Housing Influence on Churning Losses in Geared Transmissions

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In a previous paper (Changenet, and Velex, 2007, "A Model for the Prediction of Churning Losses in Geared Transmissions—Preliminary Results," ASME J. Mech. Des., ***129****(1), pp. 128–133), a series of empirical formulas were presented enabling accurate predictions of churning losses for one gear, which is typical of automotive transmission geometry. However, this formulation does not take into account the influence of flanges and deflectors. In order to extend the proposed methodology, a test rig has been set up in which several movable walls can be inserted, thus making it possible to modify the radial and axial clearances, i.e., the distances between the tested gear and the walls. Based on a qualitative evaluation of the various fluid flow regimes possible in gearboxes, the influence of the global volume of the oil sump on churning losses is analyzed. By considering a number of flange and deflector arrangements, the following conclusions are drawn: (a) Radial clearances have a weaker influence than axial clearances and (b) power losses can be minimized by properly chosen axial clearances.*
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1 Introduction

Depending on tangential and rotational speeds, grease, splash, and jet lubrications are the three techniques currently used in geared transmissions. Considering medium speed applications typical of automotive gears, splash lubrication appears as the most appropriate and economical solution provided that tangential speeds are sufficient (at least 3 m/s) to splash the lubricant from the sump onto the gear teeth and the bearings. One drawback of this lubrication process is that it generates power losses caused by the churning of the lubricant, the pumping effects by the gear teeth, the projection of oil onto the casing walls, etc. It has been shown, for instance, that churning losses in splash lubricated automotive gearboxes can be the main dissipation source leading to a significant rise in bulk temperatures [1]. In this context, the accurate prediction of churning power losses is clearly crucial for the prediction of gearbox efficiency and thermal rating.

The drag torque due to the rotation of disks submerged in a fluid has been theoretically analyzed by Daily and Nece [2], Mann and Marston [3], Soo and Princeton [4], etc. In the case of gears, comparatively fewer models have been proposed because of the intrinsic complexity of the physical situation, which corresponds to a free surface problem of fluid dynamics controlled by a large number of variable parameters. The specific studies on churning losses in the literature are therefore empirical or semiempirical. Terekhov [5] carried out numerous experiments and measured the resulting churning torque expressed in terms of a dimensionless torque C_m. The analytical expression of C_m was deduced from dimensional analysis and depends on the flow regime. Using a similar approach, but in the specific case of truck transmissions, Lauster and Boos [6] proposed a unique expression for C_m. However, it can be noticed that Terekhov's and Lauster's equations are independent of gear tooth geometry, which seems unrealistic. Boness [7] investigated the drag torque generated by disks and gears rotating in water or in oil. Here again, the dimensionless torque depends on the flow regime but it is observed that, for one regime, the drag torque becomes larger with an increasing Reynolds number, suggesting that low viscosity lubricants generate higher losses in contradiction with experimental evidence. From a number of tests, Höhn et al. [8] derived a formula independent of lubricant viscosity to estimate churning torques. Similarly, a limited influence of lubricant viscosity was reported by Luke and Olver [9] who measured churning losses on a number of meshing spur pinion-gear pairs and individual pinions for various speeds and lubricants. This conclusion seems surprising because viscous forces are generated every time a part is in contact with oil. Numerical applications of the above formulas reveal a large discrepancy between the different calculated results and, in view of their limitations, none of the formulations can be considered as being intrinsically more credible. In the continuity of previous studies [10], the present paper is therefore aimed at presenting more reliable formulas for estimating churning power losses with a particular emphasis placed on the contributions of enclosures, flanges, and deflectors.

2 Previous Results

A specific test rig was developed for churning loss measurements and more than 100 experiments were conducted covering a large range of operating conditions [10]. Dimensional analysis was used in order to generalize the results of these experiments and generate empirical equations valid for wider operating conditions and various gear dimensions. Following Boness, the churning torque C_{ch} has been expressed in terms of a dimensionless torque C_m as

$$C_{ch} = \frac{1}{2}\rho\Omega^2 R_p^3 S_m C_m \tag{1}$$

where ρ is the lubricant density, Ω is the rotational speed, R_p is the gear pitch radius, and S_m is the submerged surface area. The following analytical expressions of C_m have been proposed depending on the flow regime:
If $\Omega R_p b/\nu = \mathrm{Re}_c < 6000$

$$C_m = 1.366\left(\frac{h}{D_p}\right)^{0.45}\left(\frac{V_0}{D_p^3}\right)^{0.1}\mathrm{Fr}^{-0.6}\,\mathrm{Re}^{-0.21} \tag{2a}$$

