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SOLVING THE MULTI-OBJECTIVE FACILITY LAYOUT PROBLEM USING EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

Mohamed A. Ismail
PhD Student
Industrial and Manufacturing
Systems Engineering Department
University of Windsor
ismailm@uwindsor.ca

Attia H. Gomaa
Associate Professor
Faculty of Engineering
Cairo university, Fayoum
Branch, Egypt
attiagomaa@yahoo.com

Ashraf O. Nassef
Associate Professor
Faculty of Engineering
American University in Cairo,
Egypt
Nassef@aucegypt.edu

ABSTRACT

The multi-objective facility layout problem is defined in the literature as an extension of the famous quadratic assignment problem (QAP). Most previous mathematical models tried to combine both the quantitative and the qualitative objectives into a single objective by using weighting factors. This paper introduces a multi-objective mathematical model and solves it using the revised Strength Pareto Evolutionary Algorithm (SPEAII). The purpose of this paper is to find an efficient set of solutions "Pareto optimal set" which could be introduced to the decision maker to select the best alternative, while considering conflicting and non-commensurate objectives. A computer program is developed to define the mathematical model, code candidate solutions into genetic form, and use Evolutionary Multi-Objective Optimization algorithms (EMO) to find the efficient set of solutions. The problem model is built according to its customized data input. The suggested model and solution algorithms are applied to a wide set of different benchmark problems. Results showed the superiority of the suggested models and algorithms in terms of the quality of solution and objective space exploration..

INTRODUCTION

The facility layout problem involves the selection of the most effective arrangement of physical facilities to allow for the greatest efficiency with the combination of available resources to produce a product or provide a service. Normally the problem is formulated to satisfy a predefined objective or set of objectives such as minimizing material handling cost, smooth work flow, effective space utilization, employee satisfaction, safety, flexibility, etc. It is important to find an optimal solution in order to in order to increase the plant flexibility and decrease the production cost which will directly impact the competitiveness of products in the market. Jajodia et al [9] stated that a poor layout leads to overall inefficiency, which includes accumulation of work-in-process inventory, overloading of material handling systems, inefficient set-ups and longer queues.

Traditionally, there are two basic objectives for this problem: One aims at minimizing the sum of flows times distances while the other aims at maximizing the adjacency score. The former is a distance-based objective which is similar to the classical Quadratic Assignment Problem (QAP) [10] objective and more suitable when the input data is expressed as a from-to-chart while the latter is an adjacency-based objective and is more suitable for a relationship chart.

In this paper, the problem may have two or more objectives; each has its own data type and scoring method. Using Pareto optimization, all objectives will be optimized simultaneously. The candidate solutions are coded and recombined using genetic algorithms. The Revised Strength Pareto Evolutionary Algorithm (SPEAII) [22] is used as classifier to handle the multi-objective aspect of the problem. Finally, a set of benchmark problems are solved to test the suggested model and its solution methodology.

LITERATURE REVIEW

The traditional formulation of the multi-objective facility layout problem is a multi-objective quadratic assignment problem (MQAP). Most of the previous researches tried to find a solution methodology to handle the multi-objective aspect of the facility layout problem. They used weighting factors to convert the problem into a single objective problem as shown in the following model. By changing these factors, an efficient set of solutions is obtained.

$$\max or \min \ Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{n}\sum_{q=1}^{n} A_{ijpq} X_{ij} X_{pq} \qquad (1)$$

Subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j = 1,.....n \qquad (2)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i = 1,.....n \qquad (3)$$

where

$$X_{ij} = \begin{cases} 1 & if\ department\ i\ is\ assigned\ to\ location\ j \\ 0 & Otherwise \end{cases}$$

A_{ijpq} is the cost of locating facility i at location j and facility p to location q

X_{ij} is equal to one if facility i is assigned to location j

X_{pq} is equal to one if facility p is assigned to location q

Constraint (2) ensures that each location contains only one facility. Constraint (3) ensures that each facility is assigned to only one location. Table 1 shows different definitions of the cost term A_{ijpq} .

Rosenblatt [15] was the first to combine both quantitative and qualitative measures into a single objective function. A quite similar model was developed by Dutta and Sahu [3]. Fortenberry and Cox [5] weighted the distance between a pair of departments with their closeness rating score. Malkooti and D'souza [11] formulated the quadratic assignment problem through multiple objective programming in which the problem at hand has several objectives. Urban [21] developed a model in which the weighting of distance between facilities was improved using the sum of the flow volume and the closeness rating score multiplied by a constant. Malakooti [12] developed an interactive gradient-based approach for solving multiple-criteria facility layout problems. Malakooti and Tsurushima [13] developed an expert system for multiple-criteria facility layout problems. Their proposed approach is based on expert systems and multiple-criteria decision making (MCDM).

