Investigating mechanism of inclined CPT in granular ground using DEM

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Abstract. This paper presents an investigation on mechanism of the inclined cone penetration test (CPT) using the numerical discrete element method (DEM). A series of penetration tests with the penetrometer inclined at different angles (i.e., 0°,15°, 30°, 45° and 60°) were numerically performed under μ=0.0 and μ=0.5, where μ is the frictional coefficient between the penetrometer and the soil. The deformation patterns, displacements of soil particles adjacent to the cone tip, velocity fields, rotations of the principal stresses and the averaged pure rotation rate (APR) were analyzed. Special focus was placed on the effect of friction. The DEM results showed that soils around the cone tip experienced complex displacement paths at different positions as the inclined penetration proceeded, and the friction only had significant effects on the soils adjacent to the penetrometer side and tip. Soils exhibited characteristic velocity fields corresponding to three different failure mechanisms and the right side was easier to be disturbed by friction. Friction started to play its role when the tip approached the observation points, while it had little influence on rotation rate. The normalized tip resistance ($q_c = f / \sigma_{v0}$) increased with friction as well as inclination angle. The relationship between q_c and relative depth (y/R) can be described as $q_c = a \times (y/R)^{-b}$, with parameters a and b dependent on penetration direction. The normalized resistance perpendicular to the penetrometer axis q_p increases with the inclination angle, thus the inclination angle should be carefully selected to ensure the penetrometer not to deviate from its original direction or even be broken in real tests.

Keywords: Inclined cone penetration; Distinct element method; Tip resistance; Stress rotation; Particle rotation.

1. Introduction

The cone penetration test (CPT) is a reliable, fast and relatively economical in-situ test to obtain information about soil stratification and mechanical properties. When the cone-shaped penetrometer is pushed into the ground, the soil experiences the compression, shear deformation and plastic flow, thus making the mechanism of CPT complicated. Many investigations have been performed on the CPT mechanism in the past and they can be attributed to three methods in general: (1) analytical methods: the bearing capacity theory [1-3] and the cavity expansion theory [4,5]; (2) experimental methods: laboratory chamber calibration tests [6,7] and centrifuge methods [8]; (3) numerical analysis methods: small strain finite-element method [9], large strain finite-element method [10,11], strain path method [12] and the distinct element method (DEM) [13,14]. Nevertheless, these studies focus on the vertical CPT as an axisymmetric boundary problem.

In the in-situ test, due to the presence of existing buildings and infrastructures or lack of access, the CPT technique cannot always be performed in the vertical orientation, thus an inclined CPT is necessarily performed instead. However, it is unclear whether the penetration mechanism of an inclined CPT still keeps the same way in which the vertical penetration behaves. Therefore, a few studies have been performed on the non-vertical penetration mechanism. Among them, Broere [15] performed the CPTs horizontally and vertically in a 2 m rigid wall calibration chamber using a 36-mm cone and showed evident differences between horizontal and vertical CPT measurements. Wei et al. [10, 11] used a large-strain finite element method to analyze the effect of soil anisotropy on the inclined CPT in normally consolidated cohesive soils. The results showed that the tip resistance increases with increasing inclination angle as the coefficient of earth pressure at rest (K_0) below 1.0.

The study on the inclined CPT still remains insufficient, especially its mechanism considering the interaction between the soil and penetrometer. Therefore, the purpose of the current paper is to present the numerical analyses on the mechanism of an inclined CPT with the focus on the effect of friction. The penetration mechanism was discussed in terms of deformation pattern, velocity field, stress rotation and APR under different penetrometer-soil friction, where the penetration angle was specified to be 30°. Then the relationship between the normalized tip resistance and the inclination angle was examined with two values of coefficient of friction. Another four values of inclination angles (i.e., 0°, 15°, 45°, and 60°) were considered.

