THE NEW ADAPTIVE ACTIVE CONSTELLATION EXTENSION ALGORITHM FOR PAR MINIMIZATION IN OFDM SYSTEMS

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Abstract- In this paper, the Peak-to-Average Ratio reduction in OFDM systems is implemented by the Adaptive Active Constellation Extension (ACE) technique which is more simple and attractive for practical downlink implementation purpose. However, in normal constellation method we cannot achieve the minimum PAR if the target clipping level is much below than the initial optimum value. To get the better of this problem, we proposed Active Constellation Algorithm with adaptive clipping control mechanism to get minimum PAR. Simulation results exhibits that the proposed algorithm reaches the minimum PAR for most severely low clipping signals to get minimum PAR.

Keywords: *Peak-to-Average ratio, OFDM, adaptive, active constellation extension*

I. INTRODUCTION

OFDM is a well known method for transmitting high data rate signals in the frequency selective channels. In OFDM system, a wide frequency selective channel is sub divided in several narrow band frequency nonselective channels and the equalization becomes much simpler [1]. However, one of the major drawbacks of multitone transmission systems, such as OFDM systems has higher PAR compared to that of single carrier transmission system.

To solve this problem, many algorithms have been proposed. In some of the algorithms, modifications are applied at the transmitter to reduce the PAR. In some of the algorithms like Partial Transmit Sequence and selective mapping (SLM) the receiver requires the Side Information (SI) to receive data without any performance degradation. In some other methods the receiver can receive the data without SI; example is clipping and filtering, tone reservation [2] and ACE [3]. In the ACE method, the constellation points are moved such that the PAR is reduced, but the minimum distance between the constellation points remains the same. Thus the BER at the receiver does not increase, but a slight increase in the total average power. To find the proper movement constellation points, an iterative Projection onto Convex Set (POCS) method has been proposed [3] for OFDM systems.

Among various peak-to-average (PAR) reduction methods, the active constellation extension (ACE) technique is more attractive for down-link purpose. The reason is that ACE allow the reduction of high-peak signals by extending some of the modulation constellation points towards the outside of the constellation without any loss in data rate. This favour, however, comes at the cost of slight power penalty. For practical implementation, low complexity ACE algorithms based on clipping were proposed in [3-4]. The elementary principle of clipping based ACE (CB-ACE) involves the switching between the time domain and switching domain [5]. Clipping in the time domain, filtering and employing the ACE constraint in the frequency domain, both require iterative process to control the subsequent regrowth of the peak power. This CB-ACE algorithm provides a suboptimal solution for the given clipping ratio, since the clipping ratio is predetermined at the initial stages. Even this method has the low clipping ratio problem in that it cannot achieve the minimum PAR when the clipping level is set below an unknown optimum level at the initial stages, because many factors, such as the initial PAR and signal constellation, have an impact on the optimal target clipping level determination [3]. To the best of knowledge, a practical CB-ACE algorithm cannot predetermine the optimal target clipping level.

In this method, to solve the low clipping problem, we introduce a new method of ACE for PAR reduction. The approach combines a clipping-based algorithm with an adaptive control, which allows us to find the optimal clipping level. The rest of the paper consist OFDM system with CB-ACE, proposed ACE algorithm. Finally the last section includes the simulation results for M-QAM.

II. PAR PROBLEM WITH CB-ACE

An OFDM, the input bit stream is interleaved and encoded by a channel coder. These coded bits are mapped into complex symbols using QPSK or QAM modulation. The signal consists of the sum of N independent signals modulated in the frequency domain onto sub channels of equal bandwidth. As a continuous-time equivalent signal, the oversampled OFDM signal is expressed as

$$\mathbf{x_n} = \frac{1}{\sqrt{JN}} \sum_{k=0}^{JN-1} \mathbf{X_k} \mathbf{e}^{j2\pi\frac{k}{JN}n} \qquad (1)$$