Sevstka [19] presented a multiobjective implementation of CRAFT called MOCRAFT with enhanced facilities to consider multiple objectives and produce graphic outputs. Houshyar [8] described a Bi-Criteria approach for facility layout problems. He proposed an iterative method which uses a pair-wise comparison of layouts by decision maker, to reduce the feasible space of the weighting factor (for objectives), thereby identifying the optimum layout. Harmonosky and Tothero [7] presented a methodology that normalizes all the factors before combining them. They presented a formulation for the plant layout problem, incorporating more than two input factors that may be either qualitative or quantitative in nature. Shang [18] used the Analytic Hierarchy Process (AHP) to tackle the qualitative aspects of the facility layout problem. Chen and Sha [1] and Sha and Chen [17] defined a new measure of solution quality, dominant probability, to determine the probability that one layout is better than the others.

SUGGESTED MODEL

The suggested model is an extension of the quadratic assignment problem with a set of objectives. Each objective has its own data type (either quantitative or qualitative), optimization directive (max/min) and scoring methods (distance based/adjacency scoring).

$$Z_1 = \max f_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{n}\sum_{q=1}^{n} a_{1ijpq} X_{ij} X_{pq}$$

$$Z_2 = \min f_2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{n}\sum_{q=1}^{n} a_{2ijpq} X_{ij} X_{pq}$$

$$\vdots$$

$$Z_M = \max f_M = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{p=1}^{n}\sum_{q=1}^{n} a_{Mijpq} X_{ij} X_{pq}$$

And

$$X_{ij} = \begin{cases} 1 & \text{if department } i \text{ is assigned to location } j \\ 0 & \text{otherwise} \end{cases} \qquad (4a)$$

$$X_{pq} = \begin{cases} 1 & \text{if department } p \text{ is assigned to location } q \\ 0 & \text{otherwise} \end{cases} \qquad (4b)$$

If Z_k is a Distance – Based obejective

$$a_{kijpq} = f_{kip} d_{jq} \qquad (4c)$$

$$d_{jq} = \left(|x_j - x_q|^p + |y_j - y_q|^p \right)^{\frac{1}{p}} \qquad (4d)$$

, $p = 1$ for rectilinear distance and 2 for Eucledian distance

f_{kip} is the quantitative flow of objective k between department i,p

$$a_{kijpq} = \begin{cases} r_{kip} & if\ departments\ i\ and\ p\ are\ adjacent \\ 0 & Otherwise \end{cases} \qquad (3.4e)$$

Subject to:

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad ; j = 1,.....n \qquad (5)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad ; i = 1,.....n \qquad (6)$$

Where :

M is the number of objectives

n is the number of layout departments

f_{kip} is the quantitative coeffcient of objective k between department i and p

d_{jq} is the centoridal distance between locations j and q

r_{kip} is the qualitative coeffcient of objective k between departments i and p

TABLE 1: MULTI-OBJECTIVE QAP COST TERM EXPRESSIONS.

Author	Cost term * (Single Objective)	Author	Cost term (Single Objective)
Rosenblatt [15]	$A_{ijpq} = W_C C_{ijpq} - W_R R_{ijpq}$ C_{ijpq} = *material handling objective* R_{ijpq} = *closeness rating objective* W_C *and* W_R *are weights*. $W_C - W_R = 1$	Shang [18]	$A_{ijpq} = d_{jq}(a.f_{ik} + b.c \times r_{ip})$ *a is the sum of quantative weights.* *b is he sum of qualitative weights.* *c is const.*
Dutta and Sahu [3]	$A_{ijpq} = W_C C_{ijpq} - W_R R_{ijpq}$ $W_C - W_R = 1$	Houshyar [8]	$A_{ijpq} = W_C C_{ijpq} - (1 - W_c) R_{ijpq}$
Frontberry and Cox [5]	$A_{ijpq} = f_{ip} d_{jq} r_{ip}$	Harmonosky, and Tothero [7]	$A_{ijpq} = \sum_{m=1}^{n} w_m \frac{S_{ipm}}{\sum_i^n \sum_p^n S_{ikm}} d_{jq}$ d_{jq} *is the rectilinear disance between their locations.* S_{ipm} *is the normalized relationship value between departments i and p for objective m.*
Malkooti and D'souza [11]	$A_{ijpq} = w_1 f_1 + w_2 f_2 + ... + w_n f_k$ $\sum_{i=1}^{k} w_i = 1 \quad ; w_i > 0$	Suresh and Sahu [21]	$A_{ijpq} = W_C C_{ijpq} - (1 - W_c) R_{ijpq}$
Urban [22]	$A_{ijpq} = d_{jq}(c \times r_{ip} + f_{ip})$	Sha and Chen [17]	$A_{ijpq} = \sum_m w_m U_m$ U_m *is the normalized value for objective m*

