$$\begin{aligned}
\lim_{\tau\longrightarrow\infty}\mathbb{E}_t\left[x_{t+\tau}\right] &= \mathbb{E}_t\left[x_t + \rho\frac{1}{1-\rho}\left(x_t - x_{t-1}\right)\right] \\
&= \frac{\mathbb{E}_t\left[x_t - \rho x_{t-1}\right]}{1-\rho}.
\end{aligned}$$

Similarly,

$$\mathbb{E}_t\left[z_{t+\tau}\right] = \rho^\tau\mathbb{E}_t\left[z_t\right] \quad .$$

So

$$\lim_{\tau\longrightarrow\infty}\mathbb{E}_t\left[z_{t+\tau}\right] = 0 \quad .$$

Combining the two limits for $\mathbb{E}_t\left[x_{t+\tau}\right]$ and $\mathbb{E}_t\left[z_{t+\tau}\right]$, we obtain equality (7). ■

B Two-regime Economy

Proof. Consider an economy with two regimes described at the end of Subsection 2.1.1. Let $\delta_t \in \{0,1\}$ denote the current regime of the economy: if $\delta_t = 0$, the economy is in the first, regular regime, and if $\delta_t = 1$, the economy is in the second, technological revolution regime. Agents in the economy do not know in which regime they currently are and form an estimate from the information they observe (realized productivities and signals). γ is the probability that the economy switches from the first to the second regime. We assume that the economy starts from the first regime, i.e., $\delta_0 = 0$, and that this is public information.

We want to show that, as γ is close to 0, the agents' estimate of the permanent component of productivity, x_t, is the close to the estimate if the economy always stays in the first regime. Formally, for each t and $t' \leq t$, and all $a_t, s_t, ..., a_1, s_1$, we have the following limit:

$$\lim_{\gamma \longrightarrow 0} \left\{ \begin{array}{c} f\left(x_{t'} | a_t, s_t, ..., a_1, s_1, a_0 = 0, x_0 = 0, z_0 = 0, \delta_0 = 0\right) \\ -f\left(\begin{array}{c} x_{t'} | a_t, s_t, ..., a_1, s_1, a_0 = 0, x_0 = 0, z_0 = 0, \delta_0 = 0, \\ \delta_1 = 0, ..., \delta_t = 0 \end{array} \right) \end{array} \right\} = 0$$

where $f(X|Y)$ is the conditional probability density of variables X conditional on the observation of variables Y. In order to prove this statement, we will prove a stronger lemma, Lemma 1 below, using induction in t. The statement is a direct corollary of Lemma 1 given that x_t and x_{t-1} are linear combinations of $(x_1, x_2, ..., x_t, z_1, z_2, ..., z_t)$. ∎

Lemma 1 *For given* $(a_t, s_t, ..., a_1, s_1)$*:*

$$\begin{aligned} & \lim_{\gamma \longrightarrow 0} f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1\right) - f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1, \delta_t = 0, ..., \delta_1 = 0\right) \\ = & \ 0 \end{aligned}$$

and

$$\begin{aligned} & \lim_{\gamma \longrightarrow 0} \gamma f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1, \delta_t, ..., \delta_1\right) \\ = & \ 0 \end{aligned}$$

for all K *and* r'_k *that are linear combinations of* $(x_1, x_2, ..., x_t, z_1, z_2, ..., z_t)$.

Proof. We prove this lemma using induction in t. For $t = 0$ this is obvious because $\delta_0 = 0$ and this is public information. Suppose that the lemma holds for t, we show that it also holds for $t+1$. Indeed, without loss of generality, assume that $r_1 = \alpha_1 x_{t+1} + \beta_1 z_{t+1} + r'_1$,..., $r_K = \alpha_K x_{t+1} + \beta_K z_{t+1} + r'_K$ where $(r'_1, ..., r'_K)$ are linear combinations of $(x_1, x_2, ..., x_t, z_1, z_2, ..., z_t)$ only.

$$
\begin{aligned}
& f\left(r_1, r_2, ..., r_K, a_{t+1}, s_{t+1}, a_t, s_t ..., a_1, s_1\right) \\
= \; & f\left(\alpha_1 x_{t+1} + \beta_1 z_{t+1} + r'_1, ..., \alpha_K x_{t+1} + \beta_K z_{t+1} + r'_K, a_{t+1}, s_{t+1}, a_t, s_t ..., a_1, s_1\right) \\
= \; & f\left(\alpha_1 x_{t+1} + \beta_1 z_{t+1} + r'_1, ..., \alpha_K x_{t+1} + \beta_K z_{t+1} + r'_K, x_{t+1} + z_{t+1}, x_{t+1} + \nu_{t+1}, a_t, s_t ..., a_1, s_1\right)
\end{aligned}
$$

