Dynamic Stability of Hovercraft in Heave

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The regimes of flow governing the dynamic behavior of a two-dimensional mathematical model of an edge-jet Hovercraft in heaving motion are described and the equations associated with such regimes derived. Both the free and forced-oscillation characteristics are studied. The nonlinear nature of the system manifests itself, in the case of the forced oscillations, as a shift in the dynamic equilibrium position resulting in a loss of mean hoverheight.

Introduction

The dynamic behavior of a Hovercraft in heaving motion can be adequately described through the behavior of the edge jet and cushion. At one stage of the motion, the rate of change of the pressure in the cushion is such that the horizontal pressure force cannot be sustained by the edge jets. As a result, the cushion "bursts" causing the cushion pressure to decrease, the escaping air lifting the edge jets from the ground. Within a complete cycle a complementary state exists when the rate of change of the pressure in the cushion is such that the horizontal pressure force is insufficient to sustain the momentum flow of the edge jets. In this case the edge jet splits, part of the flow forming a compensating pumping action into the cushion. Such behavior has been described by Tulin [1][1] and Eames [2] as being akin to alternating "jet" and "piston" effects and provides the damping mechanism of the harmonic oscillation in heave. The switching from one state to the other is not a smooth instantaneous mechanism. A Coanda behavior of the edge jets results in a hysteresis effect defining a pause condition between the just described states during which the pressure change within the cushion is balanced by the compressibility of the air cushion itself and the deformation of the edge-jet contour. As the two states describe two distinct physical configurations, it is to be expected that, in order to fully describe a complete cycle, separate mathematical formulations will be required providing solutions that must be analytically continuous through the pause condition. The analysis developed by both Tulin and Eames, although initially differentiating between the two modes of operation, proceeds to develop a single-differential equation which, it is claimed, describes a mean of the separate states. Such a procedure would be justified if the damping coefficients in the two states were nearly equal. The ratio, however, can be as large as 3 (see reference [2]) which makes a separate analysis of the two states essential. For this reason the method adopted by Duke and Hargreaves [3] also can be criticized. In this work, the Hovercraft is assumed to be analogous to a mass-spring-damper system. Appropriate coefficients are derived for the two regimes of edge-jet operations and an arithmetic mean is taken to define a single-differential equation for the complete cycle. No mention is made of the switching conditions which determine the mode of operation of the edge jets and the assumption of linearity is not justified.

Each mode of jet behavior is investigated separately to clearly demonstrate the switching characteristics of the jet and the analytic continuation of such solutions forming a complete oscillation is discussed. The nonlinear character of these solutions is then adequately demonstrated by the numerical evaluation of the solutions for a discrete range of parameters.

Mathematical Model

The Hovercraft system is represented as a two-dimensional flat based craft supported on an air cushion, generated and maintained by identical edge jets. The air cushion is assumed to be isentropic and inviscid while the edge jets are, in effect, replaced by momentum lines along which the rate of change of momentum flux remains invariant. In any complete cycle of the heaving motion, the air cushion and edge jets will behave as described in the Introduction. The state following the bursting of the air cushion when the outflow lifts the edge jets from contact with the ground will be referred to as underfed. The complementary state, when the edge jets split, providing a pumping action into the air cushion, will be referred to as overfed.

[1] Numbers in brackets designate References at end of paper.