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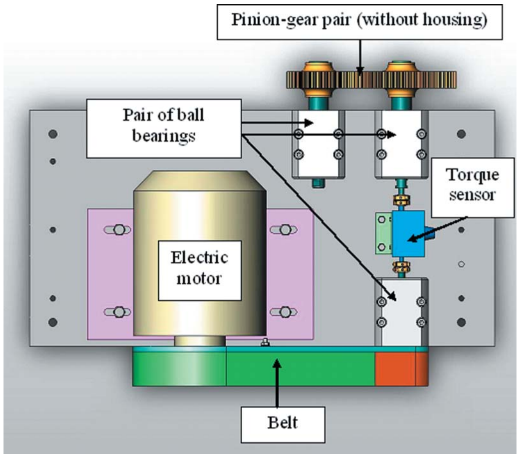

Fig. 1 Schematic representation of the test rig

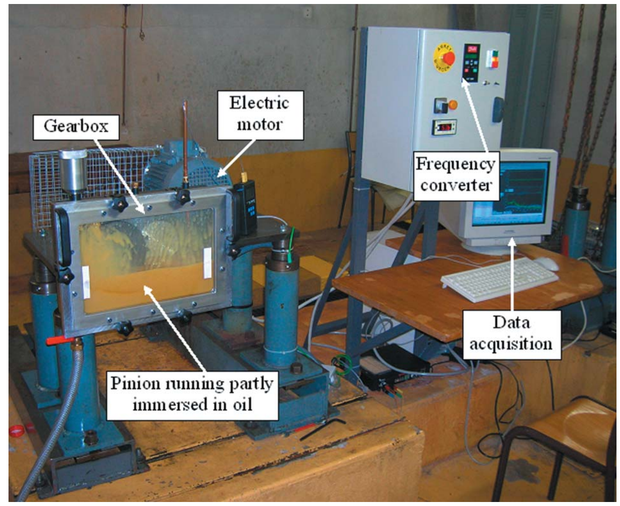

Fig. 2 Test rig

if $\mathrm{Re}_c > 9000$,

$$C_m = 3.644\left(\frac{h}{D_p}\right)^{0.1}\left(\frac{V_0}{D_p^3}\right)^{-0.35}\mathrm{Fr}^{-0.88}\left(\frac{b}{D_p}\right)^{0.85} \tag{2b}$$

where b is the tooth face width, ν is the lubricant kinematic viscosity, h is the pinion immersion depth, D_p is the gear pitch diameter, V_0 is the oil volume, Fr is the Froude number, and Re is the Reynolds number.

As far as the flow regime characterized by $\mathrm{Re}_c < 6000$ is concerned, it has been found that the dimensionless drag torque is independent of the gear geometry but depends on the Reynolds number, i.e., the viscous forces acting on the pinion or gear. On the other hand, opposite trends are observed for the second flow regime ($\mathrm{Re}_c > 9000$), namely: (i) power losses are not affected by oil viscosity because inertia forces are much larger than viscous forces and the Reynolds number can be discarded in the formulation and (ii) the gear shape ratio (b/D_p) is influential. In the transition zone ($6000 < \mathrm{Re}_c < 9000$), a linear interpolation between the two formulas is employed.

Investigations on enclosed rotating disks [2,3] have shown that chamber dimensions have an influence on drag torque and, by extrapolation, it can be reasonably inferred that the shape of the housing may affect churning losses in geared transmissions. In Eqs. (2*a*) and (2*b*), the housing contribution is indirectly introduced via a global factor related to the lubricant volume, which is not sufficient in itself to account for flanges and deflectors.

