4C3-5

Low Complexity Capacity Estimation Method for Multiuser Block Diagonalization Transmission

Riichi Kudo, Yasushi Takatori, Jiang Wenjie, Kentaro Nishimori, Ohta Atsushi, Shuji Kubota
NTT Network Innovation Laboratories, NTT Corporation
E-mail: kudo.riichi@lab.ntt.co.jp

1. Introduction

As a technique to provide further improvement in frequency utilization, multiuser (MU)-MIMO transmission was proposed [1][2] for the downlink scenario in which the number of antenna branches at the mobile terminal (MT) is restricted because of the mobility requirement while the access point (AP) can possess a larger number of antenna branches. Because MU-MIMO transmission can generate a virtual reception array antenna, a large spatial diversity effect is expected compared to the case employing single user (SU)-MIMO. This paper focuses on a high signal to noise ratio (SNR) environment and the block diagonalization (BD) algorithm [3] as a practical MU-MIMO technique. When assuming use of the BD algorithm and perfect channel state information (CSI) at the AP, the data streams transmitted to different MTs do not interfere with each other.

When there are many MTs in the service area of an AP, the AP must determine the combination of MTs for MU-MIMO transmission due to the dimensionality requirements of the signal space [4]. However, a heavy calculation load is required to estimate the transmission quality using the BD algorithm. Therefore, we propose a low complexity capacity estimation method that utilizes the correlation matrix of the signal spaces corresponding to the supported MTs. The increase in the achievable bit rate is estimated using the decay factor product based on the correlation matrix. Computer simulation is used to compare the increase estimated using the proposed method to that estimated directly using the BD algorithm.

This paper is organized as follows. Section 2 describes the SU-MIMO and the MU-MIMO transmission and explains the achievable bit rate in SU-MIMO and MU-MIMO. Section 3 presents the estimation method for the achievable bit rate and analyzes the calculation load reduction. Finally, Section 4 presents simulation results comparing the estimated ratio and the actual ratio. Throughout the paper, the superscript *, the superscript H, and $\|.\|^2$ denote the complex conjugate, the Hermitian transpose, the squared Frobenius norm, respectively. Term $|\mathbf{A}|$ is the determination of $\mathbf{A}$.

2. Transmission Scheme

The downlink transmission of a narrowband multi-user MIMO system is considered. The number of users that the AP communicates with using the same frequency and timing is assumed to be M_a. The k-th MT has $M_r(k)$ receive antenna branches and the AP has M_t transmit antenna branches. Thus, the system can be denoted as an $M_t \times (M_r(1), M_r(2), \ldots, M_r(M_a))$ system. The CSI between the AP and the k-th MT, which is channel matrix $\mathbf{H}_k \in \mathbb{C}^{Mr(k)\times Mt}$, is assumed to be known at both ends. In this paper, we assume that the number of data streams is equal to that of the receive antenna branches.

To compare the achievable bit rates of MU-MIMO and SU-MIMO, the achievable bit rate for the k-th MT in SU-MIMO is shown. By using the singular value decomposition (SVD), the channel matrix is expressed as $\mathbf{H}_k = \mathbf{U}_k(\mathbf{\Sigma}_k \quad 0)(\overline{\mathbf{V}}_k \quad \tilde{\mathbf{V}}_k)^H$. Here, $\overline{\mathbf{V}}_k \in \mathbb{C}^{Mt\times Mr(k)}$ and $\tilde{\mathbf{V}}_k \in \mathbb{C}^{Mt\times(Mt-Mr(k))}$ represent the right singular vectors for the signal space and the null space, respectively. The diagonal elements of $\mathbf{\Sigma}_k$ are the square root of the eigenvalues of $\mathbf{H}_k$, $\lambda_{k,1}, \lambda_{k,2}, \ldots \lambda_{k,Mr(k)}$. When the eigenvector transmission is applied, the achievable bit rate in SU-MIMO is expressed as