Use the evolution equation for x_{t+1}, (2) and z_{t+1}, (3), as well as the definition (1) for a_{t+1}. We obtain

$$
\begin{aligned}
& f\left(r_1, r_2, ..., r_K, a_{t+1}, s_{t+1}, a_t, s_t ..., a_1, s_1\right) \\
= \; & f\begin{pmatrix} \alpha_1\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon_{t+1}\right) + \beta_1\left(\rho z_t + \eta_{t+1}\right) + r'_1, ..., \\ \alpha_K\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon_{t+1}\right) + \beta_K\left(\rho z_t + \eta_{t+1}\right) + r'_K, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon_{t+1} + \rho z_t + \eta_{t+1}, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon_{t+1} + \nu_{t+1}, a_t, s_t ..., a_1, s_1 \end{pmatrix} \\
= \; & \int f\begin{pmatrix} \alpha_1\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon\right) + \beta_1\left(\rho z_t + \eta\right) + r'_1, ..., \\ \alpha_K\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon\right) + \beta_K\left(\rho z_t + \eta\right) + r'_K, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon + \rho z_t + \eta, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon + \nu, a_t, s_t ..., a_1, s_1, \\ \delta_{t+1} = 0, \delta_t = 0, ..., \delta_1 = 0 \end{pmatrix} dF_\epsilon\left(\epsilon\right) dF_\eta\left(\eta\right) dF_\nu\left(\nu\right), \\
+ & \sum_{\substack{\left(\delta_{t'}\right)_{t'=1}^t \\ \text{and } \delta_{t'}=1 \text{ for some } t' \leq t}} \Pr\left(\left(\delta_{t'}\right)_{t'=1}^t\right) * \\
& \int f\begin{pmatrix} \alpha_1\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon\right) + \beta_1\left(\rho z_t + \eta\right) + r'_1, ..., \\ \alpha_K\left(\left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon\right) + \beta_K\left(\rho z_t + \eta\right) + r'_K, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon + \rho z_t + \eta, \\ \left(1+\rho\right)x_t - \rho x_{t-1} + \epsilon + \nu, a_t, s_t ..., a_1, s_1, \\ \delta_1, ..., \delta_t \end{pmatrix} d\widetilde{F}_\epsilon\left(\epsilon \middle| \left(\delta_{t'}\right)_{t'=1}^t\right) dF_\eta\left(\eta\right) dF_\nu\left(\nu\right)
\end{aligned}
$$

where $F_\epsilon(\epsilon), F_\eta(\eta), F_\nu(\nu)$ are the CDFs for $\epsilon_{t+1}, \eta_{t+1}$, and, ν_{t+1}. When there is some $\delta_{t'} = 1$ before t, the stochastic process for ϵ_t will be different compared to the one under the absence of technological revolution. How exactly $\widetilde{F}_\epsilon$ differ from F_ϵ depends on the way we model ϵ after the technological revolution. $\epsilon = 1$ when the technological revolution arrives and $\epsilon = 0$ thereafter until the end of the revolution when $\epsilon = -\frac{1}{2}$. The end can arrive either stochastically or deterministically. Similarly

$$
\begin{aligned}
& f\left(r_1, r_2, ..., r_K, a_{t+1,} s_{t+1}, a_t, s_t ..., a_1, s_1, \delta_{t+1}=0, ..., \delta_1=0\right) \\
= & \int f\left(\begin{array}{c}
\alpha_1\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1^{\prime}, ..., \\
\alpha_K\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K^{\prime}, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\nu, a_t, s_t ..., a_1, s_1, \\
\delta_{t+1}=0, \delta_t=0, ..., \delta_1=0
\end{array}\right) dF_\epsilon\left(\epsilon\right) dF_\eta\left(\eta\right) dF_\nu\left(\nu\right)
\end{aligned}
$$