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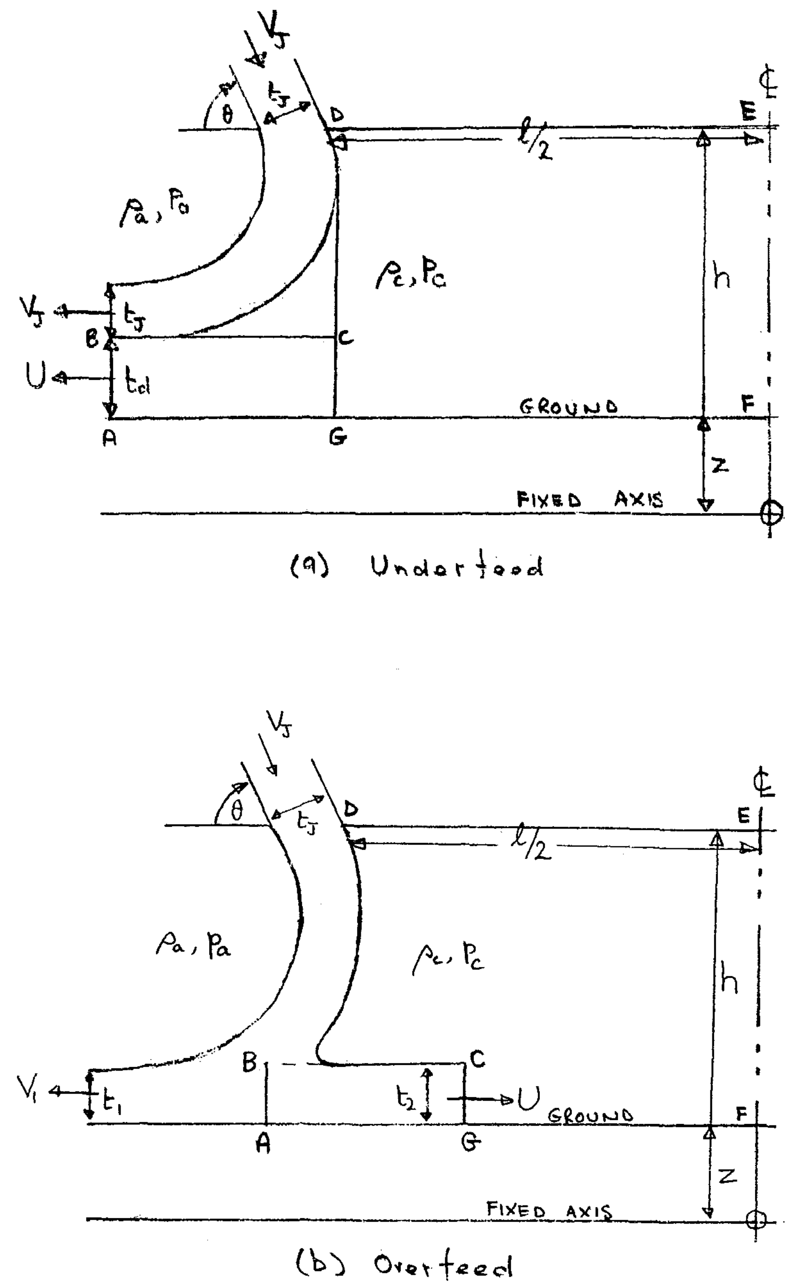