* d_{jq} *is the* distance *between location j and q*
f_{ip} *is the flow between location i and p*
r_{ip} is the closeness rating coeffcient between departments *i* and *p*

SOLUTION APPROACH

The facility layout problem has been proved to have the characteristic of NP-complete, so the Genetic Algorithms (GAs) that have the abilities of population based searching and parallel calculation are appropriate for solving the problem. To tackle the multi-objective, the revised Strength Pareto Evolutionary Algorithm (SPEAII) [23] is used as a classifier to find the efficient set of solutions "Pareto optimal set". The following algorithm demonstrates the steps of the solution:

1. Define the problem data

   -Define the problem parameters: Number of departments, objectives data and their scoring methods

   -Set the genetic Parameters (Number of generations, population size 'N', recombination probabilities)

   -Define SPEAII Archive size (Efficient set size)

2.Initialize the problem (define the initial population)

3.t=0

4.Decode chromosomes into their corresponding layouts and evaluate the objectives' values

5.Send the updated population to SPEA2 classifier, to update the archive (the efficient set)

6.Perform recombination

7.t=t+1

8.if t<N then go to 4 else go to 9

9. Display results

The next section gives a brief review of evolutionary algorithms, problem coding, and recombination operators.

THE MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS

Some evolutionary multi-objective optimization algorithms were developed in the early nineties, based on combining the ideas (suggested by Golderg, [6]) of Pareto dominance, to exploit the search space in the direction of the Pareto front, and niching techniques. Since that time several algorithms have been developed [4,19,22,23]. The main differences of these algorithms are how to evaluate individuals' fitness (Ranking of population individual), Elitisms, and how to keep diversification. In a comparative study Zitzler et al. [22] showed that their Strength Pareto Evolutionary Algorithm (SPEA) outperforms other algorithms on a variety of standard test problems.

Based on SPEA, the SPEAII [23] algorithm has some differences oriented to eliminate the possible weaknesses of its ancestor. The fitness of each individual takes into account the number of individuals it dominates and also the number of individuals it is dominated by; and also, a nearest neighbour density estimation is added to fitness. This is a more accurate fitness function in terms of comparison with the other members of the population and archive population. The archive population has a fixed size. A truncation operator is introduced instead of the clustering technique of SPEA to keep diversity in the population; it avoids the possible loss of outer solutions, preserving the whole algorithm execution and the range of Pareto solutions achieved. The SPEAII algorithm is adopted in this work as a classifier to find the Pareto front we are looking for.

PROBLEM CODING

The facilities are considered to be assigned to the possible sites, which are a rectangular pattern of locations in the plane. A chromosome represents a sequence of facilities which represents their positions in the site. Each real number represents the facility number and the size of these chromosomes is same as the number of departments or facilities to be assigned. Figure 1 represents a sample chromosome and its decoded layout.

1	4	2	5	3	6

1	4	2
5	3	6

FIG. 1: SAMPLE CHROMOSOME AND ITS DECODED LAYOUT

RECOMBINATION OPERATORS

Since the problem is represented as a permutation of integer numbers, some of the most known cross over and mutation operators can be applied to the problem directly. These operators include Partial-Mapped Crossover (PMX), Order Cross over (OX) and Cycle Crossover (CX) (Goldberg [6] and Michalewicz [14]). It is relatively easy to produce some mutation operators for permutation representation. During the past decade, several mutation operators have been proposed for permutation representation, such as inversion, insertion, displacement, and reciprocal exchange mutation. The recombination operator is selected randomly during the solution process to improve and guarantee the reliability of the solution.

TEST PROBLEMS

To handle the given types of the facility layout problems a computer program was developed. The program is called FLAp "**F**acility **L**ayout **Ap**plication". The program is used to test, benchmark, and verify the suggested model(s) and the solution methodology.

