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The Design of Quasi-Ellipsoidal Gear Ratio and Pitch Curved Surfaces

The design of the gear ratio and pitch curved surfaces of a new type of gear called as quasi-ellipsoidal gear is presented in this paper. The quasi-ellipsoidal gear can be applied to the flexible wrist of a robot and will make the wrist light and nimble, so the transmission precision will be greatly improved.

Introduction

Robot technique has developed very quickly in less than 40 years. As the key component of a robot, the flexible wrist has been greatly improved. A flexible wrist is a wrist with three DOF that can bypass certain obstacles. This kind of wrist has developed into many types, such as, linkage type, universal joint type, the wrist driven by steel string, as well as that composed of gears, and so on. But all these wrists have the following defects: a big volume, very heavy, big inertia, have clearance, bad setting accuracy and not easy to control. To overcome these defects, scientists began to develop new flexible wrists. Fig. 1 shows a new type of flexible wrist. In this wrist, there are two pairs of spherical gears. This will make the wrist simple, light, and reduce wrist inertia, so the setting accuracy will be improved. The concepts of spherical and quasi-ellipsoidal gears, as well as the design of the quasi-ellipsoidal gear ratio, and pitch curved surfaces are given in detail in the following:

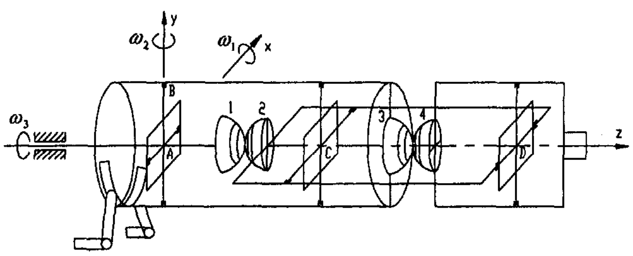

Fig. 1 A new type of flexible wrist adopting two pairs of spherical gears

1. Concepts of Spherical and Quasi-Ellipsoidal Gears

1.1 Teeth Layout of Spherical Gear. The three-dimensional drawing of spherical gears is shown in Fig. 2. The convex or concave teeth are only located in the regional area (the spherical cap) of the pitch sphere, and the method of teeth layout is: the center tooth locates on the summit, six teeth on the first latitudinal line and twelve teeth on the second latitudinal line, shown as Fig. 3. In the longitudinal direction of the pitch sphere, the arc length between the center tooth and the first circle of teeth equals to that between the first and the second circles of teeth.

1.2 Concepts of Spherical and Quasi-Ellipsoidal Gears. To make the spherical gears above mesh, the tooth pitch in the longitudinal direction of the pitch sphere of gear 1 must be equal to that of gear 2. And the tooth pitch in the latitudinal direction of the pitch sphere of gear 1 must also be equal to that of gear 2.

Shown as Fig. 4, the pitch sphere of gear 1 is Σ_1, its radius is r_1, w is a latitudinal line of Σ_1 and its radius is r_w^1. In this latitudinal line, there are z teeth. j_1 and j_2 are two longitudinal lines. M is the center tooth. N and Q are two teeth nearby in the latitudinal line w. Make $P_j^1 = MN = MQ$, $P_w^1 = NQ$, then the tooth pitch in the longitudinal line of Σ_1 is

$$p_j^1 = r_1\theta \qquad (1)$$

The tooth pitch in the latitudinal line w of Σ_1 is

$$p_w^1 = \frac{2\pi}{z} r_w^1 = \frac{2\pi}{z} r_1 \sin\theta \qquad (2)$$

For the pitch sphere Σ_2 of gear 2, r_2, φ and r_w^2 correspond respectively to r_1, θ and r_w^1 of Σ_1. In the same way, we get the tooth pitch in the longitudinal line of Σ_2 is

$$p_j^2 = r_2\varphi \qquad (3)$$

The tooth pitch in the latitudinal line of Σ_2 is

$$p_w^2 = \frac{2\pi}{z} r_w^2 = \frac{2\pi}{z} r_2 \sin\varphi \qquad (4)$$

If the gear ratio $i_{21} = r_1/r_2 = 1$, make $\varphi = \theta$, then

$$\begin{cases} p_j^1 = p_j^2 \\ p_w^1 = p_w^2 \end{cases} \qquad (5)$$

Obviously, this conforms to the right meshing condition.