where n=0,1,2,…….,JN-1, N is the number of subcarriers; X_k are the complex data symbols at kth subcarrier; J is the oversampling factor where J≥4, which is large enough to accurately approximate the peaks [6]. In matrix notation , (1) can be expressed as **x=Q*X**. where Q* is inverse discrete Fourier transform matrix of size JN × JN, ()* indicate the Hermitian conjugate, the complex time-domain signal vector $x=[x_0,x_1,\ldots,x_{JN-1}]^T$, and the complex symbol vector $X=[X_0,X_1,\ldots,X_{N/2-1},0_{1\times(J-1)N},X_{N/2},X_{N/2+1},\ldots,X_{N-1}]^T$. Here the guard interval is not considered because it does not have any impact on PAR, which is defined as x_n

$$\mathbf{PAR}(\mathbf{x}) \triangleq \frac{\max_{0\le n\le JN-1}|\mathbf{x_n}|^2}{\mathrm{E}[|\mathbf{x_n}|^2]} \qquad (2)$$

Note that (2) does not include the power of the anti-peak signal added by the PAR reduction. Let $\mathcal{L}$ be the index set of all data tones;$\mathcal{L} = \{\forall k\ s.t\ 0 \le k \le N-1\} = \{\mathcal{L}a \cup \mathcal{L}_a^c\}$, where $\mathcal{L}_a$ is the index set of active sub channels for the reducing PAR. The PAR problem in ACE is formulated as [7]

$$\min_{\mathbf{c}} ||\mathbf{x} + \mathbf{Q}^*\mathbf{C}||_\infty^2$$

Subjected to: X_k+C_k be flexible for k∈ $\mathcal{L}_a$,
$C_k=0$, for k∉ $\mathcal{L}_a$ (3)

Where C is the extension vector whose components are non zero only if k∈ $\mathcal{L}_a$. However, this optimal solution for this ACE formulation for PAR reduction is not appropriate for practical implementation due to high computational complexity. Thus, the CB-ACE algorithm is introduced [3-4].

The basic idea of the CB-ACE algorithm is to generate the anti-peak signal for PAR reduction by projecting the clipping in-band noise into the feasible extension area while removing the out-of-band distortion with filtering. Thus, the CB-ACE formulation is considered as a repeated-clipping-and-filtering (RCF) process with ACE constraint as follows;

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \boldsymbol{\mu}\tilde{\mathbf{c}}^{(i)} \qquad (4)$$

Where μ is a positive real step size that determine the convergence speed, i is the iteration index, the initial signal is $\mathrm{x}^{(0)}$, and $\tilde{\mathrm{c}}^{(i)}$ is the anti-peak at the ith iteration as follows: $\tilde{\mathrm{c}}^{(i)} = \mathrm{k}^{(i)}\mathrm{c}^{(i)}$, where $\mathrm{k}^{(i)}$ is the transfer matrix of size JN × JN and $\mathrm{k}^{(i)} = \widehat{\mathrm{Q}}^{*(i)}\,\widehat{\mathrm{Q}}^{(i)}$, and $\widehat{\mathrm{Q}}^{(i)}$ is determined by the ACE constraint that $\mathrm{X}_k^{(i)} + \mathrm{C}_k^{(i)}$ is a feasible for k∈ $\mathcal{L}_a$. Here, $\mathrm{c}^{(i)}$ is the peak signal above the predetermined clipping level A and $\mathrm{c}^{(i)} = [\mathrm{c}_0^{(i)}, \mathrm{c}_1^{(i)}, \ldots, \mathrm{c}_{JN-1}^{(i)}]^T$, where $\mathrm{c}_n^{(i)}$ is the clipping sample, which can be obtained as follows:

$$\mathbf{c_n^{(i)}} = \begin{cases} \left(|\mathbf{x_n^{(i)}}| - \boldsymbol{A}\right)e^{j\boldsymbol{\theta_n}}, & \boldsymbol{if}\ |\boldsymbol{x_n^{(i)}}| > \boldsymbol{A} \\ \mathbf{0}, & \boldsymbol{if}\ |\boldsymbol{x_n^{(i)}}| > \boldsymbol{A} \end{cases}, \quad (5)$$

Where $\theta_n = \arg(-x_n^{(i)})$. This clipping level A is related to the clipping ratio $\varkappa$ as $\varkappa = \frac{A^2}{E\{|x_n|^2\}}$. In general, we expect more PAR reduction gain with a lower target clipping level. The existing CB-ACE algorithm cannot achieve the minimum PAR for low target clipping ratios, because the reduced power by low clipping reduces the PAR reduction gain. The original constellation move towards the with the decreasing clipping ratio in [8], which places the clipping signal constellation outside the feasible extension area. The number of $\mathcal{L}_a$, corresponding to the number of reserved tones into tone reservation (TR),as in [9]. Decrease in the clipping ratio degrades the PAR reduction capacity in ACE.