$$C_s^{(M_a)} = \frac{1}{M_a}\sum_{i=1}^{M_a}\log_2\left|\mathbf{I} + \frac{\mathbf{H}_i\overline{\mathbf{V}}_i\overline{\mathbf{V}}_i^H\mathbf{H}_i^H}{M_r(i)\sigma_N^2}\right| = \sum_{i=1}^{M_a}\left(\frac{1}{M_a}\sum_{j=1}^{Mr(i)}\log_2\left(1+\frac{\lambda_{i,j}}{M_r(i)\sigma_N^2}\right)\right) = \sum_{i=1}^{M_a}C_{s,i}^{(M_a)} \tag{1}$$

where $C^{(Ma)}_{s,k}$ is the achievable bit rate for the k-th MT in time division multiplex access (TDMA) when the number of the MTs is M_a.

Next, as the MU-MIMO system, the BD algorithm transmission to the M_a MTs is considered. The received signal at the k-th MT, $\mathbf{y}_k \in \mathbb{C}^{Mr(k)\times 1}$, is expressed as

$$\mathbf{y}_k = \mathbf{H}_k\sum_{i=1}^{Ma}\mathbf{W}_i\mathbf{P}_i'\mathbf{x}_i + \mathbf{n}_k. \tag{2}$$

For fair resource allocation, the transmission power is assumed to be equally allocated among all MTs in the same time slot. Thus, $\|\mathbf{P}_1'\|_F^2 = \ldots = \|\mathbf{P}_{Ma}'\|_F^2$. Therefore, the diagonal elements of $\mathbf{P}_k'$ are expressed

ISBN: 978-4-88552-223-9 C3055©IEICE

as $p' = 1/\sqrt{M_a M_r(k)}$. Equation 1 shows that the k-th MT incurs interference from the data stream for the ($M_a - 1$) users except for the k-th MT. To avoid interference, the SVD is performed for $\mathbf{H}'_k$, as

$$\mathbf{H}'_k = \left(\mathbf{H}_1^T \quad \cdots \quad \mathbf{H}_{k-1}^T \quad \mathbf{H}_{k+1}^T \quad \cdots \quad \mathbf{H}_{Ma}^T\right)^T = \mathbf{U}'_k\left(\mathbf{\Sigma}'_k \quad 0\right)\mathbf{V}'^H_k = \mathbf{U}'_k\left(\mathbf{\Sigma}'_k \quad 0\right)\left(\overline{\mathbf{V}}'_k \quad \tilde{\mathbf{V}}'_k\right)^H \tag{3}$$

where $\mathbf{H}'_k$ is the total channel matrix except for $\mathbf{H}_k$, $\overline{\mathbf{V}}'_k$ represents the signal space of all users except for the k-th MT and $\tilde{\mathbf{V}}'_k$ represents the null space weight which does not interfere with the l-th MT, where $1 \le l \le M_a$ and $l \neq k$. Then the virtual transmission weights are calculated using the SVD for the null space channel matrix, $\mathbf{H}_k\tilde{\mathbf{V}}'_k$, as

$$\mathbf{H}_k\tilde{\mathbf{V}}'_k = \mathbf{U}''_k\mathbf{\Sigma}''_k\mathbf{V}''^H_k \quad , \tag{4}$$

where $\mathbf{\Sigma}''_k \in \mathbb{C}^{Mr(k)\times Mr(k)}$ is the diagonal matrix and the diagonal elements of $\mathbf{\Sigma}''_k$ are the square root of the eigenvalues of $\mathbf{H}_k\tilde{\mathbf{V}}'_k$, $\lambda'_{k,1}$, $\lambda'_{k,2}$, … $\lambda'_{k,Mr(k)}$. The transmission weight matrices for the k-th MT, $\mathbf{W}'_k$, are determined as $\tilde{\mathbf{V}}'_k\mathbf{V}''_k$ and the transmission qualities of their data streams correspond to the diagonal elements of $\mathbf{\Sigma}_k''$. Using the transmission weight matrix, $\mathbf{W}'_k$, Eq. 2 is rewritten as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{W}'_k\,\mathbf{P}'_k\mathbf{x}_k + \mathbf{n}_k\,. \tag{5}$$