3 Experimental Device

In an attempt to overcome the limitations of formulas (2*a*) and (2*b*), a new test rig has been developed (Fig. 1) for measuring churning losses in the presence of flanges. It is composed of an electric motor that operates a primary shaft via a belt multiplying the rotational speed up to a maximum of 7150 rpm. A frequency converter is used to modify the operating speed. The secondary shaft can be removed to perform measurements with a single pinion and both shafts are supported by two pairs of identical ball bearings of 35 mm mean diameter. Churning losses are determined from direct torque measurements by a strain-gauged, temperature compensated, contactless sensor (FGP-CD1140) of accuracy ± 0.002 N m, a sensitivity shift of $0.009\%/{}^\circ$C, and a zero shift of 0.0004 N m$/{}^\circ$C in the 5–45°C temperature range. In order to eliminate bearing loss contributions, the bearing drag torques have been experimentally determined as a function of speed (for example, it has been found that a pair of bearings generates a drag torque of 0.045 N m at 7000 rpm) and then subtracted from global torque measurements to isolate the net contributions of churning. A counter of pulses is integrated in the torque sensor, so it is used as well for speed detection. The oil sump is a parallelepiped with one face made of Plexiglas in order to visualize the oil flows around the pinions (Fig. 2). A variety of lubricants and gears with several immersion depths can be used in the test rig but the results presented in this paper are limited to spur steel gears (Table 1). The internal dimensions of the housing are $380 \times 260 \times 100$ mm^3, which, for all the gear configurations, lead to axial and radial clearances above 27 mm sufficient to minimize enclosure effects on churning losses. As shown in Fig. 3, some movable walls can be inserted in the gearbox, thus making it possible to modify the radial and axial distances between a wall and a gear face or top land. The minimum axial clearance that is possible is 1 mm while the minimum radial clearance is 5 mm. Thermocouples are used for measuring the ambient and the lubricant temperatures; they are also used to ensure that thermal equilibrium is attained. In order to carry out experiments with various lubricant temperatures, several heating covers have been placed on the external bottom face of the housing. The same experiments were carried out several times, and an overall repeatability of ± 0.035 N m was found.

Table 1 Gear data

	Gear 1	Gear 2	Gear 3	Gear 4	Gear 5	Gear 6	Gear 7
Module (mm)	1.5	1.5	3	3	5	5	5
Face width (mm)	14	14	24	24	24	24	24
Pitch diameter (mm)	96	153	90	159	100	125	150
Outside diameter (mm)	97.5	154.5	93	162	105	130	155
Base diameter (mm)	94.125	151.125	86.25	155.25	93.75	118.75	143.75
Number of teeth	64	102	30	53	20	25	30
Pressure angle (deg)	20	20	20	20	20	20	20

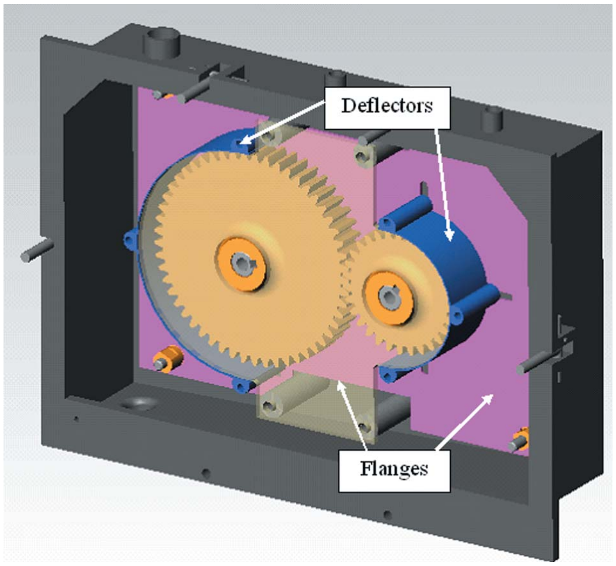

Fig. 3 Housing shape modifications

4 Experimental Results: Influence of the Enclosure and Lubricant Volumes

Since the housing dimensions are different from those used in Ref. [10], some complementary experiments were carried out in order to critically assess the validity of formulas (2*a*) and (2*b*). These tests were conducted in steady state conditions (thermal equilibrium) with several gear geometries and rotational speeds. Three lubricants (mineral oils with extreme-pressure additives), as described in Table 2, have been used, Oils 1 and 2 are employed in automotive manual transmissions, whereas Oil 3 is typical of industrial geared transmissions. The experimental and numerical churning losses are in good agreement, and it is concluded that, in the absence of movable flanges, the clearances in the system are sufficiently large to eliminate enclosure effects. A typical series of results is shown in Fig. 4, which presents churning power losses versus rotational speed for the 30 tooth (m=5 mm) gear with an immersion depth of 41 mm (h/R_p=0.55). The transition between the two flow regimes is clearly visible at about 2000 rpm where