The tested problems are grouped into sets; each set has either distinct characteristics or considered as a test set by other authors from the literature. The problem data (flow matrices, closeness ratings values, departments' requirements) and objectives scoring are taken to be identical with the original benchmark problems themselves. The original objectives' values are recalculated to define the efficient set to be compared with FLAp Pareto front. Set of each problem is

Test Problems Set # 1

In this set, the results of Rosenblatt [15], Dutta and Sahu [3], Rosenblatt and Sinuany-Stern[16], and Fortenberry and Cox[5] are compared. In both Rosenblatt's [15], Rosenblatt and Sinuany-Stern[16], and Dutta and Sahu's [3] solution algorithms, the best layout generated depends upon the weights assigned to material handling cost and closeness rating scores. Fortenberry and Cox present another model by weighting the distance between a pair of departments in the facility by the closeness rating values between departments' pairs. Table 2 and figures 2, 3 and 4 compare the results for the 6, 8 and 12 department problems.

FLAp Results: FLAp produces either a better solution (for most cases) or the same as the previously best found solutions. The efficient solutions found in the literature are very few compared with FLAp, see figures 2 to 4. Table 3 shows that the new developed algorithm explored 3, 12 and 17 new solution points for the 6, 8, and 12 departments' problem in addition to other already existing best known ones.

Rosenblatt, Dutta and Sahu, and Rosenblatt and Sinuany-Stern Results: Most closeness ratings objective scores have low values in Rosenblatt, and Dutta, and Sahu results. This is because they used single iteration qualitative computerized algorithms (construction algorithms). Most of their solution points are inefficient and are dominated by FLAp results.

Fortenberry and Cox Results: The utility function of Fortenberry and Cox may be considered an inefficient one. It couldn't discriminate the efficient solution in objective space or give misleading values. As shown in table 6, according to the Fortenberry and Cox scoring model the first chromosome should be selected, however it is the worst solution.

TABLE 2: THE SIX-DEPARTMENT PROBLEM RESULTS (P1.1):

Author	ID	Chromosome						TCS	TMHC
Rosenblatt	1*	4	3	1	5	6	2	118	184
	2	5	6	3	4	1	2	102	192
	3	4	1	5	3	2	6	94	196
	4	5	2	4	6	1	3	94	204
	5	3	5	6	2	1	4	86	220
	6	4	5	2	6	1	3	86	220
Dutta & Sahu	1	1	5	6	3	2	4	82	212
	2	4	1	6	3	2	5	86	224
	3	1	2	3	4	5	6	94	200
	4	3	2	6	4	1	5	94	196
	5	4	1	5	3	2	6	94	196
	6	1	6	5	3	2	4	98	184
	7	1	3	6	4	2	5	106	204
Rosenblatt and Sinuany-Stern	1	1	6	5	3	2	4	98	184
	2	1	2	3	5	6	4	94	196
	3	1	5	6	3	2	4	82	212
Fortenberry and Cox	1	3	1	4	2	5	6	82	212
FLAp	1*	5	6	2	4	3	1	118	184
	2**	**2**	**1**	**4**	**6**	**3**	**5**	***122***	***196***
	3**	**1**	**4**	**5**	**2**	**6**	**3**	***126***	***212***
	4**	**6**	**3**	**1**	**2**	**4**	**5**	**130**	**216**