Notice that

$$
\begin{aligned}
& f\left(\begin{array}{c}
\alpha_1\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1^{\prime}, ..., \\
\alpha_K\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K^{\prime}, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\nu, a_t, s_t ..., a_1, s_1, \\
\delta_{t+1}=0, ..., \delta_1=0
\end{array}\right) \\
= & f\left(\begin{array}{c}
\alpha_1\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1^{\prime}, ..., \\
\alpha_K\left(\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K^{\prime}, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+\epsilon+\nu, a_t, s_t ..., a_1, s_1, \\
\delta_t=0, ..., \delta_1=0
\end{array}\right) \\
& -f\left(\begin{array}{c}
\alpha_1\left(\left(1+\rho\right) x_t-\rho x_{t-1}+1\right)+\beta_1\left(\rho z_t+\eta\right)+r_1^{\prime}, ..., \\
\alpha_K\left(\left(1+\rho\right) x_t-\rho x_{t-1}+1\right)+\beta_K\left(\rho z_t+\eta\right)+r_K^{\prime}, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+1+\rho z_t+\eta, \\
\left(1+\rho\right) x_t-\rho x_{t-1}+1+\nu, a_t, s_t ..., a_1, s_1, \\
\delta_{t+1}=1, \delta_t=0, ..., \delta_1=0
\end{array}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
& f\begin{pmatrix}
\alpha_1\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1',...,\\
\alpha_K\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K',\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta,\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\nu,a_t,s_t...,a_1,s_1,\\
\delta_{t+1}=1,\delta_t=0...,\delta_1=0
\end{pmatrix}\\
= \quad & \gamma f\begin{pmatrix}
\alpha_1\left(\left(1+\rho\right)x_t-\rho x_{t-1}+1\right)+\beta_1\left(\rho z_t+\eta\right)+r_1',...,\\
\alpha_K\left(\left(1+\rho\right)x_t-\rho x_{t-1}+1\right)+\beta_K\left(\rho z_t+\eta\right)+r_K',\\
\left(1+\rho\right)x_t-\rho x_{t-1}+1+\rho z_t+\eta,\\
\left(1+\rho\right)x_t-\rho x_{t-1}+1+\nu,a_t,s_t...,a_1,s_1,\\
\delta_t=0,...,\delta_1=0
\end{pmatrix}
\end{aligned}
$$

So

$$
\begin{aligned}
& f\left(r_1,r_2,...,r_K,a_{t+1,}s_{t+1},a_t,s_t...,a_1,s_1\right)\\
& -f\left(r_1,r_2,...,r_K,a_{t+1,}s_{t+1},a_t,s_t...,a_1,s_1,\delta_{t+1}=0,...,\delta_1=0\right)
\end{aligned}
$$

$$
= \quad \int\left\{\begin{array}{c}
f\begin{pmatrix}
\alpha_1\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1',...,\\
\alpha_K\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K',\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta,\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\nu,a_t,s_t...,a_1,s_1
\end{pmatrix}\\
-f\begin{pmatrix}
\alpha_1\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_1\left(\rho z_t+\eta\right)+r_1',...,\\
\alpha_K\left(\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon\right)+\beta_K\left(\rho z_t+\eta\right)+r_K',\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\rho z_t+\eta,\\
\left(1+\rho\right)x_t-\rho x_{t-1}+\epsilon+\nu,a_t,s_t...,a_1,s_1,\\
\delta_{t+1}=0,\delta_t=0,...,\delta_1=0
\end{pmatrix}
\end{array}\right\}dF_\epsilon\left(\epsilon\right)dF_\eta\left(\eta\right)dF_\nu\left(\nu\right)
$$

$$
+\gamma\int f\begin{pmatrix}
\alpha_1\left(\left(1+\rho\right)x_t-\rho x_{t-1}+1\right)+\beta_1\left(\rho z_t+\eta\right)+r_1',...,\\
\alpha_K\left(\left(1+\rho\right)x_t-\rho x_{t-1}+1\right)+\beta_K\left(\rho z_t+\eta\right)+r_K',\\
\left(1+\rho\right)x_t-\rho x_{t-1}+1+\rho z_t+\eta,\\
\left(1+\rho\right)x_t-\rho x_{t-1}+1+\nu,a_t,s_t...,a_1,s_1,\\
\delta_t=0,...,\delta_1=0
\end{pmatrix}dF_\eta\left(\eta\right)dF_\nu\left(\nu\right)
$$