Table 2 Oil properties

Oil No.	ν at 40°C (cS)	ν at 100°C (cS)	ρ at 15°C (kg/m^3)
1	48	8.3	873
2	73.5	10.4	896
3	320	24	898

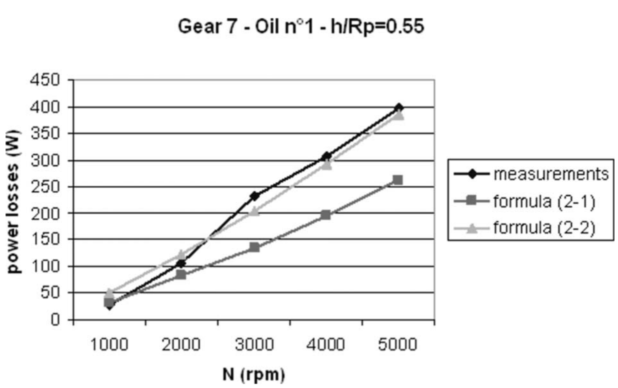

Fig. 4 Churning losses at thermal equilibrium

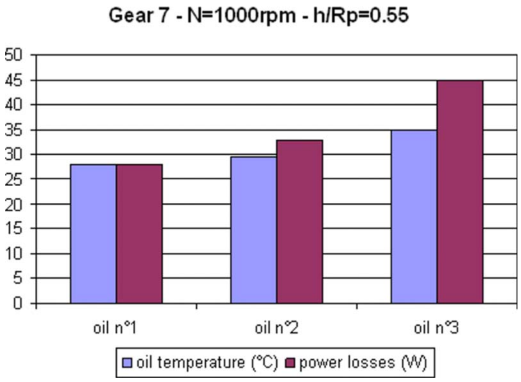

Fig. 5 Influence of oil properties

the critical Reynolds number is found to be $\mathrm{Re}_c = 2000(\pi/30)0.075\times 0.024/46\times 10^{-6}=8195$, which is within the threshold limits 6000–9000. As suggested by the authors [10], the drag torque in this zone of transition can be estimated by calculating the arithmetic average of the values from (2*a*) and (2*b*) giving a churning power loss of 103.5 W, which is very close to the experimental figure of 106 W. These complementary experiments also confirm that viscous forces have an influence on the drag torque for low values of the critical Reynolds number. The comparisons shown in Fig. 5 reveal that the highest viscosity lubricant generates larger churning losses than the others and, as a consequence, the oil temperature of equilibrium in the sump increases too.

Focusing on the influence of the enclosure, formulas (2*a*) and (2*b*) show that the drag torque is proportional to $(V_0/D_p^3)^{\psi}$. For each flow regime, the value of exponent ψ has been determined by using various gear geometries with different pitch diameters but the specific role of the global volume of lubricant in the sump V_0 has not been experimentally investigated. To this end, a flat horizontal plate is inserted in the gearbox as a new base (Fig. 6) to reduce V_0 in the proportion V_0 without plate/V_0 with plate≈ 1.6 without changing the immersion depth of the pinion.

Some tests were carried out for a given oil (Oil No. 1 in Table 2) with different gear geometries and rotational speeds but a constant immersion depth h/R_p=0.5. The corresponding results are shown in Table 3. It is found that, at 4000 rpm, churning power losses increase when oil volume decreases, whereas, at the lower speed of 2000 rpm, this evolution is no longer perceptible. These findings have been confirmed by applying formulas (2*a*) and (2*b*):

- At 4000 rpm, the second flow regime appears so that, according to (2*b*), power loss should proportionally increase to the ratio $P/P_{\mathrm{ref}}=(1.6)^{0.35}\approx 1.18$.