2. DEM modeling of CPT

2.1 Ground characteristics

The granular ground is simulated in the current study, which is composed of ten types of disks with a grain size distribution shown in Fig. 1. The maximum and minimum diameter of soil particles are 9 mm and 6 mm respectively. It has an average grain diameter d_{50} = 7.6 mm and uniformity coefficient d_{60}/d_{10} = 1.3.

The macro mechanical behavior of the ground material, which consists of 24000 particles with planar void ratio of 0.27, was investigated using the simulations of biaxial tests under a compression rate of 10%/min and confining pressures of 50 kPa, 100 kPa and 200 kPa. Fig. 2 illustrates the basic mechanical properties of the granular ground. The material shows typical characteristics of a loose ground and the peak internal friction angle of the material has found to be 15.37°.

2.2 Model setup

The dimension of the penetrometer and ground in the simulations needs to be carefully selected in order to minimize the boundary effect and obtain rational results in a DEM model with the minimum particle number. Bolton et al.[16] pointed out that the cone diameter D should be at least 20 times greater than the mean grain size, and in such simulation the possible error in q_c (tip resistance) is at most 10%. Meanwhile Jiang et al [13] suggested that there should be no less than 13 particles contacting with the tip face in order to get a steady q_c. Based on these two findings the cone diameter was set as 0.16 m in the current study. Hence, the value of D/d_{50}=21.05>20 and the penetrometer size can ensure that the tip can be always in contact with about 13 particles and thus can provide acceptable resistance values. The penetrometer was composed of rigid walls. The frictional coefficient μ between the penetrometer and the soil was chosen to be 0.0 to simulate a perfectly smooth condition and 0.5 for comparison. The parameters of the granular ground material adopted in the current simulations are presented in Table 1.

Bolton et al [16] also suggested that no apparent increase in q_c (tip resistance) for a test done with $W / R \geq 40$, where R and W are the cone radius and the width of the ground, respectively. Therefore the ground was set to be 5.0 m in width and 1.626 m in depth，resulting in a value of W / R=62.5, which satisfied the aforementioned criterion.

The multilayer under-compaction method (UCM) proposed by Jiang et al [17] was employed here to ensure homogeneity of ground sample before consolidation under gravity. Thus, five equal layers of particles were generated in a sequential way, with each layer containing 30000 particles and randomly deposited into a rectangular container to form the granular ground shown in Fig. 3(a). To achieve the target planar void ratio of 0.27, the accumulated layers of particles were compacted to an intermediate void ratio which is slightly higher than the target void ratio when each new layer was added. According to the under-compaction criterion proposed by Jiang et al. [17], the intermediate void ratios for the accumulated layers were; $e_{p(1)}$=0.29, $e_{p(1+2)=}$0.289, $e_{p(1+2+3)=}$0.284, $e_{p(1+2+3+4)=}$0.276 and $e_{p(1+2+3+4+5)=}$0.27. During the generation process, the wall-particle is frictionless in order to improve the homogeneity, while inter-particle frictional coefficient is chosen to be 1.0 in order to produce a loose packing of particles.

After the sample was generated, it was subjected to an amplified gravity field of 20g similar to the centrifuge modeling. When the equilibrium of the entire system was achieved, the penetrometer was generated at a distance of 3.0 m from the left boundary of the ground in horizontal direction and driven downward along an inclined direction at a constant rate of 1 m/s, as shown in Fig.3 (a). The relative high penetration rate was used to reduce the computational time and would not have a significant influence on the CPT results [18]. The configuration of CPT model after consolidation is illustrated in Fig. 3(a) and the layout of selected observation points accompanied by two measurement circles is illustrated in Fig. 3(b).