* Already best known solution
**Newly explored solution

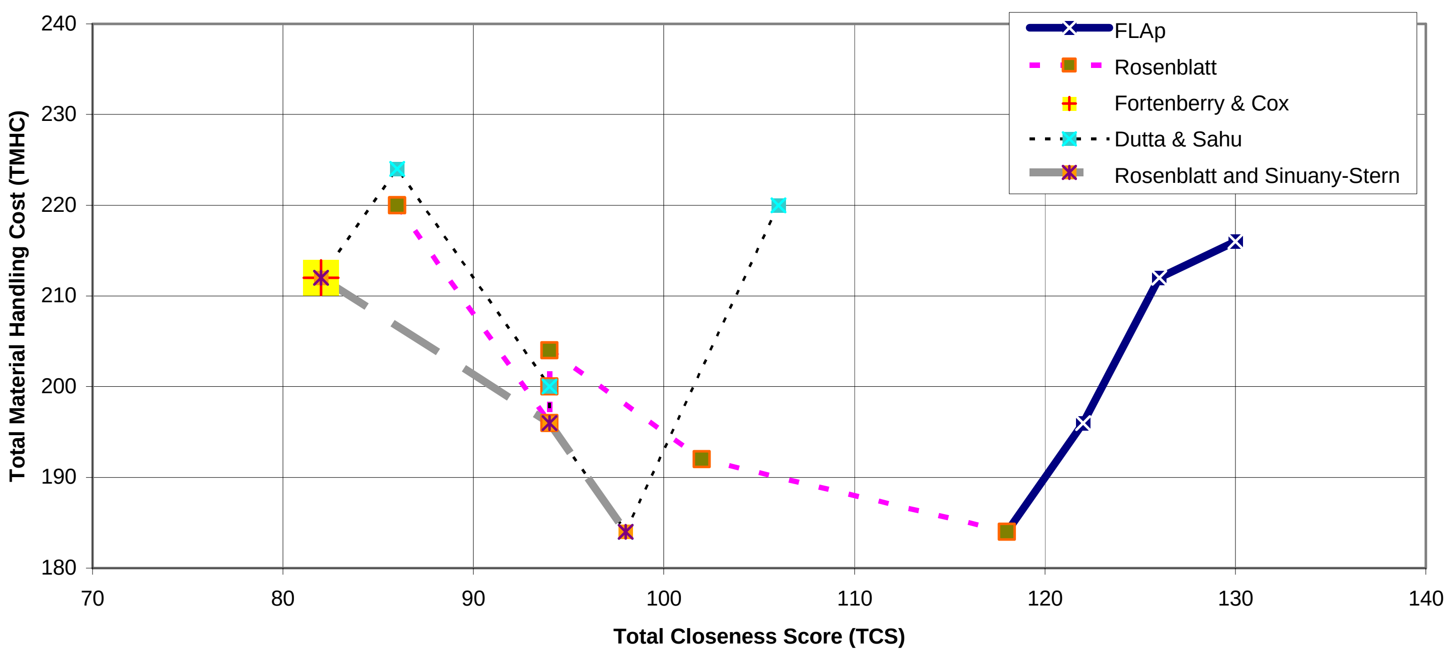

FIG. 2: THE SIX−DEPARTMENT PROBLEM (P1.1) RESULTS

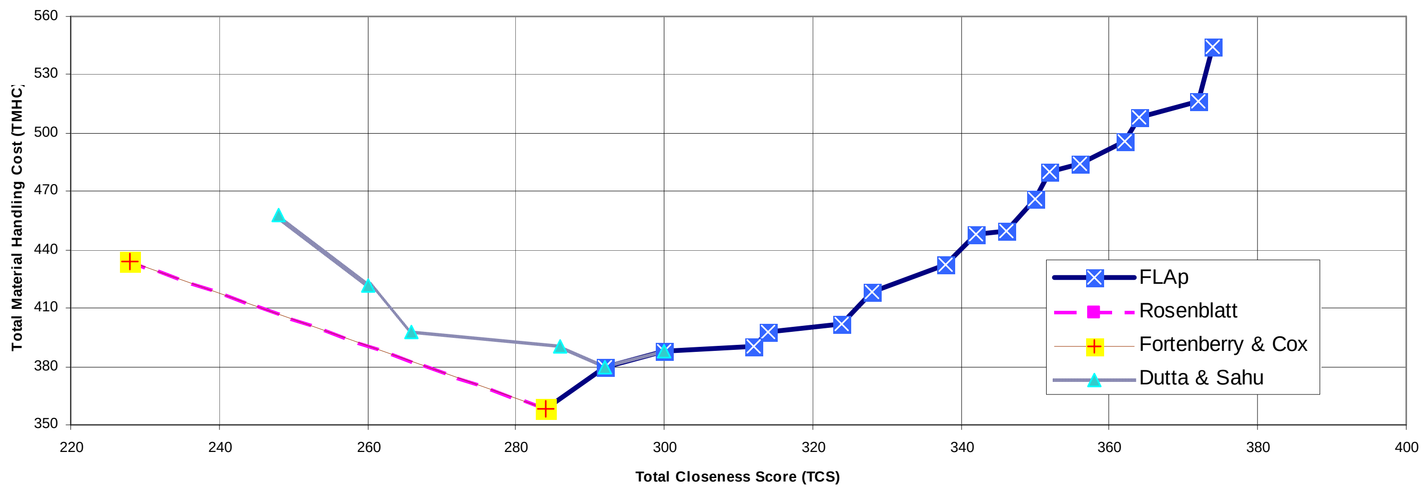

FIG. 3: THE EIGHT−DEPARTMENT PROBLEM (P1.2) RESULTS

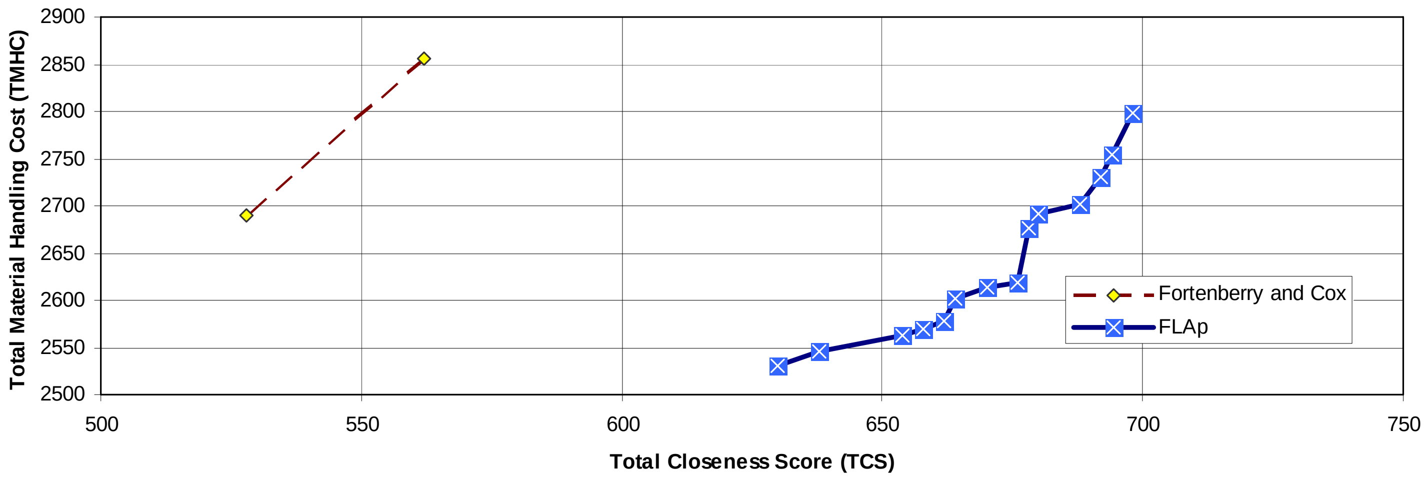

FIG. 4: THE TWELVE−DEPARTMENT PROBLEM (P1.3) RESULTS

TABLE 3: RESULTS SUMMARY OF SET #1

Problem ID	Author	# of Soln. Points	# of Efficient Points
P 1.1	Rosenblatt	6	1
	Dutta & Sahu	7	-
	Rosenblatt and Sinuany-Stern	3	-
	Fortenberry and Cox	1	-
	FLAp.	4	4
P 1.2	Dutta & Sahu	8	2
	Rosenblatt	2	1
	Fortenberry and Cox	3	1
	FLAp.	17	17
P 1.3	Fortenberry and Cox	2	-
	FLAp.	14	14

TABLE 4: FORTENBERRY AND COX RESULTS

Chromosome						TCS	TMHC	Fort. & Cox Scoring
1				2	4	82	212	368
1	6	5	3	2	4	98	184	396
5	6	2	4	3	1	118	184	476
2	1	4	6	3	5	122	196	544
1	4	5	2	6	3	126	212	584
6	3	1	2	4	5	130	216	652