$$+ \sum_{\substack{(\delta_{t'})_{t'=1}^{t} \\ \text{and } \delta_{t'}=1 \text{ for some } t' \leq t}} \Pr\left((\delta_{t'})_{t'=1}^{t}\right) *$$

$$\int f \left(\begin{array}{c} \alpha_1 \left((1+\rho) x_t - \rho x_{t-1} + \epsilon \right) + \beta_1 \left(\rho z_t + \eta \right) + r_1', ..., \\ \alpha_K \left((1+\rho) x_t - \rho x_{t-1} + \epsilon \right) + \beta_K \left(\rho z_t + \eta \right) + r_K', \\ (1+\rho) x_t - \rho x_{t-1} + \epsilon + \rho z_t + \eta, \\ (1+\rho) x_t - \rho x_{t-1} + \epsilon + \nu, a_t, s_t ..., a_1, s_1, \\ \delta_1, ..., \delta_t \end{array} \right) d\widetilde{F}_\epsilon \left(\epsilon | \left(\delta_{t'}\right)_{t'=1}^{t} \right) dF_\eta\left(\eta\right) dF_\nu\left(\nu\right)$$

The first component goes to zero as γ goes to zero and the term inside the integral goes to zero by induction

$$\lim_{\gamma \longrightarrow 0} f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1\right) - f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1, \delta_t = 0, ..., \delta_1 = 0\right) = 0.$$

The second and third component go to zero because $\Pr\left((\delta_{t'})_{t'=1}^{t}\right) \leq \gamma$ if $\delta_{t'} = 1$ for some t' and also by induction

$$\lim_{\gamma \longrightarrow 0} \gamma f\left(r_1, r_2, ..., r_K, a_{t,} s_t, ..., a_1, s_1, \delta_t, ..., \delta_1\right) = 0.$$

Combining the two limits, we obtain

$$\lim_{\gamma \longrightarrow 0} \left\{ \begin{array}{c} f\left(r_1, r_2, ..., r_K, a_{t+1,} s_{t+1}, a_t, s_t ..., a_1, s_1\right) \\ -f\left(r_1, r_2, ..., r_K, a_{t+1,} s_{t+1}, a_t, s_t ..., a_1, s_1, \delta_{t+1} = 0, ..., \delta_1 = 0\right) \end{array} \right\} = 0.$$

The proof for

$$\lim_{\gamma \longrightarrow 0} \gamma f\left(r_1, r_2, ..., r_K, a_{t+1,} s_{t+1}, ..., a_1, s_1, \delta_{t+1}, ..., \delta_1\right) = 0$$

is similar. ■

C Closed-form Solution and Limit Result for Consumption

In this Section we solve the model in closed form. Let

$$\widehat{b}_t = b_t + \frac{1 - C/Y}{1 - \beta} a_t.$$

From the intertemporal budget constraint (15), together with the budget constraint (14), we have:

$$
\begin{aligned}
\widehat{b}_t &= b_t + \frac{1-C/Y}{1-\beta}a_t \\
&= \frac{1}{\beta}b_{t-1} - \frac{1}{\beta}\frac{C}{Y}(-c_t) + \frac{1}{\beta}\frac{1-C/Y}{1-\beta}(-\Delta a_t + \beta r_t) \\
&\quad + \frac{1-C/Y}{1-\beta}a_t \\
&= \frac{1}{\beta}\widehat{b}_{t-1} - \frac{1}{\beta}\frac{C}{Y}(-c_t) + \frac{1}{\beta}\frac{1-C/Y}{1-\beta}(-a_t + \beta r_t) \\
&\quad + \frac{1-C/Y}{1-\beta}a_t \\
&= \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}c_t - \frac{1-C/Y}{\beta}a_t + \frac{1-C/Y}{1-\beta}r_t
\end{aligned}
$$

Substituting r_t from (13) into the last equality, and also using the definition of $\widehat{c}_t$, we arrive at

$$
\begin{aligned}
\widehat{b}_t &= \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}c_t - \frac{1-C/Y}{\beta}a_t + \frac{1-C/Y}{1-\beta}\psi b_t \\
&= \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}c_t - \frac{1-C/Y}{\beta}a_t \\
&\quad + \frac{1-C/Y}{1-\beta}\psi\left(\widehat{b}_t - \frac{1-C/Y}{1-\beta}a_t\right) \\
&= \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}\widehat{c}_t - \frac{1}{\beta}a_t \\
&\quad + \frac{1-C/Y}{1-\beta}\psi\left(\widehat{b}_t - \frac{1-C/Y}{1-\beta}a_t\right).
\end{aligned}
$$