Fig. 1 Jet configurations

Nomenclature

$C(t)$ = Bernoulli function

h = craft displacement from fixed axis

h_0 = static equilibrium hover-height

HF = Hovercraft-Froude number

l = craft length

L = characteristic length associated with flow out from/into cushion

p_a = ambient pressure

p_c = cushion pressure

p_0 = static equilibrium cushion pressure

t = time

$t_{d,2}$ = width of jet flow out from/into cushion

t_j = edge-jet width

U = flow velocity out from/into cushion

V = velocity of fluid along a streamline

V_j = velocity of edge jet

V_∞ = velocity of edge jet out of ground effect

$x = t_j(1 + \cos\theta)/h_0$ = edge-jet parameter

z = ground displacement from fixed axis

z_0 = maximum amplitude of ground oscillation

γ = ratio of specific heats

θ = edge-jet inclination

ρ_a = atmospheric density

ρ_c = cushion density

ρ_j = edge-jet density

ϕ = velocity potential

ω = frequency of forcing function

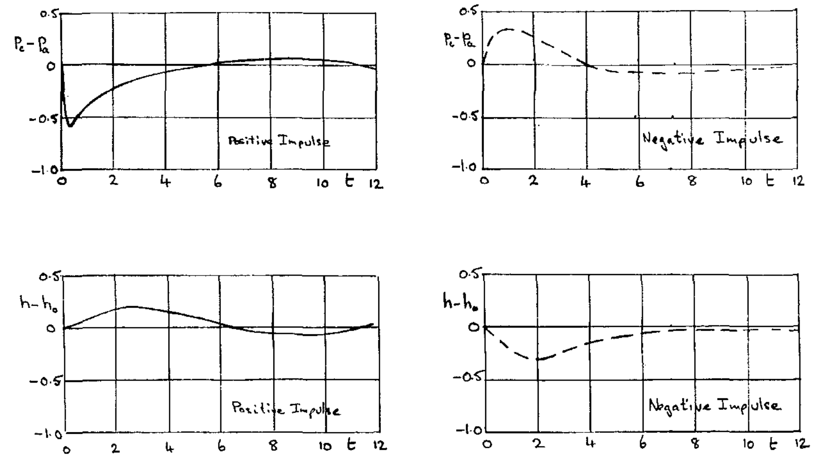

Fig. 2 Cushion pressure and hoverheight variations in free oscillations

Analysis

Underfed State. Consider the forced oscillations in heave of the Hovercraft due to the movement of the ground plane. At time t let the ground displacement as measured from a horizontal axis fixed in space be denoted by z and let the corresponding displacement of the base of the craft be h. If the rate of change of the mean cushion pressure, dp_c/dt, is greater than that necessary to maintain the edge jets in equilibrium over a height $(h - z)$ the air in the cushion will flow out in the form of a jet of thickness t_d and with velocity U; see Fig. 1(a).

Let us consider the jet alone. If it is assumed that the momentum flow in the jet remains invariant throughout the motion we see that, by comparing the displaced configuration with that of the static configuration,

$$(p_c - p_a)(h - z - t_d) = \rho_j v_j^2 t_j(1 + \cos\theta) = (p_0 - p_a)h_0$$

Now let us consider the "underfeed." In this case, the flow is time-dependent and from the equation of continuity governing the mass-flow leaving, the cushion volume is determined by

$$\frac{d}{dt}\left\{\tfrac{1}{2}\rho_c l(h - z)\right\} = -\rho_a U t_d \qquad (1)$$

where ρ_c is the mean density of the air in the cushion. By using the adiabatic relationship,

$$p_c/\rho_c^{\gamma} = p_a/\rho_a^{\gamma}$$

this equation gives, on rearrangement, the rate of change of the mean cushion pressure, p_c, in the form

$$\frac{dp_c}{dt} = -\frac{\gamma p_c}{l(h - z)}\left\{l\left(\frac{dh}{dt} - \frac{dz}{dt}\right) + 2Ut_d\left(\frac{p_a}{p_c}\right)^{1/\gamma}\right\}. \qquad (2)$$

The Bernoulli equation for the unsteady flow out from the cushion is of the form

$$\int\frac{dp}{\rho} + \tfrac{1}{2}V^2 + \frac{\partial\phi}{\partial t} = c(t) \qquad (3)$$

where, as usual, the evaluation of the acceleration potential term presents difficulties. A representative form may be obtained by following the analysis of the unsteady flow of fluid through a thin tube at the base of a tank as given in the text by Prandtl and Tietjens [4]. Thus the main cushion volume may be considered as analogous to a tank CDEFG in which the mean velocity is zero and where the underfeed is replaced by an outlet pipe ABCG of representative length L, width t_d and in which the mean velocity is $\frac{1}{2}U$; see Fig. 1(a). By substituting for $\partial\phi/\partial t$ the expression $\frac{1}{2}LDU/dt$, the Bernoulli equation gives, on rearranging,

$$\frac{dU}{dt} = \frac{2}{L}\left[\frac{\gamma}{\gamma - 1}\frac{1}{\rho_a}\left\{p_c\left(\frac{p_a}{p_c}\right)^{1/\gamma} - p_a\right\} - \tfrac{1}{2}U^2\right] \qquad (4)$$

Finally the motion of the craft itself may be described by the equation

$$md^2h/dt = (p_c - p_a)l - mg$$

where mg, the weight of the craft, may be equated to $(p_0 - p_a)$ to give

$$\frac{1}{g}\frac{d^2h}{dt^2} = \frac{p_c - p_0}{p_0 - p_a} \qquad (5)$$

The underfed phase of the motion is completely defined by the equations (1), (3)–(5). By interpreting m as a virtual and not actual mass of the craft an allowance may be made for any buoyancy effect due to the local displacement of the ground plane immediately beneath the craft in cases where such a ground plane is deformable.