2.3 Features of the ground

The distribution of initial horizontal and vertical stresses as a function of depth is illustrated in Fig. 4. As known in geo-mechanics, ground density can be calculated as:

$$\rho=\rho_S(1+w)/(1+e) \tag{1}$$

Where w is the water content and $w=0$ in the current study as only dry soils are considered; ρ_S is the particle density and $\rho_S=2600$ kg/m^3. Therefore, given the void ratio, the ground density can be obtained as 2047 kg/m^3. Thus the relationship between the initial vertical stress and the corresponding depth can be written as

$$\sigma_{v0}=\rho(20g)y=32097\times(y/R) \tag{2}$$

The measurement circles were adopted to calculate the average stress from the contact forces between particles with centroids located within the measurement circle. Two factors were considered when arranging the measurement circles: a) the measurement circle should not be too small in size so as to include enough particles to reduce the statistical error; b) the measurement circle should not be too big otherwise the localized characteristics will be smoothed and cannot be clearly discovered. Therefore the diameter of the measurement circle in the current study was chosen to be 0.18 m, which can meet the aforementioned requirements. The vertical and horizontal stresses as obtained in the measurement circles are shown in Fig. 4. It can be seen that the vertical stress increases linearly with depth from 0 to 600 kPa, and the relationship between initial vertical stress and relative depth is $\sigma_{v0}=32693\times(y/R)$, which is in good agreement with the theoretical solution in Eq.(2). The horizontal stress was observed to keep a constant ratio over the vertical stress, i.e. K_0=0.58 when y/R<27. However, it begins to deviate slightly from its initial linearity when y/R>27. This is possibly due to the kinematic constraint by the bottom boundary and similar phenomenon can also be found on retaining walls for a finite media by several researchers (e.g. [19]).The overall ground can still be assumed as a half-infinite media, though there is a slight deviation from the theoretical K_0 condition.

3 Simulation results

3.1 Deformation pattern

3.1.1 Grid deformation

The painted grid method proposed by Jiang et al [13] is employed here to investigate the grid deformation. The gird size should be carefully chose in order to capture the high gradients of variables in the soil near the penetrometer and capable of representing a 'continuum element' from the viewpoint of micro-and-macro mechanics. Hence, the width and height of grid was set close to R, which can meet the two aforementioned demands. The grid deformation in the conditions of μ=0.0 and μ=0.5 with inclination of 30° is illustrated in Fig. 5. Here, the inclination angle was defined as the vertical direction to the central axis of penetrometer. Fig. 5 shows that when the tip is driven into the ground, the penetration results in heaving of the ground surface, which is more remarkable on the left side than on the right side. The grids were stretched vertically on the left side and horizontally on the right side, which indicates that the soils on the left side underwent dilation, while the soils on the right side mainly underwent compaction. Similar phenomenon can be observed for μ=0.5, however, the grids adjacent to the penetrometer and the tip were distorted severely and the initial shape can hardly be recognized in the process of penetration. It can be concluded that the effect of friction is particularly evident in the soils adjacent to the penetrometer and the tip. Such case cannot be simulated well by the finite element method, which is only capable of dealing with small deformation problem. Therefore, the CPT simulation using by the distinct element method is of great advantage.

3.1.2 Particle trajectories

The trajectories of 48 particles were recorded until the relative depth y/R=13.5 was reached as shown in Fig. 6. In the case of μ=0.0, the particles on the left side mainly move outwards and then upwards at y/R=1.5. The particles close to the penetrometer move downwards then outwards, while other particles move outwards and then upwards at y/R =5.5, 9.5. However, the particles near the tip (y/R=13.5) only move outwards with few vertical movements. Contrasting to the movements on the left side, particles on the right side all move downwards and then outwards. These phenomena indicate that the soil on the left side tends to heave and expand laterally as observed on the ground, while the soil on the right side experience compression. This is in good agreement with the grid deformation as shown in Fig. 5. For a further comparison, the final positions of particles in the two cases were plotted together in Fig.7 to investigate the effect of friction. Figure 7 shows that the friction has little influence on soil compaction on the right side. The particles close to penetrometer were dragged down due to the drag force produced by friction and this influence is only significant along the penetrometer.