If the gear ratio $i_{21} = r_1/r_2 \neq 1$, for example, $r_2 > r_1$ (i.e., $i_{21} < 1$), in the area $\theta \in [0, \pi/2]$ the derivative of the function $f(\theta) = \sin\theta - \sin(i_{21}\theta)/i_{21}$ is

$$f'(\theta) = \frac{df}{d\theta} = \cos\theta - \cos(i_{21}\theta) < 0 \qquad (6)$$

So $f(\theta)$ reduces monotonously in the area $\theta \in [0, \pi/2]$. And $f(0) = 0$, so when $\theta > 0, f(\theta) < f(0)$, i.e.,

$$\sin\theta < \sin(i_{21}\theta)/i_{21} \qquad (7)$$

For the spherical gear, the meshing longitudinal lines are a pair of instantaneous center lines (arcs). The tooth pitches in the longitudinal direction should be equal to each other, so we get

$$r_1\theta = r_2\varphi$$

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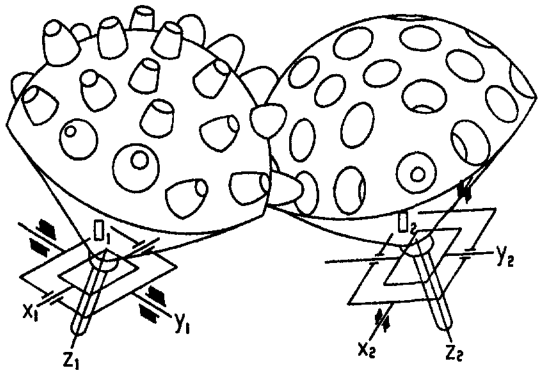

Fig. 2 Spherical gears

i.e.,

$$\varphi = \frac{r_1}{r_2}\,\theta = i_{21}\theta \tag{8}$$

Put (8) into (4), we get

$$p_w^2 = \frac{2\pi}{z}\, r_2 \sin\varphi = \frac{2\pi}{z}\, r_1 \frac{\sin(i_{21}\theta)}{i_{21}} \tag{9}$$

From (2), (7) and (9) we get

$$p_w^1 < p_w^2 \tag{10}$$

So we get such a conclusion: if the gear ratio $i_{21} = r_1/r_2 \neq 1$, when the tooth pitches in the longitudinal direction of the pitch spheres of gears 1 and 2 are equal to each other, the tooth pitches in the latitudinal direction can't be equal to each other in the meantime.

When the center teeth mesh just in the center position, the gear ratio i_{21} will become I_{21} which is named as the nominal gear ratio. So we may get another conclusion: When the nominal gear ratio $I_{21} = 1$, the pitch curved surfaces Σ_1 and Σ_2 of the gears above are two spheres and the gear ratio i_{21} is constant (i.e., $i_{21} \equiv I_{21}$) during their transmission. When the nominal gear ratio $I_{21} \neq 1$, Σ_1 and Σ_2 won't be spheres. They are not strict ellipsoids, but they are two approximate ellipsoids. So we name Σ_1 and Σ_2 under this condition as two quasi-ellipsoids and the gears above will become two quasi-ellipsoidal gears. Obviously, the gear ratio i_{21} is variable when the gears mesh in the longitudinal direction.