The total achievable data rate for the BD algorithm is given as

$$C_m^{(M_a)} = \sum_{i=1}^{M_a}\log_2\left|\mathbf{I} + \frac{\mathbf{H}_i\mathbf{W}'_i\mathbf{W}'^H_i\mathbf{H}_i^H}{M_aM_r(i)\sigma_N^2}\right| = \sum_{i=1}^{M_a}\log_2\left|\mathbf{I} + \frac{\mathbf{H}_i\tilde{\mathbf{V}}_i\tilde{\mathbf{V}}_i^H\mathbf{H}_i^H}{M_aM_r(i)\sigma_N^2}\right| = \sum_{i=1}^{M_a}\sum_{j=1}^{Mr(i)}\log_2\left(1 + \frac{\lambda'_{i,j}}{M_aM_r(i)\sigma_N^2}\right) = \sum_{i=1}^{M_a}C_{m,i}^{(M_a)} \tag{6}$$

where $C^{(Ma)}_{m,k}$ is the achievable bit rate for the k-th MT when the AP transmits signals to M_a MTs simultaneously. Although we can estimate the transmission quality using the achievable bit rate, the calculation load becomes extremely high. This is because the AP must calculate the achievable bit rate corresponding to all combinations of users.

3. Estimation Method for Achievable Bit Rate

In this section, first, the decay factors of the eigenvalues in MU-MIMO are introduced. Then, the achievable bit rate, $C^{(kl)}_{m,k}$, is estimated using the proposed method. In addition, the calculation complexity level of the derived form is compared to that for the direct calculation method. Furthermore, the proposed method is extended to an arbitrary number of users.

The relationship between the eigenvalues in MU-MIMO with BD beamforming and the original eigenvalues, i.e. eigenvalues in SU-MIMO, is defined as

$$\lambda'_{k,i} = a_{k,i}^{(kl)}\lambda_{k,i}\,, \tag{7}$$

where $a^{(kl)}_{k,i}$ is the decay factor and $0 \le a^{(kl)}_{k,i} \le 1$. In a high SNR environment, the following inequality is satisfied. $1 << a_{k,i}^{(kl)}\lambda_{k,i}/\left(2M_r(k)\sigma_N^2\right) < \lambda_{k,j}/\left(M_r(k)\sigma_N^2\right)$. Using Eqs. 1, 6, and 7, the achievable bit rate in MU-MIMO, $C^{(Ma)}_{m,k}$, and that in SU-MIMO, $C^{(Ma)}_{s,k}$, are expressed as

$$C_{m,k}^{(M_a)} \approx \log_2\left|\frac{\mathbf{H}_k\tilde{\mathbf{V}}'_k\tilde{\mathbf{V}}'^H_k\mathbf{H}_k^H}{M_aM_r(k)\sigma_N^2}\right| = \sum_{i=1}^{Mr(k)}\log_2\left(\frac{\lambda_{k,i}}{M_aM_r(k)\sigma_N^2}\right),\quad C_{s,k}^{(M_a)} \approx \frac{1}{M_a}\log_2\left|\frac{\mathbf{H}_k\mathbf{H}_k^H}{M_r(k)\sigma_N^2}\right| = \frac{1}{M_a}\sum_{i=1}^{Mr(k)}\log_2\left(\frac{\lambda_{k,i}}{M_r(k)\sigma_N^2}\right) \tag{8}$$