Test Problems Set # 2

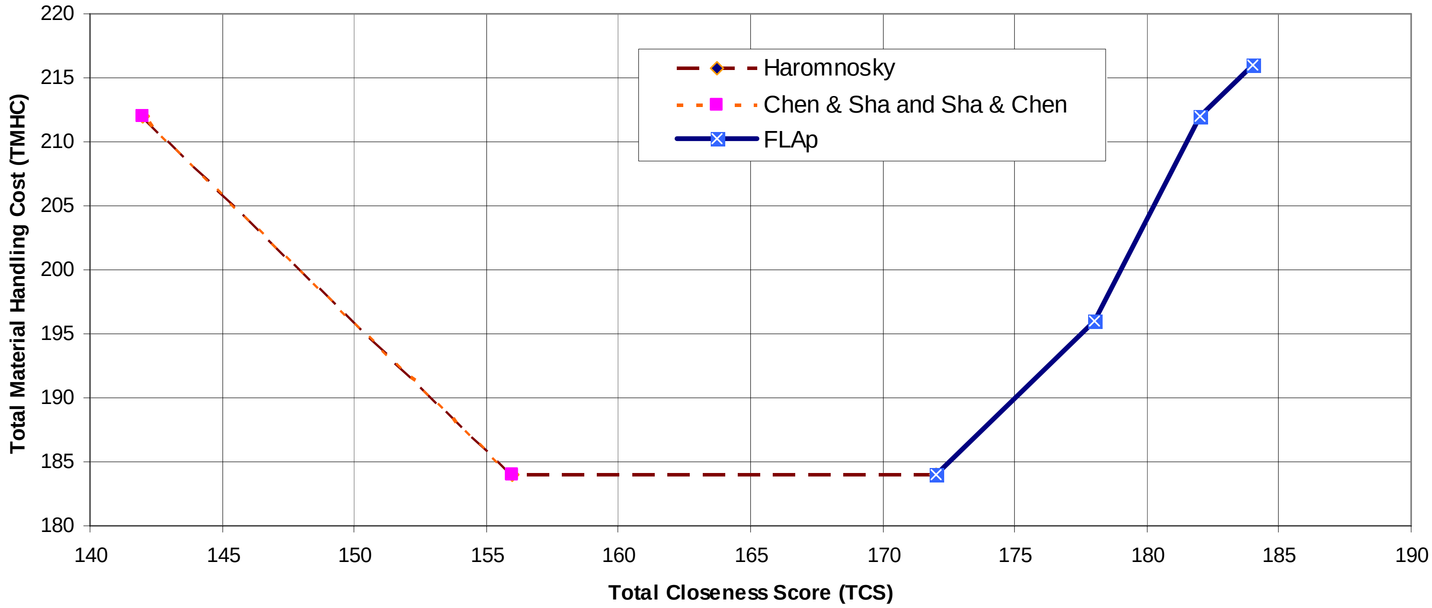

FIG. 5: THE SIX−DEPARTMENT PROBLEM (P2.1) RESULTS

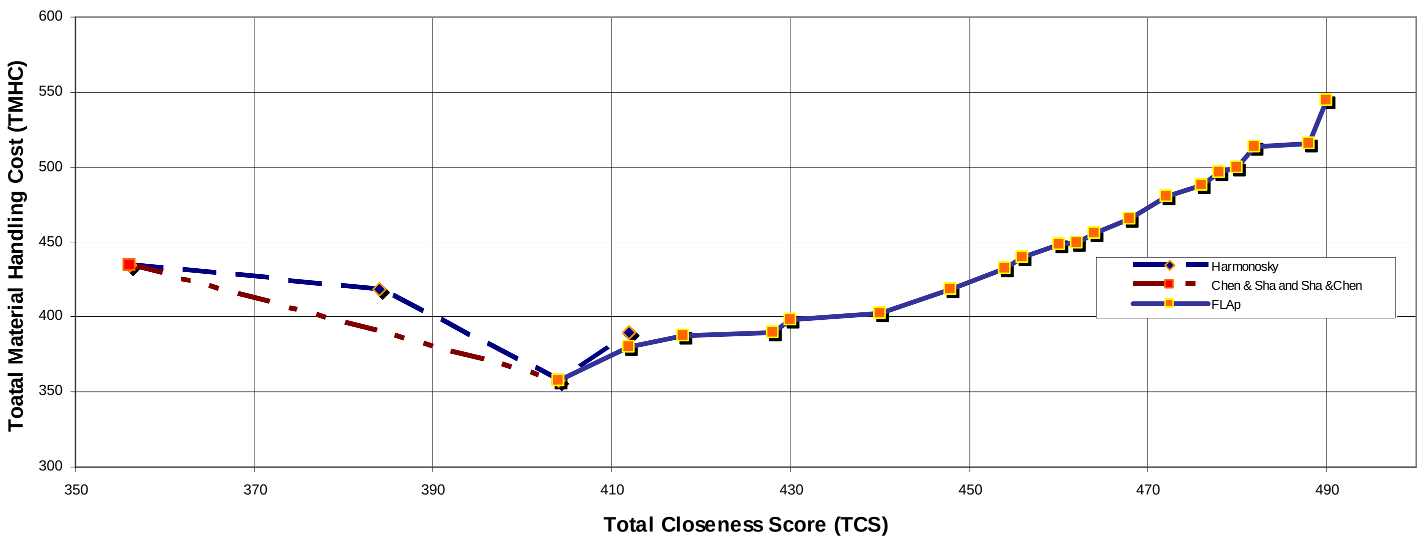

FIG. 6: THE EIGHT−DEPARTMENT PROBLEM (P2.2) RESULTS

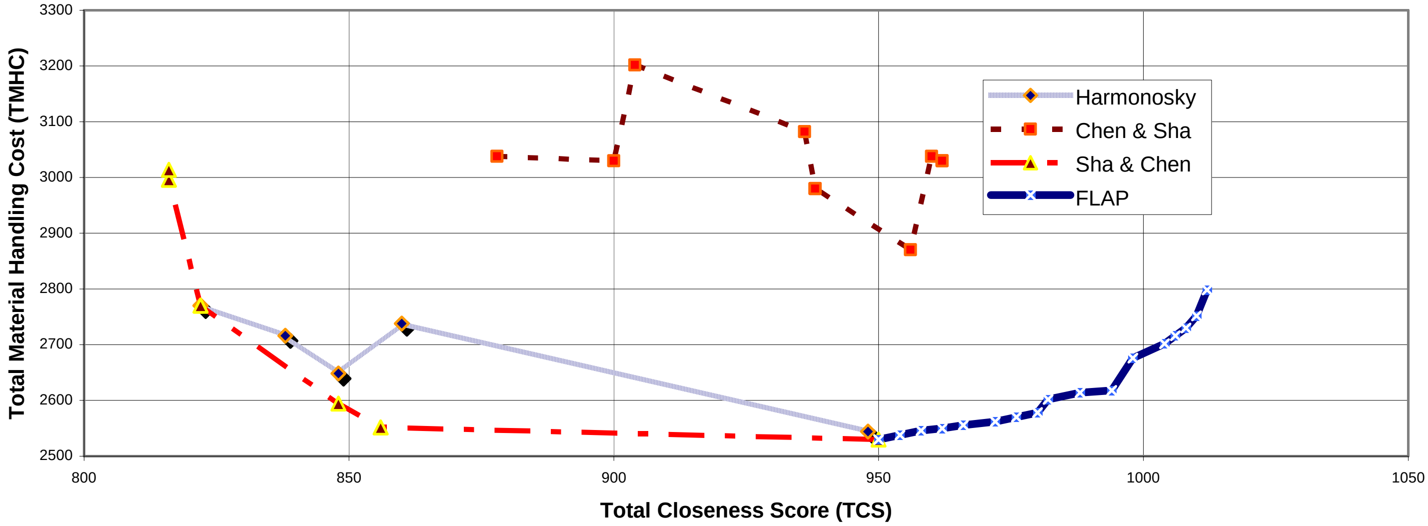

FIG. 7: THE TWELVE-DEPARTMENT PROBLEM (P2.3) RESULTS

TABLE 5: RESULTS SUMMARY OF SET #2

Problem ID	Author	# of Soln. Points	# of Efficient Points
P 2.1	Harmonosky	11	-
	Chen & Sha	11	-
	Sha & Chen	11	-
	FLAp	4	4
P 2.2	Harmonosky	11	1
	Chen & Sha	11	1
	Sha & Chen	11	1
	FLAp	20	20
P 2.3	Harmonosky	11	-
	Chen & Sha	11	-
	Sha & Chen	11	1
	FLAp	16	16

As the problem size increases the capability of FLAp to generate efficient solutions is better in both the efficient set size and the quality of solutions. Harmonosky [7], Chen & Sha [1] and Sha & Chen [17] made a mistake in finding the weighted sum by using a positive addition of the quantitative and the qualitative objectives. Instead, they should have used a negative value with either of them to find the correct weighted summation. This explains why they obtained only one efficient point for every problem

CONCLUSIONS

In this paper, a new formulation for the Multiple Objectives Layout Problem is proposed. The problem is formulated as an extension to the quadratic assignment problem. The objectives could be quantitative or qualitative and different scoring methods could be applied according to the objective's data type. The distance based objectives could be measured according to either the rectilinear or the Euclidean distance. To solve the problem, a new solution algorithm is proposed. The problem is coded into genetic algorithms; the revised Strength Pareto Evolutionary Algorithm (SPEAII) is used as a classifier to find the efficient set of solutions "Pareto Optimal set". A computer program is developed to build the mathematical model and implement the solution algorithms.

The proposed models and the solution approach outperformed all the bench mark problems obtained from the literature. All of the best known solutions have been improved and a lot of solutions are explored for the first time.

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