Overfed State. In this phase of the motion, the rate of change of the cushion pressure is insufficient to balance the momentum flow in the edge jet. Part of this momentum is then deflected into the cushion in the form of a compensating flow of velocity U and jet width t_2. The main part of the jet still flows outward and it is assumed that its velocity remains V_j through a decreased jet width t_1 (see Fig. 1(b)), such that

$$V_j t_j = V_j t_1 + U t_2.$$

The momentum balance equation for the edge jet can then be written in the form

$$(p_c - p_a)(h - z) = (p_0 - p_a)h_0 - \rho_j t_2 U(U + V_j) \qquad (6)$$

By assuming that the acceleration potential occurring in Bernoulli's equation takes the identical form to that assumed for the underfed state the energy equation yields

$$\frac{dU}{dt} = \frac{2}{L}\left(\frac{p_a}{\rho_j} + \tfrac{1}{2}V^2 - \frac{p_c}{\rho_j} - \tfrac{1}{2}U^2\right) \qquad (7)$$

whereas the edge-jet fluid is regarded as incompressible, the flow into the cushion is regarded as an adiabatic continuous process resulting in the equation

$$\frac{dp_c}{dt} = -\frac{\gamma p_c}{(h - z)}\frac{1}{l}\left\{l\left(\frac{dh}{dt} - \frac{dz}{dt}\right) - 2t_2U\left(\frac{p_a}{p_c}\right)^{1/\gamma}\right\} \qquad (8)$$

The overfed phase of the motion is completely defined by equations (5)–(8).

Switching Conditions. The conditions which determine in which

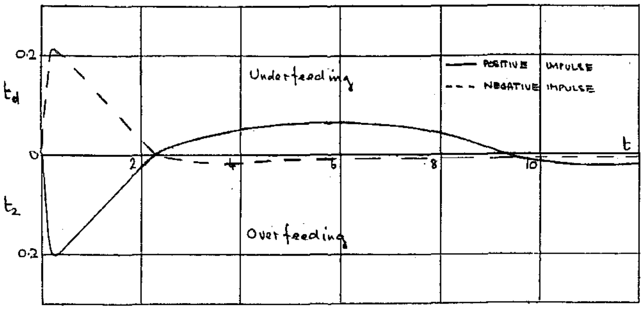

Fig. 3 Extent of over and underfeeding

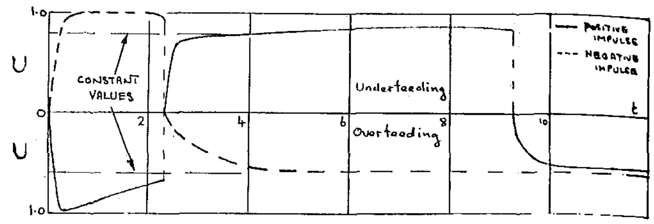

Fig. 4 Velocity variation of over and underfeeding

phase of the motion the mathematical model should be are as follows. When the edge jet is in the underfed state, the switch occurs when $t_d = 0$; whereas when the edge jet is in the overfed state, the switch occurs when $t_2 = 0$; see Fig. 3. These conditions are both equivalent to

$$(p_c - p_a)(h - z) - (p_0 - p_a)h_0 = 0$$

The hysteresis effect occurring in the finite time phase of the switch is neglected resulting in a discontinuous jump from the overfed to underfed stages.

Nondimensional Equations

Preparatory to the computational work, the equations governing the modes of motion are nondimensionalized. The characteristic pressure is taken to be the dynamic head of the edge-jet out-of-ground effect; i.e., $\frac{1}{2}\rho_j V_\infty^2$ where V_∞ is the velocity of the edge jet in this mode of operation and is used as the characteristic velocity. A reference length is defined as xl where

$$x = t_j(1 + \cos\theta)/h_0.$$

Such a length is characteristic of the system in that it combines all relevant lengths and angles, viz., edge-jet width; t_j, edge-jet inclination, θ; craft length, l; and static equilibrium hoverheight, h_0, in a well-known form occurring in the simple static equilibrium theory.

Thus, if nondimensional quantities are denoted by a bar, we shall write

$$\bar{h} = h/xl, \qquad \bar{V} = V/V_\infty, \qquad \bar{t} = tV_\infty/xl,$$

$$\bar{p} = p/\tfrac{1}{2}\rho_j V_\infty^2, \quad \text{and} \quad \bar{\rho} = \rho/\rho_j.$$