Therefore, $C^{(Ma)}_{m,k}$, is rewritten as

$$\begin{aligned} C_{m,k}^{(M_a)} &\approx \sum_{j=1}^{Mr(k)}\left(\log_2\left(\frac{a_{k,j}^{(M_a)}\lambda_{k,j}}{M_aM_r(k)\sigma_N^2}\right)\right) = \sum_{j=1}^{Mr(k)}\left(\log_2\left(\frac{\lambda_{k,i}}{M_r(k)\sigma_N^2}\right)\right) + \sum_{j=1}^{Mr(k)}\log_2\frac{a_{k,j}^{(M_a)}}{M_a} \\ &= M_aC_{s,k}^{(M_a)} - M_r(k)\cdot\log_2 M_a + \log_2\prod_{j=1}^{Mr(k)}a_{k,j}^{(M_a)} = M_aC_{s,k}^{(M_a)} - M_r(k)\cdot\log_2 M_a + \log_2\Lambda_k^{(M_a)} \end{aligned} \tag{9}$$

where $\Lambda^{(Ma)}_k$ is the decay factor product expressed as

$$\Lambda_k^{(M_a)} = \prod_{j=1}^{Mr(k)}a_{k,j}^{(M_a)}. \tag{10}$$

The achievable bit rate for all MTs, $C^{(Ma)}_m$, can be estimated as

$$C_m^{(M_a)} \approx C_m'^{(M_a)} = M_a\sum_{i}^{M_a}C_{s,i}^{(M_a)} - \sum_{i=1}^{M_a}M_r(i)\cdot\log_2 M_a + \sum_{i=1}^{M_a}\log_2\Lambda_i^{(M_a)} \tag{11}$$

where $C'^{(Ma)}_m$ is the estimated achievable bit rate in the MU-MIMO system. Because $C^{(Ma)}_{s,k}$, M_a, and $M_r(k)$ are fixed numbers, the decay factor product, $\Lambda^{(Ma)}_k$, must be estimated to obtain $C^{(kl)}_{m,k}$. The amount of $\Lambda^{(Ma)}_k$ depends on the combination of M_a MTs.

Next, we focus on the decay factor product $\Lambda^{(Ma)}_k$. The determinant of the correlation matrix among received signals at the k-th MT using the BD algorithm is expressed as

$$\left|\mathbf{H}_k\tilde{\mathbf{V}}'_k\tilde{\mathbf{V}}'^H_k\mathbf{H}_k^H\right| = \prod_{i=1}^{Mr(k)}a_{k,i}^{(Ma)}\lambda_{k,i} = \Lambda_k^{(Ma)}\prod_{i=1}^{Mr(k)}\lambda_{k,i}\,. \tag{12}$$

The left side of Eq.12 is rewritten as

$$\left|\mathbf{H}_k\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\mathbf{H}_k^H\right| = \left|\mathbf{U}_k\mathbf{\Sigma}_k\overline{\mathbf{V}}_k^H\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\mathbf{\Sigma}_k\mathbf{U}_k^H\right| = \left|\mathbf{\Sigma}_k\overline{\mathbf{V}}_k^H\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\mathbf{\Sigma}_k\right| = \left|\overline{\mathbf{V}}_k^H\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\right| \cdot \prod_{i=1}^{Mr(k)}\lambda_{k,i}\,. \quad (13)$$

Here, since matrix, $\left(\overline{\mathbf{V}}_k \quad \tilde{\mathbf{V}}_k\right)$ is the unitary matrix, the relationship $\left|\overline{\mathbf{V}}_k{}^H\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\right| = \left|\mathbf{I} - \overline{\mathbf{V}}_k{}^H\overline{\mathbf{V}}_k'\overline{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\right|$ is satisfied. Therefore, the following relationship is derived from Eqs. 17 and 18.

$$\Lambda_k^{(Ma)} = \left|\overline{\mathbf{V}}_k^H\tilde{\mathbf{V}}_k'\tilde{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\right| = \left|\mathbf{I} - \overline{\mathbf{V}}_k^H\overline{\mathbf{V}}_k'\overline{\mathbf{V}}_k'^H\overline{\mathbf{V}}_k\right|. \quad (14)$$

Therefore, the decay factor product, $\Lambda^{(Ma)}{}_k$, is calculated using the signal space of the k-th MT and the signal space of all MTs except for the k-th MT.