3.2 Velocity fields

The evolution of maximum particle velocity is shown in Fig. 8, where each datum plotted represents the maximum particle velocity in the granular ground at the time when the tip reaches specific relative depth during the penetration. Fig. 8 shows that when the tip was initially pushed into the ground, the soil particles started to move from a static state, which resulted in an abruptly increase in velocity followed by fluctuations around a steady value, indicating a stable state of penetration. The particles were able to move along with the penetrometer due to the frictional drag force in the case of μ=0.5, where the maximum velocity approached the speed of penetrometer (1m/s). However, in a perfectly smooth case, the maximum velocity was only 0.63 m/s.

Normalized by the corresponding maximum velocity in each case (values can be found in Fig. 8), all velocities of particles were divided into seven groups of magnitudes and rendered with different colors as shown in Fig. 9.The velocity vectors described by different colors represent the sliding lines of particles, which in turn can reflect the failure mechanism. Fig. 9(a) to Fig. 9(c) shows that the maximum velocity group appears near the tip of the penetrometer, while the particles next to both sides of the penetrometer all move at relative low velocity. The zone of the maximum velocity group on the left side is larger than that on the right side. As illustrated in Fig. 9, the velocity fields at different relative depths show different shapes. Previous research on the vertical CPT [20] demonstrated that these velocity fields can be classified as three typical failure mechanisms [1, 21-24], as illustrated in Fig. 10. By comparing the velocity fields near the tip in the perfectly smooth case as shown in Fig. 9(a) to Fig. 9(c) with the sliding lines in Fig. 10, it can be found that soils in the inclined CPT also experience three failure mechanisms successively as the depth increases, i.e., Terzaghi mechanism for shallow penetration followed by Biarez and Hu mechanism for medium penetration, and finally Berezantev and Vesic mechanism for deep penetration. All the three mechanisms are observed on the left side, while only the second and third mechanisms are captured on the right side, as seen from Fig. 9(a) to Fig. 9(c), since the right-half of the tip disturbs deeper soils than the left-half. In contrast to the perfectly smooth case, particles adjacent to both sides of the penetrometer exhibit relative high velocities due to the effect of friction, while only a very small region is influenced by the penetration. The failure mechanism on the right side retains the same as that in the case of μ=0, while on the left side, Terzaghi mechanism remains for the shallow penetration and then only Berezantev and Vesic mechanism is observed at the medium and deep penetration.

3.3 Stress rotation and APR

Two measurement circles as shown in Fig. 3(b) were arranged to investigate the stress rotation of soil. Three factors were considered in determining the position: 1) the observation points should be placed at a depth when the penetration gets steady; 2) the position should be close enough to the central axis in order to capture the features of the stress variation of soils adjacent to the penetrometer; 3) the area covered by the measurement circles should be guaranteed not to be overlapped by the penetrometer when it passes by. As mentioned before, the penetration reached stable soon after the tip is pushed into the ground, thus the locations of the measurement circles at a relative depth y/R=13.5 can ensure a steady penetration before the tip approaches that depth. The other two factors were checked to be reasonable in the simulation process.

Fig. 11 provides the inclination angles of the major principal stresses with respect to the vertical direction as measured in the measurement circles 29 and 32 during penetration. The initial orientation of the major principal stress is in the vertical direction, i.e. inclination angle = 0°. A positive angle represents an anticlockwise stress rotation and vice versa. Both frictional case and smooth case are considered in Fig. 11.

Fig.11 shows that in the case of μ=0, the principal stresses in both measurement circles undergo large rotations with values of over 180° on the left side and nearly 180° on the right side. Before the penetration started, the major principal stresses all head vertically as K_0=0.58. When the tip was initially pushed into the ground, the soil along the central axis line of penetrometer contacted tightly because of compaction, and the principle stresses on both sides of penetrometer tended to be parallel to penetration direction. Therefore, the major principal stress moved from the vertical to the compaction direction. That's why the two observation points initially rotated clockwise when penetration occurred at shallow depth. When the tip approached the two observation points, the influence of the tip face became significant. The principle stress at the observation points tended to become perpendicular to the tip face, as a result, the principal stress at the left observation point continually rotated clockwise, while the principal stress at the right observation point began to rotate counterclockwise. When the tip passed over the two observation points, the penetrometer side began to take effect instead of tip, thus resulting in an apparent leap. After that, the stress rotation tends to be constant, especially on the right side. From these observations it can be inferred that the effect of side friction on the stress rotation of the soil adjacent to penetrometer is constant once penetration gets steady. This phenomenon is almost the same in the case of μ=0.5 except more rotation on the right side.