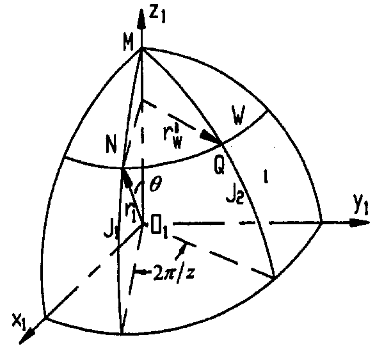

Fig. 4 The pitch sphere of gear 1

2 The Design of the Quasi-Ellipsoidal Gear Ratio and Pitch Curved Surfaces

2.1 The Vector Equation of the Quasi-ellipsoidal Gear. Shown as Fig. 5, O_1 and O_2 are respectively the rotatory centers of convex gear 1 and concave gear 2. A is their center distance. $O_1x_1y_1z_1$ and $O_2x_2y_2z_2$ are two coordinate systems fixed respectively to gear 1 and gear 2. Pitch curved surfaces Σ_1 and Σ_2 are two surfaces of revolution rotating respectively around axises z_1 and z_2. The parameter in the longitudinal direction of Σ_1 is θ and the parameter in the latitudinal direction is α_1. The parameter in the longitudinal direction of Σ_2 is φ and the parameter in the latitudinal direction is $\alpha_2 (\alpha_2 = \alpha_1)$. $\mathbf{r}_1$ and $\mathbf{r}_2$ are the vector radiuses corresponding respectively to P_1 of Σ_1 and P_2 of Σ_2 (P_1 and P_2 are the corresponding teeth). So, the vector equation of Σ_1 is

$$\mathbf{r}_1 = r_1(\theta)(\mathbf{i}_1 \sin\theta\cos\alpha_1 + \mathbf{j}_1 \sin\theta\sin\alpha_1 + \mathbf{k}_1\cos\theta) \tag{11}$$

The vector equation of Σ_2 is

$$\mathbf{r}_2 = r_2(\varphi)(\mathbf{i}_2 \sin\varphi\cos\alpha_2 + \mathbf{j}_2 \sin\varphi\sin\alpha_2 + \mathbf{k}_2\cos\varphi) \tag{12}$$

Here $\varphi = \varphi(\theta)$, Obviously,

$$\frac{\partial \mathbf{r}_1}{\partial\theta}\cdot\frac{\partial \mathbf{r}_1}{\partial\alpha_1} = \frac{\partial \mathbf{r}_2}{\partial\varphi}\cdot\frac{\partial \mathbf{r}_2}{\partial\alpha_2} = 0 \tag{13}$$

So the parameter curves θ and α_1 of Σ_1 are perpendicular to each other. And the parameter curves φ and α_2 of Σ_2 are also perpendicular to each other. The tangential vector $(\partial\mathbf{r}_1/\partial\theta)$ $(\partial\mathbf{r}_1/\partial\alpha_1)$ and the normal vector $(\partial\mathbf{r}_1/\partial\theta)\times(\partial\mathbf{r}_1/\partial\alpha_1)$ of Σ_1 are three vectors perpendicular to each other. Their corresponding unit vectors are $\mathbf{i}_n$, $\mathbf{j}_n$, $\mathbf{k}_n$. $\mathbf{i}_n$, $\mathbf{j}_n$, $\mathbf{k}_n$ will form a regional space coordinate system at the point P_1.