Now, $\overline{\mathbf{V}}_k'$ is calculated using the QR decomposition as

$$\left(\overline{\mathbf{V}}_1 \ \cdots \ \overline{\mathbf{V}}_{k-1} \ \ \overline{\mathbf{V}}_{k+1} \ \cdots \ \overline{\mathbf{V}}_{M_a}\right) = \overline{\overline{\mathbf{V}}}_k = \mathbf{Q}_k\begin{pmatrix}\mathbf{R}_k \\ \mathbf{0}\end{pmatrix} = \left(\overline{\mathbf{V}}_k' \quad \tilde{\mathbf{V}}_k'\right)\begin{pmatrix}\mathbf{R}_k \\ \mathbf{0}\end{pmatrix}, \quad (15)$$

where $\overline{\overline{\mathbf{V}}}_k$ is $\left(\overline{\mathbf{V}}_1 \ \cdots \ \overline{\mathbf{V}}_{k-1} \ \ \overline{\mathbf{V}}_{k+1} \ \cdots \ \overline{\mathbf{V}}_{M_a}\right)$, $\mathbf{R}_k \in \mathbb{C}^{Mr'(k)\times Mr'(k)}$ is the upper triangular matrix, $\mathbf{Q}_k \in \mathbb{C}^{Mt\times Mt}$ denotes the unitary matrix, the column vectors of $\overline{\mathbf{V}}_k' \in \mathbb{C}^{Mt\times Mr'(k)}$ correspond to the diagonal elements of $\mathbf{R}_k$, and the column vectors of $\tilde{\mathbf{V}}_k' \in \mathbb{C}^{Mt\times(Mt\ -\ Mr'(k))}$ correspond to zero. Here, $M'_r(k)$ is expressed as $M_r'(k) = \sum_{i=1, i\neq k}^{M_a} M_r(i)$. Therefore, $\overline{\mathbf{V}}_k'$ is expressed as $\overline{\mathbf{V}}_k' = \overline{\overline{\mathbf{V}}}_k\mathbf{R}_k^{-1}$. Using Eqs. 14 and 15, the decay factor product, $\Lambda^{(Ma)}{}_k$ is expressed as

$$\Lambda_k^{(Ma)} = \left|\mathbf{I} - \overline{\mathbf{V}}_k^H\overline{\overline{\mathbf{V}}}_k\mathbf{R}_k^{-1}\left(\mathbf{R}_k^{-1}\right)^H\overline{\overline{\mathbf{V}}}_k^H\overline{\mathbf{V}}_k\right| = \left|\mathbf{I} - \overline{\mathbf{V}}_k^H\overline{\overline{\mathbf{V}}}_k\left(\mathbf{R}_k^H\mathbf{R}_k\right)^{-1}\overline{\overline{\mathbf{V}}}_k^H\overline{\mathbf{V}}_k\right|. \quad (16)$$

Furthermore, $\mathbf{R}^H{}_k\mathbf{R}_k \in \mathbb{C}^{Mr'(k)\times Mr'(k)}$ is rewritten as

$$\mathbf{R}_k^H\mathbf{R}_k = \begin{pmatrix} \mathbf{I} & \overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_2 & \cdots & \overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{k-1} & \overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{k+1} & \cdots & \overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{M_a} \\ \left(\overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_2\right)^H & \mathbf{I} & & \overline{\mathbf{V}}_2^H\overline{\mathbf{V}}_{k-1} & \overline{\mathbf{V}}_2^H\overline{\mathbf{V}}_{k+1} & \cdots & \overline{\mathbf{V}}_2^H\overline{\mathbf{V}}_{Ma} \\ \vdots & & \ddots & & \vdots & & \vdots \\ \left(\overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{k-1}\right)^H & & & \mathbf{I} & \overline{\mathbf{V}}_{k-1}^H\overline{\mathbf{V}}_{k+1} & \cdots & \overline{\mathbf{V}}_{k-1}^H\overline{\mathbf{V}}_{Ma} \\ \left(\overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{k+1}\right)^H & & & & \mathbf{I} & & \overline{\mathbf{V}}_{k+1}^H\overline{\mathbf{V}}_{Ma} \\ \vdots & & & & & \ddots & \vdots \\ \left(\overline{\mathbf{V}}_1^H\overline{\mathbf{V}}_{M_a}\right)^H & \cdots & & & & \cdots & \mathbf{I} \end{pmatrix} \quad (17)$$