III. SUGGESTED ACE ALGORITHM

The main objective of the proposed algorithm is to control both clipping level and convergence factor at each iteration and to iteratively minimize the peak power signal greater than the target clipping level. The basic cost function is defined as

$$\boldsymbol{\xi}(\boldsymbol{I}^{(i)}) \triangleq \min_{\boldsymbol{\mu},\boldsymbol{A}} \left\| \boldsymbol{x}^{(i)} + \boldsymbol{\mu}\tilde{\boldsymbol{c}}^{(i)} - \boldsymbol{A}e^{j\boldsymbol{\Phi}^{(i)}} \right\|_2^2 \qquad (6)$$

Where $\Phi^{(i)}$ is the phase vector of $x^{(i)} + \mu\tilde{c}^{(i)}$ at the ith iteration and $I^{(i)}$ represents the set of time indices at the ith iteration, $I^{(i)} = \{\forall n\ s.t\ n \in [0, JN-1]\}$. The summary the proposed algorithm is,

Step 1: Initialize the parameters
(a) Selecting the target clipping level A
(b) Setting the maximum number of iterations L.

Step 2: Set$i = 0$, $x^{(0)} = x$ and $A^{(0)} = A$.

Step 3: Computing the clipping signal in (5); if there is no clipping signal, transmit signal, $x^{(i)}$.

Step 4: Transfer the clipping signal into anti-peak signal subjected to ACE constraint;
(a) Convert $\mathrm{c}^{(i)}$ into$\mathrm{C}^{(i)}$.
(b) Removing the out-of-band of $\mathrm{C}^{(i)}$ by projecting $\mathrm{C}^{(i)}$ onto the feasible region in ACE.
(c) By taking the IDFT obtain $\tilde{c}^{(i)}$.

Step 5: Update $\mathrm{x}^{(i)}$ in (4) and minimizing (6).
(a) Computing the optimal step size μ,

$$\boldsymbol{\mu} = \frac{\Re[\langle \mathbf{c}^{(i)}, \tilde{\boldsymbol{c}}^{(i)} \rangle]}{\langle \tilde{\boldsymbol{c}}^{(i)}, \tilde{\boldsymbol{c}}^{(i)} \rangle} \qquad (7)$$

Where $\Re$ defines the real part and ⟨,⟩ is the complex inner-product.
(b) Adjust the clipping level A
(c)

$$\boldsymbol{A}^{(i+1)} = \boldsymbol{A}^{(i)} + \boldsymbol{\nu}\boldsymbol{\nabla_A} \qquad (8)$$

Where the gradient with respect to A is

$$\nabla_A = \frac{\sum_{n \in I_1^{(i)} \cup I_3^{(i)}} \left| c_n^{(i)+1} \right|}{N_p} \qquad (9)$$

and ν is the step size with $0 \leq \nu \leq 1$ and N_p is the number of peak signals larger than A.

Step 6: Increase the iteration counter $i = i + 1$. If $i < L$, go to Step 3 and repeat ; otherwise, transmit signal, $x^{(i)}$.

Compared to the existing CB-ACE with complexity of order $\mathcal{O}\,(JN \log JN)$, the complexity of the proposed algorithm slightly increases whenever the adaptive control is calculated in (8).however this complexity is negligible compared to that of order $\mathcal{O}\,(JN \log JN)$.

IV. SIMULATION RESULTS

In this section, we illustrate the performance of our proposed algorithm using computer simulations. In the simulations, we use an OFDM system with 2048 sub carriers and M-QAM constellation on each subcarrier. To approximate the continuous-time peak signal of an OFDM signal, the oversampling rate factor J = 8 is used in (1).