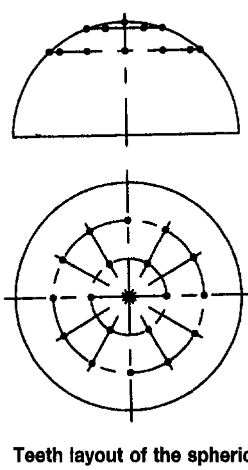

Fig. 3 Teeth layout of the spherical gear

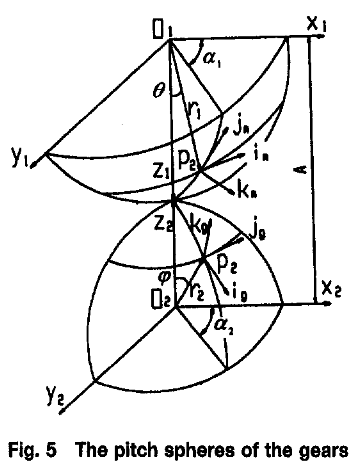

Fig. 5 The pitch spheres of the gears

The tangential vectors $\partial \mathbf{r}_2/\partial\varphi$, $\partial \mathbf{r}_2/\partial\alpha_2$ and the normal vector $(\partial \mathbf{r}_2/\partial\varphi) \times (\partial \mathbf{r}_2/\partial\alpha_2)$ of Σ_2 are also three vectors perpendicular to each other. Their corresponding unit vectors are $\mathbf{i}_g$, $\mathbf{j}_g$, $\mathbf{k}_g$. $\mathbf{i}_g$, $\mathbf{j}_g$, $\mathbf{k}_g$ will form another regional space coordinate system at the point P_2. The convex and concave tooth shape equations will be built respectively in the two regional space coordinate systems. Every tooth on the pitch curved surface corresponds to its own regional space coordinate system. When the position (θ, α_1) or (φ, α_2) of the tooth on the pitch curved surface is defined, the position of its corresponding tooth surface in the pitch curved surface will be attained. Limited to the paper length, the design of tooth shape will be omitted in this paper.

2.2 The Design of the Pitch Curved Surfaces Σ_1 and Σ_2

2.2.1 The Design Criterion of Pitch Curved Surfaces Σ_1 and Σ_2

(1) The Right Meshing Condition

Pitch curved surfaces Σ_1 and Σ_2 meshing with each other in the longitudinal direction is similar to the transmission of a pair of noncircular gears that aren't close. Pitch curved surfaces Σ_1 and Σ_2 meshing with each other in the latitudinal direction is similar to the transmission of a pair of cone gears whose transmission ratio is 1. Therefore, the arc length in the longitudinal direction of pitch curved surface Σ_1 must be equal to that of pitch curved surface Σ_2 and the arc length in the latitudinal direction of pitch curved surface Σ_1 must be equal to that of pitch curved surface Σ_2.

(2) The Smooth Condition

Pitch curved surfaces Σ_1 and Σ_2 must be smooth and convex curved surfaces, there should not exist odd points (i.e., tips and pits) on them.

2.2.2 The Design of Pitch Curved Surfaces Σ_1 and Σ_2 Pitch curved surfaces Σ_1 and Σ_2 are two surfaces of revolution. If a mother curve is designed, the pitch curved surface will be attained. Shown as Fig. 6, Γ_1 and Γ_2 are the mother curves (longitudinal lines) of pitch curved surfaces Σ_1 and Σ_2. The equation of Γ_1 and Γ_2 are $r_1 = r_1(\theta)$, $r_2 = r_2(\varphi)$. The center distance between two gears is A and the nominal gear ratio is $I_{21} = r_{10}/r_{20}$ (r_{10}, r_{20} are the radiuses when the center teeth mesh just in the center position) the practical gear ratio is $i_{21} = r_1/r_2 = f(\theta)$.

Suppose s_1, s_2 are the arc length of Γ_1, and Γ_2, then

$$\begin{cases} s_1 = \int_0^\theta r_1 d\theta \\ s_2 = \int_0^\varphi r_2 d\varphi \end{cases} \tag{14}$$

Because the arc length in the longitudinal direction of Σ_1 and Σ_2 equal to each other, we get

$$\int_0^\theta r_1 d\theta = \int_0^\varphi r_2 d\varphi \tag{15}$$

Extract the derivative of (15) to θ, by arranging, we get

$$\varphi = \int_0^\theta \frac{r_1}{r_2} d\theta = \int_0^\theta i_{21} d\theta \tag{16}$$

Besides, the arc length in the latitudinal direction of Σ_1 and Σ_2 also equal to each other, we get

