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Clinical monitoring with fuzzy automata

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Abstract: In this paper, a framework for an intelligent bedside monitor is presented. The monitor derives an abstraction of the current status of a patient by performing fuzzy state transitions on pre-processed input continuously supplied by clinical instrumentation. So far, an implementation called DiaMon-1 has been used for off-line evaluation of data of patients suffering from the adult respiratory distress syndrome (ARDS).

Keywords: Fuzzy automata; monitoring; abstraction; intensive care; ARDS.

1. Introduction

As new devices for on-line sampling of physiological data become available, intensive care monitors display more and more information. However, instead of the clinical staff being relieved, they are faced with a different problem, that of monitoring the monitor. Perception and interpretation of a multitude of time-varying parameters is difficult for humans [2, 5, 10], even if the parameter values are displayed in an ergonomic style. The situation is further complicated when parameters interact such that only certain constellations provide hints for critical conditions, or when the meaning of a value depends on the patient history, i.e., on what has happened before. Context-specific alarms for which no absolute thresholds can be established are a good example of this [2, 9].

This paper presents a formal framework for the design of monitors that exhibit the following properties:

- abstraction from objectively observed (quantitative) parameters to (qualitative) stages of a disease,
- early indication of improvement in or deterioration of the patient's state by providing smooth transitions between stages, and
- consideration of previous events, i.e., history-based interpretation of data.

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2. Definition of a state monitor

A state monitor as defined in the following employs concepts that stem from automata theory [3]: inputs, states, and transitions. A *state monitor* is an instrument that traces the patient's change of state with time, i.e., that records the progress of his illness and, hopefully, of his improvement. In this context, a *state* is considered to be an abstraction of the patient's status which accounts for a specific stage of a disease. *Transitions* provide possible paths from one state to the next. They depend on *input,* events that need to occur or conditions that need to be satisfied for a transition to take place. The input is obtained by processing objectively and preferably automatically acquired data.

The design of a state monitor is dominated by the nature of the monitored disease: it is intended to be an abstract model of the medical knowledge in that specific area. Because medical decision making is based on knowledge that has to take uncertainty such as physiological variability into account, judgement of the current state of a patient is often a matter of degree [1]. Consequently, transition from one stage of a disease to the next is hardly ever abrupt but usually smooth. To allow for smooth transitions, a state monitor is based on a fuzzy automaton rather than on a conventional one.

Definition 1 (fuzzy automaton). A fuzzy automaton is a quadruple

$$\tilde{A} = (Q, \tilde{q}_0, I, \delta)$$

where

Q is a finite set of states,
$\tilde{q}_0$ is a fuzzy subset of Q called the fuzzy initial state,
I is a finite set of input symbols, and
$\delta : Q \times I \rightarrow Q$ is a transition function that maps states and inputs onto states.

δ is extended to fuzzy arguments by the extension principle [4, 12] such that

$$\tilde{q}_{t+1} = \tilde{\delta}(\tilde{q}_t, \tilde{i}) = \sum_{Q \times I} \min(\mu_{\tilde{q}_t}(q), \mu_{\tilde{i}}(i))/\delta(q, i)$$

where for each $q \in Q$

$$\mu_{\tilde{q}_{t+1}}(q) = \begin{cases} \max\limits_{\delta(q',i)=q} \min(\mu_{\tilde{q}_t}(q'), \mu_{\tilde{i}_t}(i)), \\ 0 \quad \text{if } \delta^{-1}(q) = \emptyset. \end{cases} \tag{1}$$

If $\tilde{\mathcal{P}}(A)$ denotes the set of all fuzzy subsets of A, $\tilde{\delta} : \tilde{\mathcal{P}}(Q) \times \tilde{\mathcal{P}}(I) \rightarrow \tilde{\mathcal{P}}(Q)$. A fuzzy state $\tilde{q}$ is said to be included in the fuzzy state $\tilde{q}'$ ($\tilde{q} \subseteq \tilde{q}'$) if $\forall q \in Q$: $\mu_{\tilde{q}}(q) \leq \mu_{\tilde{q}'}(q)$. $\tilde{q}$ is said to be strictly included in $\tilde{q}'$ ($\tilde{q} \subset \tilde{q}'$) if $\tilde{q} \subseteq \tilde{q}' \wedge \tilde{q} \neq \tilde{q}'$. A state $\tilde{q}$ is said to be empty if $\forall q \in Q$: $\mu_{\tilde{q}}(q) = 0$. A sequence of fuzzy states is denoted by $\langle \tilde{q}_t \rangle$ and said to be increasing if $\tilde{q}_{t+1} \supseteq \tilde{q}_t$ for all t.