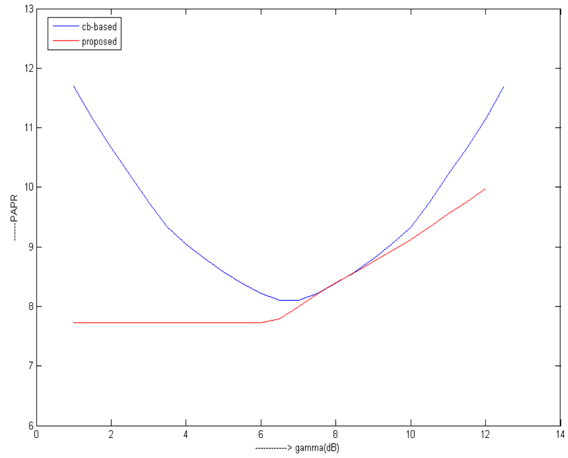

Figure1. The achievable PAR of CB-ACE and the proposed algorithm for an OFDM signal with a 12dB PAR, for different target clipping levels.

Fig. 1 compares the achievable PAR of CB-ACE with the optimal adaptive scaling with that of our proposed algorithm for an OFDM signal with an initial 11.7 dB PAR and 16-QAM modulation, for different target clipping ratios   from 0dB to 12dB. In the case when CB-ACE is applied, we find the minimum achievable PAR, 7.72Db, is obtained with a target clipping ratio of 6dB, which shows that CB-ACE depends on the target clipping ratio, as we mentioned in the previous section. The PAR reduction gain becomes smaller with a decreasing target clipping ratio from the optimal value of 6dB. Thus, we must carefully select the target clipping ratio for CB-ACE. On the other hand, we observe that our proposed algorithm can achieve the lower minimum PAR even when the initial target clipping ratio is set below the CB-ACE optimal value of 6.4dB, It is obvious that our proposed algorithm solves the low target clipping ratio problem associa`ted with the CB-ACE, as shown in Fig.1.

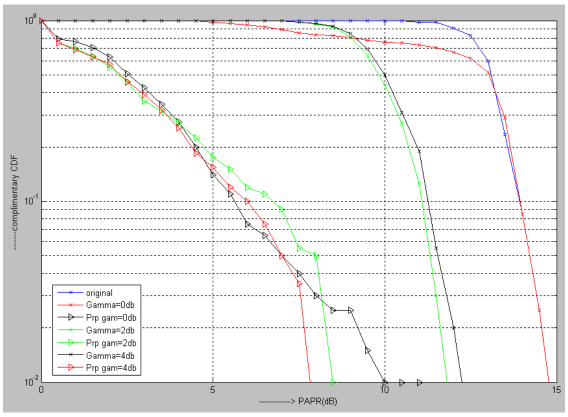

Figure2. PAR CCDF comparison of the CB-ACE and proposed method for different initial target clipping ratios: =0dB, 2dB, and 4dB, L=10.

Fig,2 considers two algorithms: CB-ACE and our proposed method for three different initial target clipping ratios,   =0dB, 2dB, and 4db, in terms of their complementary cumulative density function (CCDF). The solid line curve at the right is plotted for the original OFDM signal. The marked lines correspond to the PAR reduced signals of CB-ACE and our proposed method after 10-iterations, which we have confirmed is sufficient for convergence. For a 10^{-3} CCDF, CB-ACE with initial target clipping ratios of   =0dB, 2dB, and 4dB can be achieve a 0.14dB,0.89dB, and 2.95dB PAR reduction from the original PAR of 11.7dB,respectively. In other words, when the target clipping ratio is set low, the achievable gain in PAR reduction decreases, which is opposite to out general expectation, but is consistent with the trend shown in Fig.1.On other hand, our proposed algorithm shows about a 4dB reduction gain in PAR at10^{-3} CCDF for all three of the initial low target clipping ratios.

V. CONCLUSION

In this paper, we proposed a new CB-ACE algorithm for PAR reduction using adaptive clipping control. We observed that the existing CB-ACE depends on initial target clipping ratio. The lower the initial target clipping ratio is from the optimal clipping value, the smaller the PAR reduction gain. However, our proposed algorithm provided the minimum PAR even when the initial target clipping ratio was set below the unknown optimum clipping point.

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