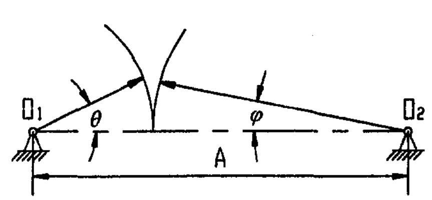

Fig. 6 The mother curves of the pitch curved surfaces

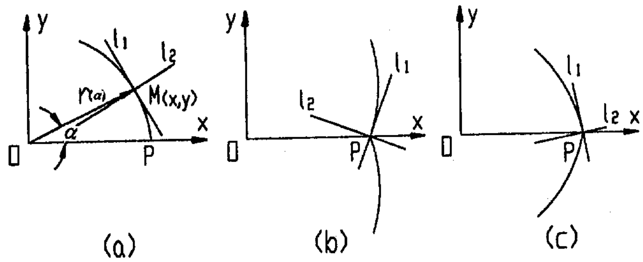

Fig. 7 The shape of the mother curves

$$r_1 \sin\theta = r_2 \sin\varphi$$

i.e.,

$$\varphi = \arcsin(i_{21} \sin\theta) \tag{17}$$

From (16) and (17) we get

$$\int_0^\theta i_{21} d\theta = \arcsin(i_{21} \sin\theta) \tag{18}$$

Extract the derivative of (18) to θ, we get

$$i_{21} = \frac{i_{21}' \sin\theta + i_{21}\cos\theta}{\sqrt{1-(i_{21}\sin\theta)^2}} \tag{19}$$

By arranging, we get

$$i_{21}' = \frac{di_{21}}{d\theta} = \frac{i_{21}[\sqrt{1-(i_{21}\sin\theta)^2} - \cos\theta]}{\sin\theta} = f(\theta, i_{21}) \tag{20}$$

Obviously, this is an ordinary differential equation. To get the numerical solution, the initial condition must be defined. When $\theta = 0$, $i_{21} = I_{21}$. So the initial condition is

$$i_{21}|_{\theta=0} = I_{21} \tag{21}$$

However, when $\theta = 0$, the derivative of (20) doesn't exist. To make the numerical calculation continue favorably, the initial value of i_{21}' must be defined first. Shown as Fig. 7, l_1 is the tangential line at the point M and l_2 is the normal line at the same point. The slope of l_1 is k_1 and the slope of l_2 is k_2 then

$$\begin{cases} k_1 = \dfrac{dy}{dx} = \dfrac{(dr/d\alpha)\cdot\sin\alpha + r\cos\alpha}{(dr/d\alpha)\cdot\cos\alpha - r\sin\alpha} \\ k_2 = -\dfrac{1}{k_1} = -\dfrac{(dr/d\alpha)\cdot\cos\alpha - r\sin\alpha}{(dr/d\alpha)\cdot\sin\alpha + r\cos\alpha} \end{cases} \tag{22}$$

When $\alpha = 0$, if $k_2 = 0$, the normal line of Γ coincides with the parameter axis x, the surface of revolution rotated by Γ will be smooth. If $k_2 < 0$, there will exist a pit in Σ, shown as Fig. 7(b). If $k_2 > 0$, there will exist a tip in Σ, shown as Fig. 7(c). To conform to the smooth condition, we must make

$$k_2|_{\alpha=0} = -\frac{dr/d\alpha}{r} = 0$$

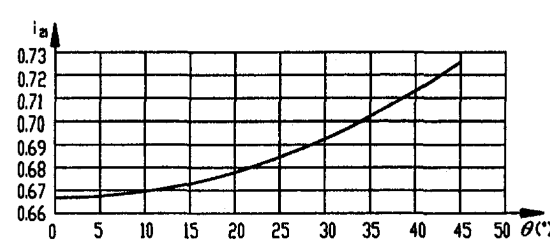

Fig. 8 Calculation of the gear ratio i_{21}

i.e.,

$$\left.\frac{dr}{d\alpha}\right|_{\alpha=0} = 0 \tag{23}$$

For Σ_1 and Σ_2, we get

$$\begin{cases} \left.\dfrac{dr}{d\theta}\right|_{\theta=0} = 0 \\ \left.\dfrac{dr_2}{d\varphi}\right|_{\varphi=0} = 0 \end{cases} \tag{24}$$

As $\varphi = \varphi(\theta)$, so

$$\frac{dr_2}{d\theta} = \frac{dr_2}{d\varphi}\cdot\frac{d\varphi}{d\theta} = 0 \tag{25}$$

And

$$i'_{21} = \frac{dr_{21}}{d\theta} = \frac{d}{d\theta}\left(\frac{r_1}{r_2}\right) = \frac{\dfrac{dr_1}{d\theta} r_2 - r_1 \dfrac{dr_2}{d\theta}}{r_2^2} \tag{26}$$