A set of final states that can usually be found in the definition of automata has been omitted on purpose, as monitoring is a continuous process that is rather terminated on the exhaustion of input data than on arrival at a certain state. As opposed to other definitions of fuzzy automata [3, 4, 8, 11], the transition function is not itself fuzzy; instead, the uncertainty expressed in a fuzzy input alone results in a partial transition from one state to another, without any possibility for a general statement about how strongly two states can at most be related. Thus, the design of a fuzzy automaton is not itself fuzzy: apart from the fuzzy initial state the fuzzification of the automaton is completely covered by the extension principle. Non-fuzzy tabular and graphical forms of defining an automaton can still be used – all the fuzziness is brought into the system through fuzzy inputs (compare Figure 2).

Despite its very similar definition, a fuzzy automaton exhibits properties quite different from those of its crisp counterpart. Firstly, being a fuzzy set, the current state is a distribution over several crisp states. Consequently, the automaton can perform different (partial) transitions simultaneously and therefore track parallel paths. Secondly, while crisp automata report an error on input not accounted for at the current state, a fuzzy automaton reacts on low or zero membership grades in the fuzzy input with continuously decreasing membership grades in its current state as obtained by (1) and depicted in Figure 3(b), a depletion of certainty that seems coherent with all repeated applications of fuzzy set operations.

However, a fuzzy automaton alone is not very appropriate to perform monitoring: firstly, automatically acquired data is generally precise and hence no source of fuzzy input as required by the automaton defined above. Secondly, if every single parameter value acquired represented an input on its own, the automaton would (a) explode because of an excessive number of possible input symbols all needing to be accounted for, and (b) continuously change its state in order to react to a certain input (if it did not, the input would remain unconsidered and thus become lost). The data is therefore pre-processed by a function that abstracts from single input parameters by generating fuzzy events that are passed on to the automaton.

Definition 2 (state monitor). If $\tilde{A} = (Q, \tilde{q}_0, I, \delta)$ is a fuzzy automaton as defined above, n is the number of parameters observed, R_1 through R_n are the parameter ranges, $P = R_1 \times \cdots \times R_n$ is the parameter value space, and $f : P \rightarrow \tilde{\mathcal{P}}(I)$ is a function that maps parameter tuples to fuzzy subsets of the input alphabet of $\tilde{A}$, then $\tilde{M} = (\tilde{A}, P, f)$ is a *state monitor.*

Technically, $\tilde{\mathcal{P}}(I)$ specifies the interface between pre-processing of data through f and interpretation of input through $\tilde{A}$. f can therefore be replaced by any computable method that yields a fuzzy set $\tilde{i}$ suitable for input to $\tilde{A}$, regardless of being a function or some other evaluation such as trend analysis, integration, or any other.

3. Peak hold

The definition of $\tilde{A}$ and the extension of δ in (1) guarantee that state membership values