Fig.12 presents the average pure rotation rates (APR) within the measurement circles 29 and 32 during penetration. 12. APR is denoted by w_3^c and defined in [25] as

$$\omega_3^c = \frac{1}{N}\sum_{k=1}^{N}\dot{\theta}^k = \frac{1}{N}\sum_{k=1}^{N_c}\left[\frac{1}{r^k}(\dot{\theta}_1^k r_1^k + \dot{\theta}_2^k r_2^k)\right] \tag{3}$$

where the summation is over the N_c particle contacts in a measurement circle. Contact k is between two particles with the radii of r_1^k and r_2^k, and the angular velocities of $\dot{\theta}_1^k$ and $\dot{\theta}_2^k$ (positive denoting counter-clockwise rotation), respectively. r^k is the common radius defined as

$$r^k = \frac{2r_1^k r_2^k}{r_1^k + r_2^k} \tag{4}$$

APR is a microscopic kinematic variable to describe the rotation features of particles, which is important but neglected in continuum mechanics. Fig. 11 shows that friction has no apparent effect on the rotation rate. Therefore, only two APRs in perfectly smooth penetration are investigated here. It is interesting to note that the sign of APRs are generally the same with the principal stress rotation angles. Moreover, the magnitudes of APRs are closely associated with the rotation angle of the principal stresses. These observations indicate that the continuum-based qualities such as the principal stress direction may be related to the micro-scale particle behavior to a certain extent, which is worth further study.

3.5 Normalization of tip resistance in the inclined penetration

For geotechnical engineers, the tip resistance q_c in a typical CPT is of great interest since q_c is important and useful in determining the bearing capacity and relative density of a ground. In addition to the previous simulations with an inclination angle of 30°, the study is extended further to examine the effect of the inclination angle with values of 0°, 15°, 45° and 60°. Every penetration was performed with two different coefficients of friction between penetrometer and particles. The tip resistance q_c is obtained by the summation of the contact force components exerted on the tip parallel to the central axis of the penetrometer divided by the penetrometer diameter or a half. For convenience in the analysis, normalized tip resistance was adopted in this paper in our post process, as shown in Eqs. (5)-(7):

$$q_c = \frac{f_{c.left} + f_{c.right}}{D \cdot \sigma_{vo}} \tag{5}$$

$$q_{c.left} = \frac{f_{c.left}}{(D/2) \cdot \sigma_{vo}} \tag{6}$$

$$q_{c.right} = \frac{f_{c.right}}{(D/2) \cdot \sigma_{vo}} \tag{7}$$

where $f_{c.left}$ and $f_{c.right}$ correspond to the summation of the contact force components exerted on the tip parallel to the central axis of the penetrometer, respectively. D is the cone diameter and σ_{v0} is the initial vertical stress in the ground, as shown in Fig. 4.