The fundamental equations (1), (3)–(5), governing the underfed state, respectively, take on the following nondimensional forms:

$$(\bar{p}_c - \bar{p}_a)(\bar{h} - \bar{z} - \bar{t}_d) = (\bar{p}_0 - \bar{p}_a)\bar{h}_0 \tag{9}$$

$$\frac{d\bar{p}_c}{d\bar{t}} = -\frac{\gamma\bar{p}_c}{\bar{l}(\bar{h} - \bar{z})}\left\{\bar{l}\left(\frac{d\bar{h}}{d\bar{t}} - \frac{d\bar{z}}{d\bar{t}}\right) + 2\bar{U}\bar{t}_d\left(\frac{\bar{p}_a}{\bar{p}_c}\right)^{1/\gamma}\right\} \tag{10}$$

$$\frac{d\bar{U}}{d\bar{t}} = \frac{1}{\bar{L}}\left[\frac{\gamma}{\gamma - 1}\frac{1}{\bar{p}_a}\left\{\bar{p}_c\left(\frac{\bar{p}_a}{\bar{p}_c}\right)^{1/\gamma} - \bar{p}_a\right\} - \bar{U}^2\right] \tag{11}$$

and

$$\frac{d^2\bar{h}}{d\bar{t}^2} = \frac{1}{(\mathrm{HF})^2}\frac{(\bar{p}_c - \bar{p}_0)}{(\bar{p}_0 - \bar{p}_a)} \tag{12}$$

where $\mathrm{HF} = (V_\infty^2/x^2 g)^{1/2}$ is the Froude number for the gravitational system associated with the Hovercraft.

Similarly the fundamental equations (6)–(8) governing the overfed state, respectively, take on the following nondimensonal forms:

$$(\bar{p}_c - \bar{p}_a)(\bar{h} - \bar{z}) = (\bar{p}_0 - \bar{p}_a)\bar{h}_0 - 2\bar{t}_2\bar{U}(\bar{U} + \bar{V}_j) \tag{13}$$

$$\frac{d\bar{U}}{d\bar{t}} = \frac{1}{\bar{L}}(\bar{p}_a + 1 - \bar{p}_c - \bar{U}^2) \tag{14}$$

$$\frac{d\bar{p}_c}{d\bar{t}} = -\frac{\gamma\bar{p}_c}{\bar{l}(\bar{h} - \bar{z})}\left\{\bar{l}\left(\frac{d\bar{h}}{d\bar{t}} - \frac{d\bar{z}}{d\bar{t}}\right) - 2\bar{t}\,\bar{U}\left(\frac{\bar{p}_a}{\bar{p}_c}\right)^{1/\gamma}\right\} \tag{15}$$

together with equation (12).

Computation

The program was written in Elliott 803 Autocode with a step length of $0.05\bar{t}$ and an error bound of 0.01 giving results correct to four significant figures. As the solutions are in tabulated form the switching condition is taken in the form $\bar{t}_d > 0$. An automatic check at the end of each iterative step is built into the program to insure the correct choice of phase equations for the following step.

A detailed investigation was carried out to determine whether the switching conditions given in the section, "Switching Conditions" were the same as $\dot{\bar{h}} \gtrless 0$. The results showed that when $\dot{\bar{h}} = 0$, $\bar{t}_{d,2} = 0$ and hence the two conditions are not the same, thereby giving rise to the pause condition.

Both the free oscillation ($z = 0$) of the system and the response to a sinusoidal input ($z = z_0 \sin\omega t$) are investigated using the data shown in Table 1.

Table 1

Characteristic Parameters:

$\rho_j, \rho_a = 0.002378$ slugs/cu ft; $\frac{1}{2}\rho_j V_\infty^2 = 43.93$ lb/sq ft
$xl = 28.452$ ft; $V_\infty = 193$ ft/sec

Nondimensional Parameters:

$\mathrm{HF} = 6.376$; $\bar{p}_a = 47.82$
$\bar{h}_0 = 0.1054$; $\bar{p}_0 = 48.50$
$\bar{l} = 1.757$; $\bar{V}_j = 0.775$
$\bar{L} = 0.0618$; $\bar{z}_0 = 0.00527$
$\omega = 0, 1.571, 3.142, 6.283$

It will be observed that the value of $\bar{h}_0$ indicates a static clearance of about 3 ft which was an early claim for an unskirted 50-ft craft.

The value of $\bar{z}_0$, giving a "ground" forcing amplitude of about 2 in. was dictated by the necessity of working within the range of validity of the simple static hovering theory. In order to demonstrate the nonlinear effects, it was felt that the frequency of input and not the amplitude was the important parameter. To increase the amplitude would necessitate the use of the exponential form of the static hovering theory [5].