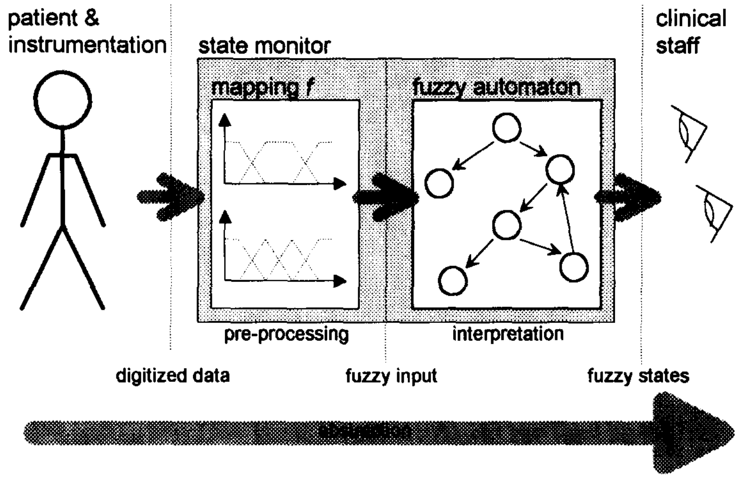

Fig. 1. Environment, components, and data flow of a state monitor.

$\mu_{\tilde{q}_t}(q)$ other than those of $\tilde{q}_0$ can only be introduced through fuzzy input $\tilde{i}$. If the set of all fuzzy inputs fed to a fuzzy automaton is finite, then the set of fuzzy states it can take on is also finite. Particularly:

Lemma 1. *If a fuzzy automaton $\tilde{A}$ is repeatedly fed with constant fuzzy input $\tilde{i}$, the set of fuzzy states it transitions between is finite.*

Proof. Follows directly from (1).

Note: If the range of membership grades is discrete, the set of fuzzy states of $\tilde{A}$ is also finite (proof omitted).

The *min* in the definition of $\tilde{\delta}$ further implies that no state can become more evident than the most evident of its predecessors and is further limited by the fuzzy input, as depicted in Figure 3(b). The sequence $\langle \text{hgt}(\tilde{q}_t) \rangle$, where $\text{hgt}(\tilde{q}_t) = \max_q \mu_{\tilde{q}_t}(q)$ is called the height of the current state, is therefore decreasing reflecting a continuous loss of certainty in the automaton. In practice the situation is further aggravated when the monitor is provided input in rapid succession, as the height can fall rapidly even if the input does not change, and once the current state is the empty set, it can never recover. In fact, if the automaton does not contain any feedback loops, i.e., does not provide circular transitions, it will arrive at the empty state after at most as many steps as there are states, as demonstrated in Figure 2(b). This is clearly not a desired property of a state monitor. However, instead of leaving the responsibility for providing appropriate feedback loops to the designer of the automaton, the following defines a property that overcomes this inadequate behaviour.

Definition 3 (peak hold). A fuzzy automaton is said to provide a *peak hold* if there is a transition for every state to itself on every input that leads to the state, i.e., if

$$\forall q', i, q: \quad \delta(q', i) = q \rightarrow \delta(q, i) = q. \qquad (2)$$

The condition implies that no state can be entered and left on the same input, otherwise δ would no longer be deterministic. Semantically, the peak hold guarantees that the maximum evidence for a state provided by its predecessors is memorized and held as long as input of ingoing transitions can support it, as shown in Figure 3(c). However, because a state does not remember its predecessor, the peak hold may also be sustained by an input other than the one that initially led to that state, and consequently the grade of membership can unintentionally remain high. Careful design of the state monitor is therefore necessary.

A positive side effect of the peak hold is that the automaton cannot oscillate on constant input

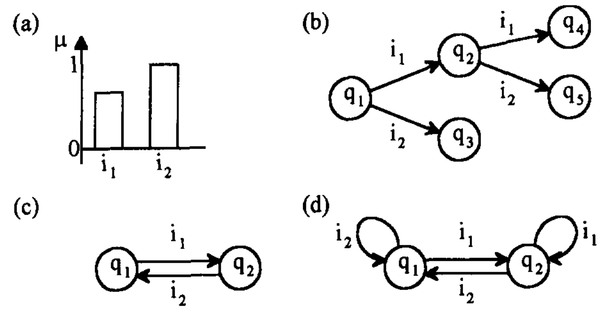

Fig. 2. (a) Fuzzy input $\tilde{i}$ supported by two (crisp) input symbols i_1 and i_2, (b) an automaton that is empty after at most three transitions on any input, (c) circular transitions that can lead to oscillations on repeated inputs of $\tilde{i}$, (d) automaton with peak hold that cannot oscillate.