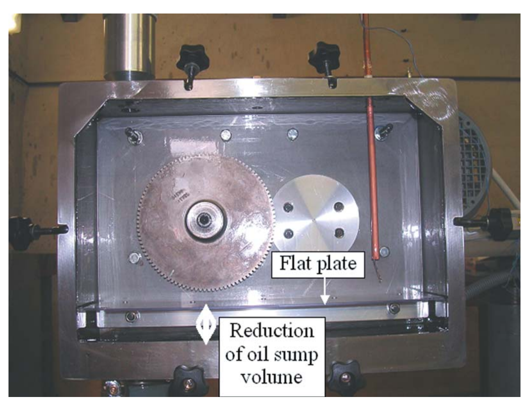

Fig. 6 Modification of oil sump volume

Table 3 Influence of oil volume on power losses

	Gear 4 N=4000 rpm	Gear 2 N=4000 rpm	Gear 2 N=2000 rpm
P/P_{ref}	1.17	1.13	1
Re_c	32,800	11,000	3500

- 2000 rpm corresponds to the first flow regime and from (2a), power loss should remain approximately constant since $P/P_{ref}=(1.6)^{-0.1}\approx 0.96$.

More physical insight into the phenomenon can be gained by considering the Froude number Fr* for free surface flows (Fig. 7) defined as

$$\mathrm{Fr}^* = \frac{V}{\sqrt{gy}} \tag{3}$$

where V is the mean flow velocity, y is the height/depth of the flow, and g is the acceleration of the gravity field.

Fr* can be interpreted as the ratio of the inertial to gravity forces in the flow or as the ratio between the mean flow velocity and the speed of an elementary gravity wave traveling on the fluid surface. Free surface flows caused by a submerged source have been studied by Vanden-Broeck [11] who has shown that a wave train is created on the free surface whose amplitude increases with the value of Fr*. In a gearbox, the mean flow velocity is related to the gear circumferential speed, and gravity waves are observed at the oil surface, as visualized in Fig. 8. The introduction of the flat plate reduces the effective height y and leads to a higher Froude number. As a consequence, the wave train is more marked than that with the original bottom face and it increases the pinion drag torque. On the other hand, for a rotational speed of 2000 rpm (first flow regime), viscous forces are still prevalent over inertial forces and, as discussed in Ref. [11], they introduce some damping that limits the wave train amplitudes. Consequently, the drag torque and the associated churning losses are not strongly affected compared to the situation with the original sump volume. In order to confirm this hypothesis, additional tests were carried out with a 53 tooth (m=3 mm) gear at the same speed of 2000 rpm. Since the tooth face width is larger, power losses are higher and oil viscosity at thermal equilibrium is lower. Due to these evolutions, the critical Reynolds number increases (Re_c=7800) so that viscous forces

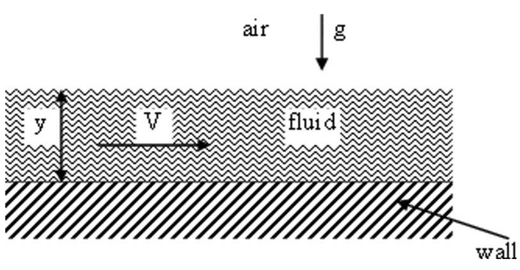

Fig. 7 Free surface flow

Fig. 8 Gravity waves traveling over the oil surface

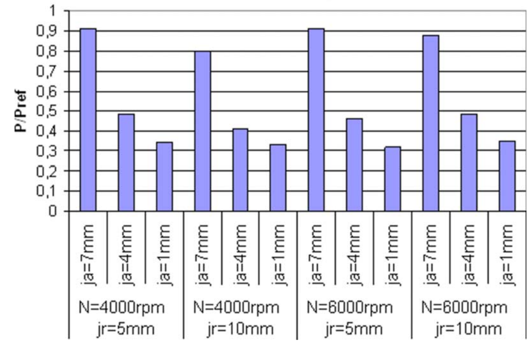

Fig. 9 Influence of axial and radial clearances on churning losses

become of secondary importance compared to the inertial forces. Contrary to the first set of results obtained with a thinner gear, the churning power loss with the additional plate (or modified bottom face) is found to be higher than that with the initial casing geometry: P/P_{ref}=1.14. It can therefore be concluded that the effects of the global oil sump volume on churning losses are largely dependent on viscous forces.

Finally, it should be emphasized that, in real conditions, the variations in the lubricant volume are not independent of the immersion depth. The two groups of parameters (V_0/D_p^3) and (h/D_p) in (2a) and (2b) are then modified and, in general, churning losses do not increase by reducing the oil volume: An experiment has shown that it is easier to drain the oil sump when the flat plate is added; in this case, the gear immersion depth is lower, compared to the one without flat plate, and, as a consequence, oil churning losses decrease.