So

$$
\begin{aligned}
\widehat{b}_t\left(1 - \frac{1-C/Y}{1-\beta}\psi\right) &= \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}\widehat{c}_t \\
&\quad - \left(\frac{1}{\beta} - \psi\left(\frac{1-C/Y}{1-\beta}\right)^2\right)a_t
\end{aligned}
$$

From the Euler equation (12), we have

$$
\begin{aligned}
\widehat{c}_t &= -\psi b_t + E_t[\widehat{c}_{t+1}] \\
&= -\psi\widehat{b}_t + \psi\frac{1-C/Y}{1-\beta}a_t + E_t[\widehat{c}_{t+1}].
\end{aligned}
$$

Again we conjecture that

$$\widehat{c}_t = D_b \widehat{b}_{t-1} + D_k \mathbf{X}_t,$$

where the state variable $\mathbf{X}_t$ is defined in the proof above and solve for the coefficients D_b and D_k using the method of undetermined coefficients.
Indeed, from the Euler equation:

$$\begin{aligned}
\widehat{c}_t &= -\psi \widehat{b}_t + \psi \frac{1 - C/Y}{1-\beta} a_t + E[\widehat{c}_{t+1}] \\
&= -\psi \widehat{b}_t + \psi \frac{1 - C/Y}{1-\beta} a_t + E[D_b \widehat{b}_t + D_k \mathbf{X}_{t+1}] \\
&= (D_b - \psi) \widehat{b}_t + \psi \frac{1 - C/Y}{1-\beta} a_t + E[D_k \mathbf{X}_{t+1}] \\
&= (D_b - \psi) \frac{1}{1 - \frac{1-C/Y}{1-\beta}\psi} \begin{pmatrix} \frac{1}{\beta}\widehat{b}_{t-1} + \frac{1}{\beta}\frac{C}{Y}\widehat{c}_t \\ -\left(\frac{1}{\beta} - \psi \left(\frac{1-C/Y}{1-\beta}\right)^2\right) a_t \end{pmatrix} \\
&\quad + \psi \frac{1 - C/Y}{1-\beta} a_t + D_k \mathbf{A} \mathbf{X}_t.
\end{aligned}$$

Where the second equality comes from applying the conjectured solution for c_{t+1}, the dynamics of shocks, and the formula for the Kalman filter presented in BLL Appendix 5.1, from which we have

$$E_t[\mathbf{X}_{t+1}] = \mathbf{A}\mathbf{X}_t,$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1+\rho & -\rho & \rho \\ 0 & 1+\rho & -\rho & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho \end{pmatrix}.$$

So

$$
\begin{aligned}
& \left(1-\left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi} \frac{1}{\beta} \frac{C}{Y}\right) \widehat{c}_t \\
= \quad & \left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi} \frac{1}{\beta} \widehat{b}_{t-1} \\
& -\left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\left(\frac{1}{\beta}-\psi\left(\frac{1-C/Y}{1-\beta}\right)^2\right) a_t \\
& +\psi \frac{1-C/Y}{1-\beta} a_t+D_k \mathbf{A} \mathbf{X}_t .
\end{aligned}
$$

Comparing coefficient-by-coefficent to the initial conjecture of $\widehat{c}_t$, we obtain the system of equations on D_b and D_k:

$$
\left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi} \frac{1}{\beta}=\left(1-\left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi} \frac{1}{\beta} \frac{C}{Y}\right) D_b
$$

and

$$
\begin{aligned}
& \left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\left(\frac{1}{\beta}-\psi\left(\frac{1-C/Y}{1-\beta}\right)^2\right)\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \\
& +\left(1-\left(D_b-\psi\right) \frac{1}{1-\frac{1-C/Y}{1-\beta}\beta\psi} \frac{1}{\beta} \frac{C}{Y}\right) D_k \\
= \quad & \psi \frac{1-C/Y}{1-\beta}\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \\
& +D_k \mathbf{A}
\end{aligned}
$$

The first equation is a quadratic equation in D_b:

$$
D_b^2+\left(\frac{1}{C/Y}-\left(1-\frac{1-C/Y}{1-\beta}\psi\right) \beta \frac{1}{C/Y}-\psi\right)-\psi \frac{1}{C/Y}=0 .
$$