[11] (compare Figures 2(c) and (d)), a property that would clearly not be acceptable in the clinical context, as stable (i.e., not changing) input should be reflected in stable output. Instead, the following theorem holds.

Theorem 1 (stability). *The fuzzy state of a fuzzy automaton with peak hold always becomes stable after a finite number of repeats of the same input.*

Proof. The proof is performed by showing that there is a step r such that

$$\tilde{q}_t \subseteq \tilde{q}_{t+1} \subseteq \cdots \subseteq \tilde{q}_{t+r} = \tilde{q}_{t+r+1} = \cdots \qquad (3)$$

in two steps:

(1) $\langle \tilde{q}_t \rangle$ is increasing, i.e., $\tilde{q}_t \subseteq \tilde{q}_{t+1} \subseteq \cdots$,

(2) $\exists r$: $\tilde{\delta}(\tilde{q}_{t+r}, \tilde{i}) = \tilde{q}_{t+r}$. For all subsequent states (3) follows from $\tilde{\delta}$ being a function.

(1) For every fuzzy state $\tilde{q}_t$ following the initial state $\tilde{q}_0$ and every input $\tilde{i}$ (1) implies: For every state q there is a transition that determines its membership value, i.e.

$$\forall q,\ \delta^{-1}(q) \neq \emptyset: \exists q', i:$$
$$\delta(q', i) = q \wedge \mu_{\tilde{q}_t}(q) = \min(\mu_{\tilde{q}_{t-1}}(q'), \mu_{\tilde{i}}(i)).$$

Thus, $\mu_{\tilde{q}_t}(q) \leqslant \mu_{\tilde{i}}(i)$ and following $\min(\mu_{\tilde{q}_t}(q), \mu_{\tilde{i}}(i)) = \mu_{\tilde{q}_t}(q)$. Repeated input of $\tilde{i}$ and $\delta(q, i) = q$ then implies

$$\mu_{\tilde{q}_{t+1}}(q) = \max(\mu_{\tilde{q}_t}(q), \max_{\delta(q',i)=q} \min(\mu_{\tilde{q}_t}(q'), \mu_{\tilde{i}}(i))),$$

$$\mu_{\tilde{q}_{t+1}}(q) \geqslant \mu_{\tilde{q}_t}(q),$$

which is justifying the term *peak hold,* and consequently

$$\tilde{q}_{t+1} \supseteq \tilde{q}_t.$$

(2) (indirect) Lemma 1 implies that there is no infinite sequence of fuzzy states $\langle \tilde{q}_t \rangle$ such that $\tilde{q}_{t+1} = \tilde{\delta}(\tilde{q}_t, \tilde{i}) \wedge \tilde{q}_{t+1} \supset \tilde{q}_t$. Therefore, there has to be a step r after which $\tilde{q}_{t+r} \subseteq \tilde{q}_{t+r-1}$. Because $\langle \tilde{q}_t \rangle$ is increasing, $\tilde{q}_{t+r}$ must equal $\tilde{q}_{t+r-1}$.

In particular, the proof shows that $\langle \tilde{q}_t \rangle$ does not converge to the empty state. The reason why it can take several steps until $\tilde{A}$ is stable is basically the fuzzy input $\tilde{i}$ which, when supported by more than one input symbol, can cause a propagation of higher grades of membership along a sequence of transitions. Note that non-fuzzy deterministic automata with peak hold are stable after one step. This is yet another example of how fuzzification yields more general results.

4. Active states

Despite the peak hold property, the height of the current state of a fuzzy automaton is still decreasing, as high grades of membership cannot be regained once they are lost. A particular source of loss is a situation where the grade of membership of one state decays while its successor's rises, as depicted in Figure 3(c).

This behaviour does not model the natural decision process correctly: once a decision has been made, it is usually pursued rather uncritically until there is sufficient evidence for another decision to be made.