5 Modification of Radial and Axial Clearances

In order to quantify the influence of deflectors and flanges on churning losses, several tests were carried out with axial clearances from 1 mm to 10 mm and radial clearances within the 5–10 mm range. Axial clearances correspond to the distance between a flange and a gear face and are identical on both sides of the gear. On the other hand, radial clearance is defined as the separation between a deflector and the gear tip radius. The deflector extends over half the gear circumference by analogy with differential drive housings in automotive manual gearboxes. The inner part of the housing can be assimilated to a half cylinder, which acts as a circular deflector covering half the differential wheel circumference. By contrast, flanges cover the entire lateral surface of the pinion. First, some experiments were conducted with Gear No. 2 and Oil No. 1 (Tables 1 and 2) whose results are synthesized in Fig. 9. The evolutions of churning loss with respect to the case of reference, i.e., no flanges, no deflectors, are quantified by the ratio (P/P_{ref}). Figure 9 reveals that, when the speed varies between 4000 rpm and 6000 rpm, churning is almost insensitive to the radial clearance but that power losses are strongly reduced when the axial clearance is reduced from 7 mm to 1 mm. The experiments on bladed disks by Mann and Marston [3] have already shown that the radial clearance has a small influence on the drag torque. According to Daily and Nece [2], four basic flow regimes can take place within the axial gap between a rotating disk and a casing wall, which are governed by both the Reynolds number and axial separation. For the considered geometries and operating conditions, the regime is characterized by a flow with two separate boundary layers, i.e., the combined thickness of the boundary layers on the rotor and the stator is smaller than the axial gap j_a. In such conditions, for either laminar or turbulent flows, the frictional torque acting on an enclosed disk is, in theory, proportional to $(j_a)^{1/10}$ [2]. However, the experimental evidence in

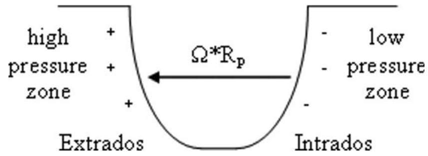

Fig. 10 Extrados and intrados surfaces

Fig. 9 reveals a stronger influence of the axial clearance attributed to the suction effect by the teeth. By analogy with impeller blades in centrifugal pumps, tooth extrados surfaces generate high pressure zones, whereas intrados surfaces create low pressure or suction areas at the back of the teeth, as schematically represented in Fig. 10. It is therefore postulated that a reduction in the axial distance between the rotating gear and a plate or flange can decrease churning losses if it is sufficient for lowering or eliminating the suction of lubricant by the teeth. It is to be noted that a similar phenomenon has been reported for windage losses [12,13] for which it has been demonstrated (a) that air is axially pumped and radially expelled by the teeth and (b) that enclosures reduce power losses.

In order to generalize the observation above and establish formulas that include the influence of axial clearances on churning losses, some additional measurements have been made in steady state thermal conditions and without deflector (whose influence was found negligible). The speed range was 1000–6000 rpm, and the various gear geometries under consideration are defined in Table 1. It has been found that the contribution of the axial clearance depends on the fluid flow regime encountered.

At low-medium rotational speeds ($\mathrm{Re}_c < 6000$), since viscous forces are influential on the drag torque, the lubricant was heated to impose an identical temperature for all the tests and, in this way, the same oil viscosity. It has been found that the dimensionless ratio (P/P_{ref}) does not depend on parameters related to speed or gear tooth geometry (Fig. 11) while the gear diameter appears as a prominent factor since churning power loss reductions are more significant for larger gears. By using dimensional analysis (whose parameters are defined in Fig. 12), the experimental results lead to the following relationship:

$$\frac{P}{P_{\mathrm{ref}}} = 2.17\left(\frac{D_p}{D_f}\right)^{3/4}\left(\frac{j_a}{R_p}\right)^{0.383(D_p/D_f)} \tag{4}$$

where P_{ref} is determined using Eqs. (1) and (2*a*).