Fig.13 provides the relationship between the normalized resistance and the relative depth (y/R) in different penetration directions for the two values of friction. In each figure, the resistances on both sides together with resultant resistance are included. It is shown in the figure that similar to the field tests, the resistances in the simulations are quite fluctuating. The resistances on both sides show similar developing trend and are virtually equal in vertical penetration due to the symmetric stress condition. On the contrary, $q_{c,right}$ tends to be larger than $q_{c,left}$ at shallow depth when inclined penetration occurs and this phenomenon is more significant as inclination angle increases. Further investigation shows that the tip resistances on both sides finally approach a same value at a relatively deep depth. This may be explained in view of stress conditions in which the side experienced: when the inclined CPT initially began, the stress condition was quite different where the stress was larger on the right side and this resulted in a higher resistance as shown in Figure 13. As penetration continued, the stress difference tended to be smaller and the resistances then grew synchronously. Same as $q_{c,right}$, q_c also decays with penetration depth in a decreasing rate. Fitting curves are proposed in the form of $q_c = a\times(y/R)^{-b}$, where a and b are two parameters varying with penetration direction. At the same penetration depth, q_c gradually increases as the penetration direction changes gradually from a vertical direction to 60°. These observations are consistent with the investigation described in [15], where the tip resistance measured in the horizontal direction is about 20% larger than that in the vertical. The similar phenomenon observed in DEM simulation and chamber tests can be explained by the soil stress state K_0=0.58, i.e. the vertical stress is higher than the horizontal stress. Nevertheless, it is evident in the figure that the friction results in higher tip resistance, which can be easily explained as that more energy is required to compensate the work done by the frictional force.

Curves shown in Fig. 14 were given to compare the evolution trend, from which it can be easily found that the difference of normalized tip resistance tends to decrease with increasing depth regardless of friction. The relationship between parameters (a,b) and inclination angles is shown in Fig. 15. In the smooth condition, parameter a has an evident increase as the penetration direction changes from 0° to 60° while in the case of μ=0.5, the value in vertical penetration show some inconsistency. Parameter b also exhibits increasing trend, but on a smaller scale in both cases, also accompanied by inconsistency in the case of vertical penetration when μ=0.5.

In addition to the force aligned along the axis of the penetrometer, there is also a force perpendicular to the penetrometer axis as soon as the test is inclined, which is always ignored in the analysis of traditional cone penetration tests as the forces are balanced in axisymmetric condition. However, this force in an inclined

CPT is of great importance from a practical view as it may deviate or even break the penetrometer in real tests. Therefore, the evolution of the normalized resistance perpendicular to the penetrometer axis, which is denoted q_p in this paper, is also investigated. Its definition is as follow:

$$q_p = \frac{f_{p.left} + f_{p.right}}{\frac{\sqrt{3}}{2} \cdot D \cdot \sigma_{vo}} \tag{8}$$

Where $f_{p.left}$ and $f_{p.right}$ correspond to the summation of the contact force components exerted on the tip perpendicular to the central axis of the penetrometer, respectively. D is the cone diameter and σ_{v0} is the initial vertical stress in the ground, as shown in Fig. 4.

Fig. 16 shows the relationship between the normalized resistance perpendicular to the penetrometer axis q_p and the relative depth (y/R) in different penetration directions for the two values of friction. As shown in the figure, q_p approximately equals zero when performed in vertical direction as the two sides of tip experienced equal and opposite reaction. However, it increases significantly with the inclination angle at shallow depth in the same way as the normalized resistance q_c. One apparent difference between q_c and q_p lies in the deep penetration where equal values on both sides do not appear in normalized resistance q_p. The unbalanced force applied perpendicular to the penetrometer axis may deviate the cone from its desired penetration direction. The phenomenon described here is limited to the cone tip which should be the same to the penetrometer side, thus q_p on both sides of penetrometer is not included in this paper. Based on the above analysis, when performing inclined cone penetration tests, the inclination angle should be carefully selected to ensure the penetrometer not to deviate from its original direction or even be broken in real tests. Same as the normalized resistance q_p, higher friction results in higher normalized resistance $q_{p.}$.