The state monitor can be modified to adopt this kind of inertia in its behaviour: if a state is said to be *active* when its grade of membership in the current state exceeds a certain threshold α, then an active state is defined to remain active until there is a transition that induces activity of one of its successors, i.e.,

$$\mu_{\tilde{q}_{t+1}}(q) = \begin{cases} \mu_{\tilde{q}_t}(q) & \text{if } \mu_{\tilde{q}_t}(q) \geqslant \alpha \wedge \neg\exists q', i: \\ & \quad \delta(q, i) = q' \wedge \mu_{\tilde{i}_t}(i) \geqslant \alpha, \\ (1) & \text{else.} \end{cases} \qquad (4)$$

In other words, once a state has gained a certain grade of membership, it keeps it until a transition can pass it on to one of its successors, a behaviour that is illustrated in Figure 3(d). It implies that the height of the current state is

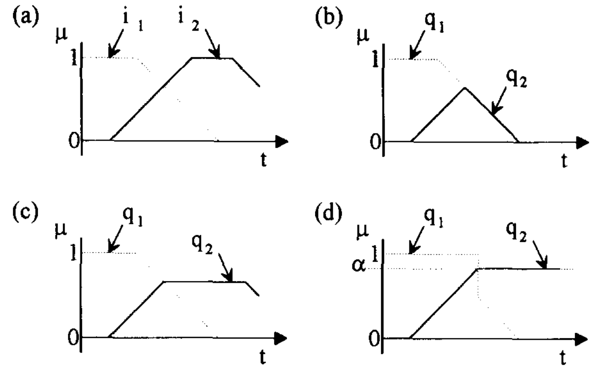

Fig. 3. (a) Membership course of input signals i_1 and i_2, (b) membership grades of q_1 and q_2 where $\mu(q_0) \equiv 1$, $\delta(q_0, i_1) = q_1$, and $\delta(q_1, i_2) = q_2$, (c) same as (b) with peak hold, and, (d) same as (b) with peak hold and active state level α.

always greater than α, a certain level of certainty thus always being maintained.

For $\alpha = 1$ (4) implies that $\tilde{q}_t$ is always normalized, i.e., there is always at least one state q such that $\mu_{\tilde{q}_t}(q) = 1$. This accounts for the fact that the patient is considered to be at least in one state at a time, even if only no successor with more evident support could yet be determined.

Note that the proof of Theorem 1 is only slightly affected by (4):

(1) $\langle \tilde{q}_t \rangle$ is still increasing, as the peak hold also works for $\mu_{\tilde{q}_t}(q) \geq \alpha$ and $\mu_{\tilde{q}_t}(q)$ can only once drop below α, namely on the first input of $\tilde{i}$, and

(2) still holds because Lemma 1 is not affected.

Also note that (4) without peak hold, although keeping the height above α, cannot prevent the automaton from oscillation.

5. Conclusion

A formal framework has been presented that allows a clinical monitor to be defined which abstracts from a continuous flow of input parameters by deriving a current state that comprises both the actual input and the previous states of a patient. Based on the simple concept of deterministic automata or finite state machines, a state monitor is easily designed and straightforward to implement. It competes among other fuzzy (e.g., [9]) and non-fuzzy (e.g., [5, 10]) approaches.

Because automata do not provide adequate means of performing complex mathematical operations, trend and artefact detection as well as evaluation of derived parameters are best placed in a function that pre-processes input before it is fed to the automaton. This work clearly separates the idea of state-based interpretation of events from rather general problems such as trend analysis quite common to other medical expert systems by introducing a layered architecture as suggested in [2, 5].

Originally, a diagnostic monitor DiaMon-1 very similar to the one introduced, with peak hold but with a restriction to only one active state was developed and implemented to retrospectively analyze data of patients suffering from the adult respiratory distress syndrome (ARDS) [7]. The idea arose from the demand for standardized criteria for the different stages of ARDS together with an objective evaluation technique, as widely differing mortality rates were considered to be due to different definitions of the syndrome itself as well as varying entry criteria for its possible therapies. The monitor is currently used in a multi-centre study with the aim of standardizing the ARDS criteria of different clinical centres. Independently, the concept of fuzzy events resulting in smooth transitions between states was judged to model human decision making naturally, and prospective operating of the monitor promises to be of valuable help to clinical staff.

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