It should be emphasized that (4) is valid only when flange diameters are larger than gear diameters; otherwise, the suction of the lubricant by the teeth is not affected and no loss reduction is observed. The fact that power reductions are expected, i.e., $P/P_{\mathrm{ref}} \leq 1$, implies that the axial clearance is limited by the condition

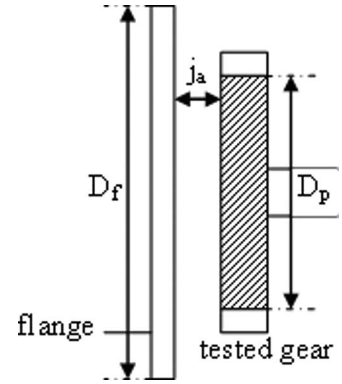

Fig. 12 Geometrical parameters for the influence of flanges

$$j_a \leq R_p\left(\frac{1}{2.17}\left(\frac{D_f}{D_p}\right)^{3/4}\right)^{2.61(D_f/D_p)} \tag{5}$$

For the gear geometries considered in this paper, the range of validity approximately corresponds to $j_a \leq 10$ mm. For higher axial clearances, this parameter has no more influence on churning losses.

At higher rotational speeds ($\mathrm{Re}_c > 9000$), the dimensionless ratio (P/P_{ref}) is still found to be largely independent of rotational speed but gear geometry appears as influential (Fig. 13) with a more pronounced reduction in churning losses when tooth faces and modules are larger. In this context, the following equation is proposed:

$$\frac{P}{P_{\mathrm{ref}}} = 0.76\left(\frac{D_f}{D_p}\right)^{0.48}\left(\frac{j_a}{\sqrt{mb}}\right)^{0.548(D_p/D_f)} \tag{6}$$

where P_{ref} is calculated from (1) and (2*b*).

Using a similar reasoning as for Eq. (4), the condition on the clearance amplitude reads

$$j_a \leq \sqrt{mb}\left(\frac{1}{0.76}\left(\frac{D_p}{D_f}\right)^{0.48}\right)^{1.82(D_f/D_p)} \tag{7}$$

For the gear geometries considered in this paper, the range of validity approximately corresponds to $j_a \leq 5$ mm for small gears, and up to $j_a \leq 15$ mm when tooth faces, modules, and diameters are larger.

All the tests were conducted with the same axial clearance on both sides of the gear. The case of different clearances has been examined (Fig. 14) and it has been observed that (P/P_{ref}) mostly depends on the smallest clearance. For example, it has been found that an axial clearance of 1 mm on one side and 10 mm on the opposite side leads to the same power loss reduction as two

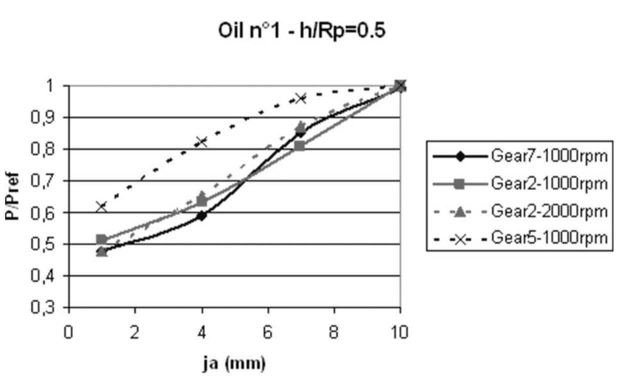

Fig. 11 Influence of axial clearances at low rotational speeds on experimental churning losses

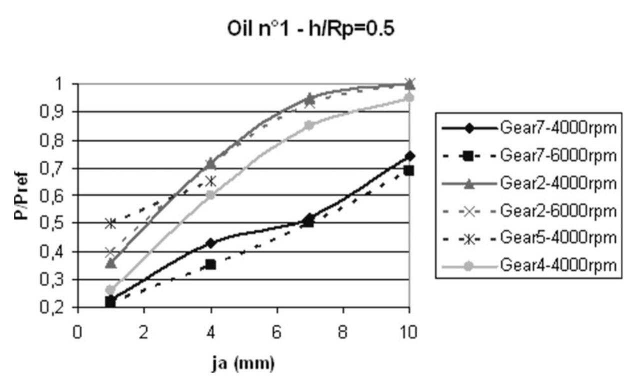

Fig. 13 Influence of axial clearances at high rotational speeds on experimental churning losses