So

$$i'_{21}\,|_{\theta=0} = \frac{di_{21}}{d\theta}|_{\theta=0} = 0 \tag{27}$$

When i_{21} is defined, Γ_1 and Γ_2 will be defined with the following equation

$$\begin{cases} r_1 = \dfrac{Ai_{21}}{1 + i_{21}} \\ r_2 = \dfrac{A}{1 + i_{21}} \end{cases} \tag{28}$$

Make Γ_1 and Γ_2 rotate 360 deg around their own axes, the pitch curved surfaces Σ_1 and Σ_2 will be attained.

3 Calculation Example

With the method presented above, we may design the gear ratio i_{21} and the pitch curved surfaces Σ_1 and Σ_2 of the quasi-ellipsoidal gears whose nominal gear ratio $I_{21} = \frac{2}{3}$, the results are shown respectively as Fig. 8 and Fig. 9.

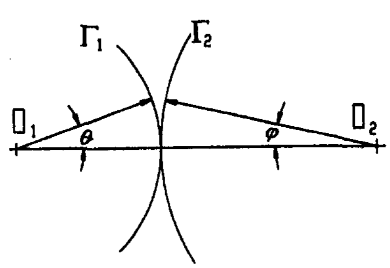

Fig. 9 Mother curves of the pitch curved surfaces

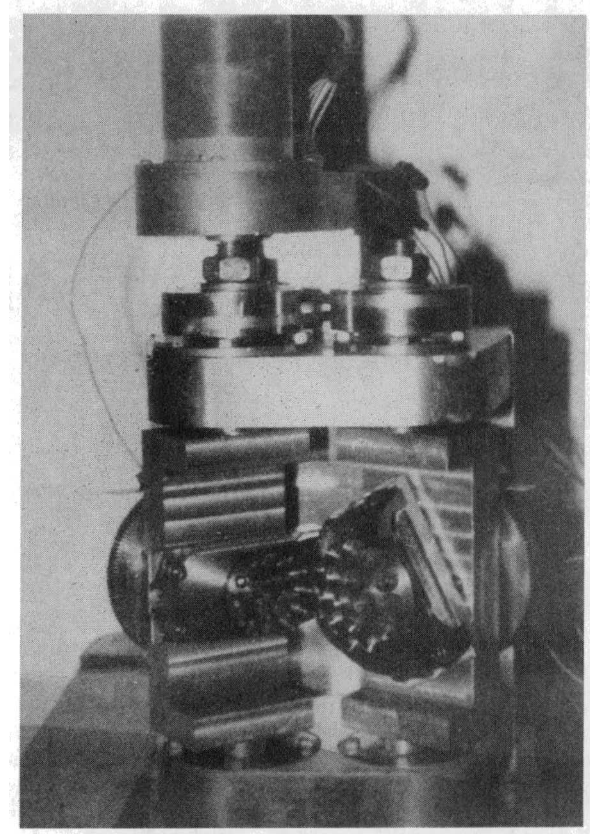

Fig. 10 This picture shows the EDM device of quasi-ellipsoidal gears. There is a pair of gears on it. The left is a convex gear, and the right is a concave one.

4 Conclusion

A new type of gear called as quasi-ellipsoidal gear is presented in this paper. When the nominal gear ratio $I_{21} = 1$, the quasi-ellipsoidal gear will become spherical gear and the transmission ratio i_{21} is constant (i.e., $i_{21} \equiv I_{21}$) during their transmission. When the nominal gear ratio $I_{21} \neq 1$, the gear ratio i_{21} is variable when the gears mesh in the longitudinal direction. The design of the gear ratio and pitch curved surfaces of the quasi-ellipsoidal gears is presented in this paper. The quasi-ellipsoidal gear can be applied to the flexible wrist of a robot and will make the wrist light and nimble, so the transmission precision will be greatly improved. Fig. 10 shows the EDM of the quasi-ellipsoidal gears.

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