where $\mathbf{R}^H{}_k\mathbf{R}_k$ is a Hermitian matrix and consists of the matrices calculated from the signal space of each MT. Although, the size of $\mathbf{R}^H{}_k\mathbf{R}_k$ is large, the inverse matrix of $\mathbf{R}^H{}_k\mathbf{R}_k$ does not requires $M_r'(k)^3$-order calculations due to the special form of the $\mathbf{R}^H{}_k\mathbf{R}_k$. When the number of MTs is three, the inverse matrix of $\mathbf{R}^H{}_k\mathbf{R}_k$ is expressed as

$$\left(\mathbf{R}_k^H\mathbf{R}_k\right)^{-1} = \begin{pmatrix} \mathbf{I} & \overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n \\ \left(\overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\right)^H & \mathbf{I}\end{pmatrix}^{-1} = \begin{pmatrix} \left(\mathbf{I} - \overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\overline{\mathbf{V}}_n^H\overline{\mathbf{V}}_m\right)^{-1} & -\overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\left(\left(\mathbf{I} - \overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\overline{\mathbf{V}}_n^H\overline{\mathbf{V}}_m\right)^{-1}\right)^H \\ -\left(\overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\right)^H\left(\mathbf{I} - \overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\overline{\mathbf{V}}_n^H\overline{\mathbf{V}}_m\right)^{-1} & \left(\left(\mathbf{I} - \overline{\mathbf{V}}_m^H\overline{\mathbf{V}}_n\overline{\mathbf{V}}_n^H\overline{\mathbf{V}}_m\right)^{-1}\right)^H \end{pmatrix} \quad (18)$$

where m and n satisfy that $1 \leq m < n \leq 3$ and $k \neq m \neq n$. Therefore, the estimated achievable bit rate, $C_m'^{(3)}$, in Eq. 13 is estimated as

$$C_m'^{(3)} = 3C_s^{(3)} - \sum_{i=1}^{3}M_r(i)\log_2 3 + \sum_{i=1}^{3}\log_2\det\left(\mathbf{I} - \overline{\mathbf{V}}_k^H\overline{\overline{\mathbf{V}}}_k'\left(\mathbf{R}_k^H\mathbf{R}_k\right)^{-1}\overline{\overline{\mathbf{V}}}_k'^H\overline{\mathbf{V}}_k\right). \quad (19)$$

4. Effectiveness of Proposed Method

In this section, we evaluate the effectiveness of the proposed method in terms of the calculation load and the estimation accuracy. The number of MTs that communicate with the AP simultaneously is assumed to be three.

4.1. Calculation Load

We assume that the AP has already transmitted signals to all MTs using the eigenvector beamforming in SU-MIMO. Thus, the AP knows $\overline{\mathbf{V}}_k$, the eigenvalues, $\lambda_{k,1}, \ldots, \lambda_{k,Mr(k)}$ corresponding to all MTs, and the achievable bit rate in SU-MIMO, $C_{s,k}$. The calculation load of the proposed method is compared to that for the direct estimation in the BD algorithm. As a parameter to express the calculation load, the number of multiplications of the complex number is applied. Table 1 shows the calculation load in the direct estimation and the proposed method, respectively. The calculation load of the proposed estimation method is proportional to the number of the transmit antenna branches while that of the direct estimation in the BD algorithm is proportional to the cube of the number of the transmit antenna branches.