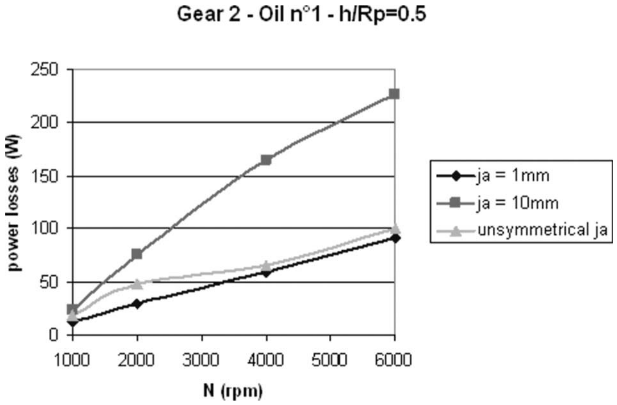

Fig. 14 Effect of unsymmetrical clearances on experimental churning losses

flanges mounted at the same distance of 1 mm from the gear faces. It seems therefore that one single flange can be sufficient to reduce churning losses. However, this point has yet to be investigated by considering various gear and flange combinations with, in particular, the influence of helical gears, which, depending on the sense of the helix, may induce a privileged sense for the lubricant suction by the teeth. Finally, it should be emphasized that the experiments in this paper were carried out with a single pinion. The presence of a pinion-gear pair certainly modifies the oil flow (for example, by creating an additional axial flow near the meshing teeth) and may affect the influence of clearances on churning losses.

6 Conclusion

A new specific test rig has been exploited to measure churning losses in gearboxes with special emphasis on the contributions of enclosures. In order to quantify the influence of the clearances between housing walls and gear faces, various movable walls can be inserted in the test rig housing including a flat base, flanges parallel to gear faces, and circular deflectors enclosing the gears. It has been verified that the formulation proposed by the authors in Ref. [10] is valid for this new test rig and that the effect of the lubricant volume is correctly apprehended in the churning loss equations. Considering the particular flange and deflector geometries tested in this paper, the following conclusions have been drawn: (a) The influence of radial clearances is weaker than that of axial clearances and (b) power losses can be substantially reduced by mounting flanges close to the gear lateral faces. It seems that the pumping effect by the teeth is one of the physical mechanisms at the origin of churning loss and that obstacles deviating the lubricant flow between the teeth can improve the system efficiency. Based on the dimensional analysis, a correction formula has been derived, which accounts for the contribution of axial clearances. The contributions of flanges on churning losses depend on the Reynolds number related to the gear geometry and, for high Reynolds numbers, viscous forces can be neglected compared to inertia forces. This modification of force balance within the lubricant flow has an influence on the reduction in churning losses. The experiments in this paper were conducted with a single pinion but, from the results in Ref. [10], the influence of flanges and deflectors on churning losses may depend on the sense of rotation when a pinion-gear pair is considered. Further research is currently under way in order to systematically analyze the churning losses associated with this situation as well as the modifications brought by helical teeth on the flows around rotating gears.

Acknowledgment

The authors want to gratefully thank Vincent Ricol and Louis Bartolomé for their important contributions to the test rig design and construction.

Nomenclature

b = tooth face width (m)
C_{ch} = churning torque (N m)
C_m = dimensionless drag torque
D_f = outer diameter of flange (m)
D_p = pitch diameter (m)
Fr = Froude number depending on gear parameters $=\Omega^2 R_p/g$
Fr* = Froude number for free surface flows$=V/\sqrt{gy}$
g = acceleration of gravity $(\mathrm{m/s^2})$
h = immersion depth of a pinion (m)
j_a = axial clearance (m)
j_r = radial clearance (m)
m = module (m)
N = rotational speed (rpm)
P = churning power losses (W)
P_{ref} = churning losses for a state of reference (W)
R_p = pitch radius (m)
Re = Reynolds number$=\Omega R_p^2/\nu$
Re_c = critical Reynolds number$=\Omega R_p b/\nu$
S_m = immersed surface area of the pinion (m^2)
V = velocity (m/s)
V_0 = oil volume (m^3)
y = vertical length scale (m)
υ = kinematic viscosity $(\mathrm{m^2/s})$
ρ = fluid density $(\mathrm{kg/m^3})$
Ω = rotational speed (rad/s)

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