This equation has two roots, but we pick the negative root to ensure the stability of the dynamic system:

$$
D_b=\frac{-\left(\frac{1}{C/Y}-\left(1-\frac{1-C/Y}{1-\beta}\psi\right) \beta \frac{1}{C/Y}-\psi\right)-\sqrt{\left(\frac{1}{C/Y}-\left(1-\frac{1-C/Y}{1-\beta}\psi\right) \beta \frac{1}{C/Y}-\psi\right)^2+4\psi \frac{1}{C/Y}}}{2} .
$$

Given D_b, we solve for the coefficients D_k using the second equation. First, the coefficient on a_t:

$$\begin{aligned}
&(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\left(\frac{1}{\beta}-\psi\left(\frac{1-C/Y}{1-\beta}\right)^2\right) \\
&+\left(1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}\right)D_{k,1} \\
=\ &\psi\frac{1-C/Y}{1-\beta}
\end{aligned}$$

So

$$D_{k,1}=\frac{\psi\frac{1-C/Y}{1-\beta}-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\left(\frac{1}{\beta}-\psi\left(\frac{1-C/Y}{1-\beta}\right)^2\right)}{\left(1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}\right)}.$$

The coefficient on $z_{t|t}$:

$$\begin{aligned}
&\left(1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}\right)D_{k,4} \\
=\ &\rho D_{k,1}+\rho D_{k,4}
\end{aligned}$$

so

$$D_{k,4}=\frac{\rho D_{k,1}}{1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}-\rho}.$$

The coefficients on $x_{t|t}$ and $x_{t-1|t}$:

$$\begin{aligned}
&\left(1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}\right)D_{k,2} \\
=\ &(1+\rho)D_{k,1}+(1+\rho)D_{k,2}+D_{k,3}
\end{aligned}$$

and

$$\begin{aligned}
&\left(1-(D_b-\psi)\frac{1}{1-\frac{1-C/Y}{1-\beta}\psi}\frac{1}{\beta}\frac{C}{Y}\right)D_{k,3} \\
=\ &-\rho D_{k,1}-\rho D_{k,2}\quad.
\end{aligned}$$

So

$$\begin{bmatrix}\rho+\widetilde{x} & 1\\ \rho & 1-\widetilde{x}\end{bmatrix}\begin{pmatrix}D_{k,2}\\ D_{k,3}\end{pmatrix}=-\begin{pmatrix}1+\rho\\ \rho\end{pmatrix}D_{k,1}$$

where $\widetilde{x} = (D_b - \psi) \frac{1}{1 - \frac{1-C/Y}{1-\beta}\psi} \frac{1}{\beta} \frac{C}{Y}$. Thus

$$\begin{aligned}
\begin{pmatrix} D_{k,2} \\ D_{k,3} \end{pmatrix} &= -\frac{1}{(1-\rho-\widetilde{x})\,\widetilde{x}} \begin{bmatrix} 1-\widetilde{x} & -1 \\ -\rho & \rho+\widetilde{x} \end{bmatrix} \begin{pmatrix} 1+\rho \\ \rho \end{pmatrix} D_{k,1} \\
&= -\frac{D_{k,1}}{\widetilde{x}} \frac{1}{(1-\rho-\widetilde{x})} \begin{pmatrix} 1-\widetilde{x}(1+\rho) \\ -\rho+\rho\widetilde{x} \end{pmatrix}.
\end{aligned}$$

C.1 Limit Result

We provide the proof of Proposition 1 for two cases $C/Y = 1$ and $C/Y \neq 1$.

Proof of Proposition 1 when $C/Y = 1$. Given that $C/Y = 1$,

$$D_b = \frac{-(1-\beta-\psi) - \sqrt{(1-\beta-\psi)^2 + 4\psi}}{2}$$

and

$$\begin{aligned}
D_{k,1} &= -\frac{x}{1-x} \\
D_{k,2} &= \frac{(1-x)(1+\rho)-\rho}{(1-x)(1-\rho-x)} \\
D_{k,3} &= -\frac{\rho}{1-\rho-x} \\
D_{k,4} &= \frac{D_{k,1}\rho}{1-\rho-x}
\end{aligned}$$

where $x = (D_b - \psi)\frac{1}{\beta}$. We have $\lim_{\psi\longrightarrow 0} D_b = -(1-\beta)$ so $\lim_{\substack{\psi\longrightarrow 0 \\ \beta\longrightarrow 1}} D_b = 0$. At the same time

$$\lim_{x\longrightarrow 0} \begin{pmatrix} D_{k,1} \\ D_{k,4} \end{pmatrix} = 0$$

and

$$\lim_{x\longrightarrow 0} \begin{pmatrix} D_{k,2} \\ D_{k,3} \end{pmatrix} = \frac{1}{1-\rho} \begin{pmatrix} 1 \\ -\rho \end{pmatrix} \quad .$$