Results

Free Oscillations. The response to the two initial states $dh_0/dt \gtrless 0$ are investigated. The degree of nonlinearity of the system is demonstrated by the dissimilarity of the cushion pressure variation curves shown in Fig. 2 which also demonstrates the critical nature of the damping. The difference between the modes of operation, under and overfeed, is shown in Fig. 4. It will be noticed that the value of U at the end of any one mode of operation is not zero. This is the impulsive effect referred to in the Introduction due to the neglect of the pause condition. Superimposed on this figure are the constant values of U assumed by

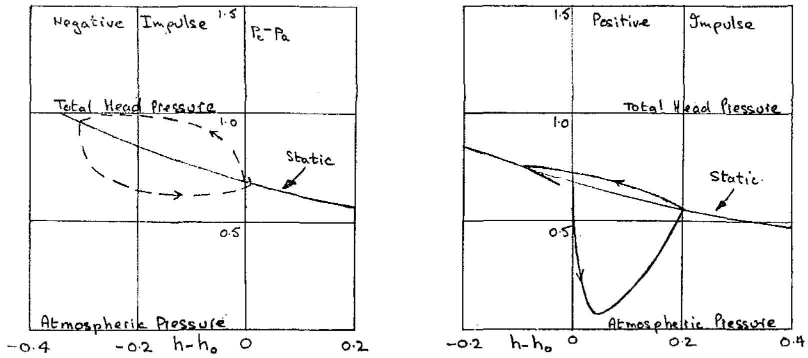

Fig. 5 Comparative static and dynamic cushion variations in free oscillations

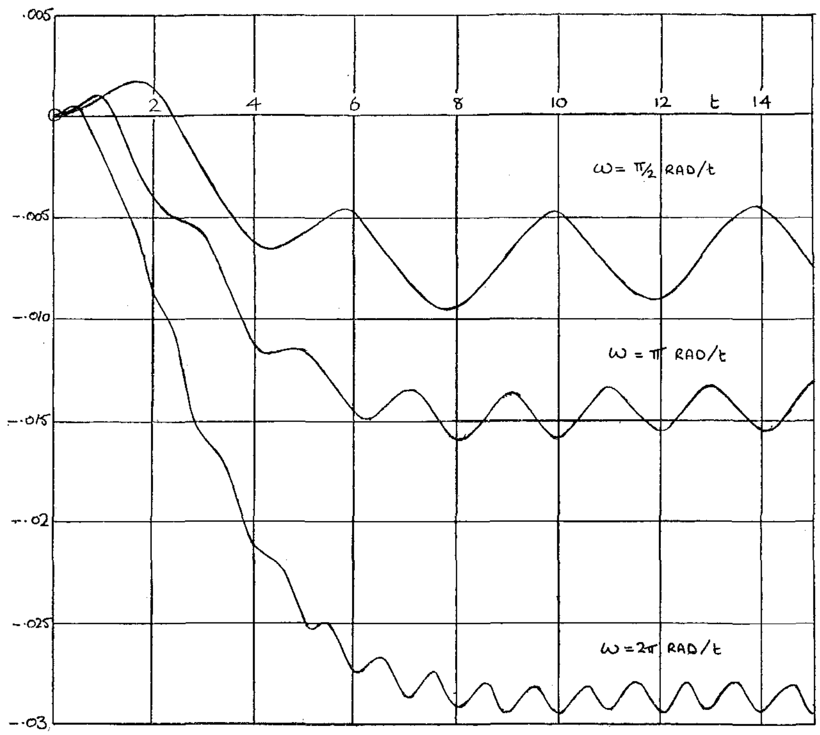

Fig. 6 Variation of center of oscillation with frequency of input

previous authors [1–3]. It can be seen beyond an initial time interval the approximation is good which is in accord with the pressure variations shown in Fig. 2 where it is seen that, in the corresponding time interval, the pressure is nearly equal to that given under static equilibrium conditions. In Fig. 5 the pressure variations corresponding to static and dynamic conditions are plotted against hoverheight illustrating the damping mechanism of the system as described in the Introduction; i.e., the rate of change of pressure with height is greater or less than that given under static conditions.