5. Discussions

The material used in the simulations has quite different internal friction from the real materials. The internal friction angle considered in this paper is only 15.37° and corresponds to a typical loose sample with low relative density. Such a small value is normal with models that ignore the possibility of particle rolling resistance at contacts [26, 27]. There are two available approaches in DEM analyses which can increase the friction angle for the material considered: The first approach is to use irregular grains such as clustered disks/spheres, polygon/polyhedron or ploy-ellipsoids etc. This may significantly increase the internal friction angle but require more computational time in contact detection, making it difficult to apply to large-scale boundary value problem. Alternately, the rolling resistance may be preferred without considering the details at the particle scale such as the particle shape. However, it can simultaneously satisfy the demand of improving internal friction angle and computational efficiency [26]. In addition, there have been many researches investigating the relationship of tip resistance and relative density [16, 28, 29] or internal friction angle [30-32] and several empirical formulas have been proposed. Thus results obtained from the low internal friction angle material may be used to predict the responses of more frictional material once the relative density or internal friction angle are given.

In this paper, we mainly focused on the tip resistance as previous works [32-34] have shown that the sleeve friction is small compared to the tip resistance, only around 10% or even smaller. Besides, the friction effect on sleeve friction has been investigated in our previous papers [13, 20] hence only the tip resistance is included in our analysis for simplicity.

Cone penetration is actually a three-dimensional problem however it is simulated in plane-strain conditions in the current study. It is obvious that a two-dimensional simulation cannot accurately represent a three-dimensional deposit of a granular material that consists of spherical particles. However, there is no intention in this paper to link the result of numerical simulations to field CPT quantitatively. The results presented herein will be analyzed strictly from a mechanism point of view. In terms of investigating the mechanism of inclined CPT, 2D DEM is still a reasonable option for our analysis. This is because: (a) Both 2-D and 3-D assemblies are a type of mechanical system, they must obey and share basic laws. It is these laws that would enhance understanding the behavior of natural soils and subsequently establishing their practical macro-constitutive models. Hence, the mechanism of particle movement obtained from a two-dimensional simulation is expected to be similar to that from a three-dimensional simulation. (b) To simulate large-scale boundary-value problems in geotechnical engineering using current PCs, the size effect and boundary effect must be reduced to the minimum, which requires an extremely large number of particles hence possible by 2D DEM for current PCs. (c) 2D DEM has been proved to be efficient in describing soil behavior qualitatively with numbers of studies.

Therefore, the soil in 3D simulations should also experience dilation and compression during the penetration as observed in this paper. However, quantitative comparison of failure mechanisms is impossible in this paper, since rigid plasticity is assumed in the three typical failure mechanisms proposed by Terzaghi, Biarez and Berezantev etc.[1,21-24], but it is not true for granular materials in the simulations. The stress rotation described in this paper is restricted to in-plane while the out-of-plane rotation is not considered. Besides, the out-of-plane constraint necessary to enforce a state of plane strain is not present in 2D DEM and this may results somewhat different tip resistance. For those reasons, the stress rotation and tip resistance measured in 2D DEM should be properly modified when extrapolated to 3D problems. Alternatively, three-dimensonal problem like CPT maybe reduced to a particular 2-dimensonal case by limiting the size of the media domain as has been introduced in [35].

6. Concluding remarks

The distinct element method was used to investigate the effect of friction on the inclined cone penetration mechanism in this paper. Based on the numerical simulations, the following conclusions can be made:

(1) Soils on the left side of the inclined penetration experience dilation, while on the right side undergo compaction. The effect of friction is particularly evident in the region adjacent to the penetrometer and the tip.

(2) Soils experience three different failure mechanisms successively during the penetration as the depth increases. The friction mainly affects the failure mechanism on the left side of the tip.

(3) The principal stresses of soils around the cone tip undergo large rotation accompanied by apparent particle rotations, and this rotation is nearly independent on friction.

(4) The normalized tip resistance increases with friction as well as inclination angle. The relationship between the normalized resistance ($q_c = q_c / \sigma_{v0}$) and relative depth (y/R) can be described by $q_c = a \times (y/R)^{-b}$, with parameters a and b dependent on the penetration direction.

(5) The inclination angle should be carefully selected to ensure the penetrometer not to deviate from its original direction or even be broken in real tests.

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