In the end, the limit dynamics of consumption are

$$\widehat{c}_t = \frac{1}{1-\rho}\left(x_{t|t} - \rho x_{t-1|t}\right).$$

■

Proof of Proposition 1 when $C/Y \neq 1$. From the closed form expressions, it is easy to verify that as β goes to 1 and $\frac{\psi}{(1-\beta)^2}$ goes to 0: $D_b, D_{k,1}, D_{k,4}$

go to zero and $\begin{pmatrix} D_{k,2} \\ D_{k,3} \end{pmatrix}$ goes to $\frac{1}{C/Y}\frac{1}{1-\rho}\begin{pmatrix} 1 \\ -\rho \end{pmatrix}$. ■

Notice, then, that the limit result requires that ψ goes to zero faster than $1-\beta$.

D A Two-country Open Economy Model

The model in Section 2 can be extended to two countries. For each variable X of the home country, denote X^* the corresponding variable for the foreign country. The interest rate equation (10) is modified to:

$$R_t = R_t^* + \psi\left\{e^{\frac{B_t}{Y_t}-b} - 1\right\} \tag{16}$$

Let m and m^* denote the population sizes of the home and foreign country respectively.

An equilibrium is a set of choices $\{C_t, N_t, B_t, C_t^*, N_t^*, B_t^*\}_{t=0}^{\infty}$ and equilibrium interest rates $\{R_t, R_t^*\}_{t=0}^{\infty}$ such that

$$mB_t + m^*B_t^* = 0$$

and the interest rate spread $R_t - R_t^*$ follows (16).

We assume that the two countries have the same steady state growth rate so in steady state:

$$R = R^* = \frac{1}{\beta} \quad .$$

In the loglinearized version of this model, we replace the interest rate equations for the home and the foreign countries, equation (13), by:

$$r_t = r_t^* + \psi \cdot b_t \quad . \tag{17}$$

Moreover, we need to add the linearization for the bond market clearing conditions:

$$mb_t + m^*b_t^* = 0 \quad . \tag{18}$$

It is straightforward to show that Proposition 1 generalizes to this model. Therefore, for the standard parametrization in the literature, our main results can also be obtained in a two country model.

E Data Appendix

In the case of the Great Recession, the series for productivity is constructed by dividing GDP by labor input. GDP is measured by taking the series for Real GDP from the Bureau of Economic Analysis (available through the Federal Reserve Bank of Saint Louis online database). The labor input is measured by the employment series (Bureau of Labor Statistics online database, series IDs LNS12000000Q). The series for per capita net exports is constructed by dividing Real Net Exports by Population. Real Net Exports are measured by the difference between Real Exports and Real Imports from the St. Louis Fed database (series IDs EXPGSC96 and EXPGSC96 respectively). Population is from the BLS (series IDs LNS10000000Q). The series for Real Personal Consumption Expenditures is from the St. Louis Fed database (series IDs PCEC96). The series for TFP was downloaded from John Fernald's website ("A Quarterly, Utilization-Adjusted Series on Total Factor Productivity", Fernald 2012b, supplement, series dtfp_util).

In the case of Japan, all series come from the OECD website. Real GDP, Exports and Imports are contained in the measure named VOBARSA. Employment comes from the OECD website. It is published in monthly frequency, and thus its frequency was changed to quarterly by computing the quarterly arithmetic average at every quarter. Population comes from the ALFS Summary tables in annual frequency, and thus a linear interpolation was performed to obtain quarterly frequency data.

In the case of the Great Depression, GDP, consumption, exports and imports were obtained from Robert Gordon's website. The labor input series was obtained from Kendrick 1961, Appendix A, Table XXIII, 2nd column ("Persons Engaged"). (Gordon (2000) uses the same measure.)

Our data set is available upon request.