Forced Oscillations. Three frequencies of input are considered: $\omega = \frac{1}{4}, \frac{1}{2}$, and 1. The well-known characteristic of the input frequency dependence of the dynamic equilibrium position of a nonlinear system is shown to occur in Fig. 6. The magnitude of the offset in terms of percentage loss of hoverheight is plotted against frequency of input in Fig. 7(*a*). As far as the authors are aware the fact that the graph is a straight line is purely fortuitous. The magnification factor, defined as the ratio of the mean amplitude of the forced oscillation to the amplitude of the input, is plotted against time in Fig. 7(*b*) and the phase-shift of the forced response is shown in Fig. 7(*c*). The nonlinear character of the system is again apparent in Fig. 8 where the cushion pressure variation with hoverheight clearance is seen to present a closed

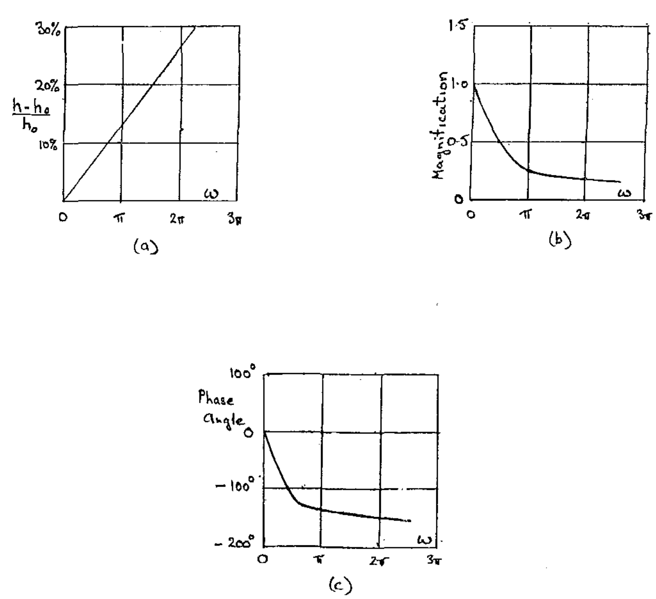

Fig. 7 Parameter variations with frequency of input

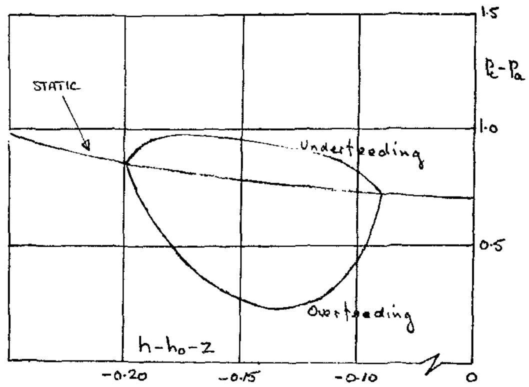

Fig. 8 Comparative static and dynamic cushion variations in forced oscillations

loop for the two phases of the motion. It appears that during the phase when the edge jet overfeeds there is a greater loss of pressure than there is of gain during the underfeed phase. The ratio of gain to loss of cushion pressure in one complete cycle of the steady-state oscillation is approximately 3. It may be recalled that the ratio of the damping factors found by Eames [2] was also approximately 3. The variation of cushion pressure, hoverheight, and craft heave velocity during the first cycle of the input is shown in Fig. 9 where the dominance of the negative values indicates the steady downward trend of lost hoverheight.

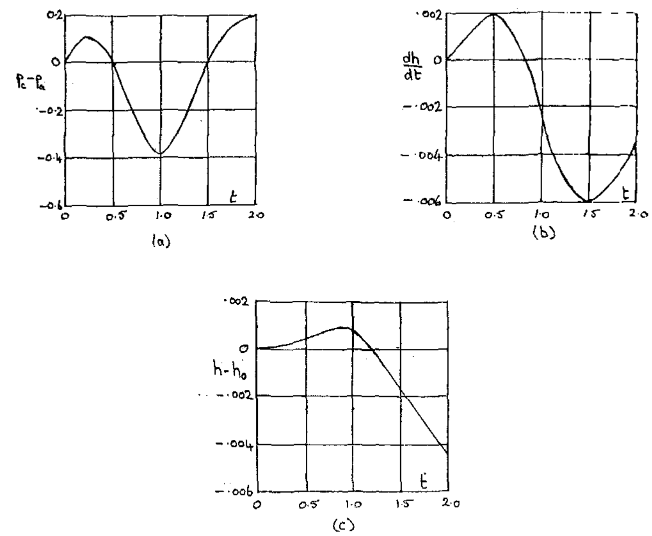

Fig. 9 First-cycle variations in forced oscillation

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