4.2. Estimation Accuracy

The effectiveness of the proposed method is evaluated by computer simulation. The channel matrix between the AP and the k-th MT, $\mathbf{H}_k$, is assumed to be a complex Gaussian variable with zero mean (random i.i.d.). $||\mathbf{H}_k||^2$ is expressed as $M_t \times M_r(k) \times \Gamma$ and σ_N^2 is assumed to be one. Thus, Γ expresses the expectation of the received SNR between the AP and the k-th MT in a single input single output (SISO) channel. We assume that the environment was quasi-static and the numbers of the transmit and receive antenna branches are 8 ($M_t = 8$), and 2 ($M_r = 2$), respectively. The MU-MIMO transmission for 3 MTs ($M_u = 3$) is considered. In the computer simulation, we calculate the actual achievable bit rate in MU-MIMO and the estimated achievable bit rate. Computer simulations are performed 1000 times when Γ is 25 dB. Figure 1 shows the actual achievable bit rate versus the estimated one. The proposed estimation method is very effective in a high achievable bit rate region. In a low achievable bit rate region, the error of the estimation increases. The reason for this is that the approximation in Eq. 11 does not hold true when $a^{(kl)}_{k,i}\lambda_{k,i}$ becomes low. Figure 1 clarifies that the error decreases as both the achievable bit rate becomes high. Thus, the proposed method estimates the capacity ration accurately when the MU-MIMO effect becomes high. For the user selection in MU-MIMO, the estimation accuracy of the proposed method is sufficient for practical use because the AP selects the MTs that have a high achievable bit rate.

5. Conclusion

This paper proposed a low complexity capacity estimation method for the BD algorithm in MU-MIMO systems. Theoretical analysis showed that the achievable bit rate in MU-MIMO was obtained from the determinant of the correlation matrix corresponding to the signal space of each MT. The proposed method reduced the calculation load compared to the direct estimation in the BD algorithm. The number of complex number multiplications is proportional to the number of the transmit antenna branches while that of the direct estimation is proportional to the cube of the number of the transmit antenna branches. Furthermore, the simulation results confirmed that the proposed estimation method estimate the achievable bit rate in MU-MIMO with sufficient accuracy for practical systems.

Acknowledgments

This work is supported by the Ministry of Internal Affairs and Communications, Japan, under the grant, “Research and development of fundamental technologies for advanced radio frequency spectrum sharing in mobile communication systems.”

References

[1] Q.H. Spencer, C.B. Peel, A. L. Swindlehurst, M. Haardt, “An introduction to the multi-user MIMO downlink,” *IEEE Commun. Magazine*, vol. 42, no. 10, pp. 60-67, Oct. 2004.

[2] M. Costa, “Writing on dirty paper,” *IEEE Trans. Inform. Theory*, vol. 29, no.3, pp. 439-441, May 1983.

[3] Q.H. Spencer, A.L. Swindlehurst, M. Haardt, “Zero-Forcing Methods for Downlink Spatial Multiplexingin Multiuser MIMO Channels,” *IEEE Trans. Sig. Processing*, vol. 52, issue 2, Feb. 2004

[4] S. Sigdel and W. A. Krzymien, “Antenna and User Subset Selection in Downlink Multiuser Orthogonal Space-Division Multiplexing,” *WPMC 2006 proceeding*, pp. 671-675, Sep. 2006.

TABLE 1: ROUND NUMBERS OF COMPLEX NUMBER MULTIPLICATIONS WHEN Ma = 3

Direct estimation	*Proposed method*
$3(2Mt^3 - 5Mr^2Mt^2 + (6Mr^2 - Mr)Mt)$	$3(3Mr^2Mt + 9Mr^3)$

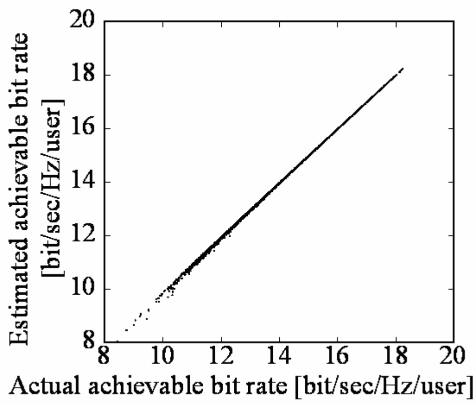

Fig. 1. Estimated achievable bit rate versus actual bit rate.