Analytical Comparison of Two Different Redundancy Concepts for Switched Reluctance Machines

Dieter Gerling, Andreas Schramm

Institute for Electrical Drives
University of Federal Defense, Munich
Werner-Heisenberg-Weg 39
D-85577 Neubiberg, Germany
phone: +49 89 6004 3708, fax: +49 89 6004 3718
e-mail: Dieter.Gerling@unibw-muenchen.de

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Abstract
In this paper two different, fully redundant SRM (switched reluctance machine) topologies are compared: Firstly a 6/4-topology with two identical motors on one axis, and secondly a 12/8-topology, where one 3-phase-system uses every other stator tooth (and the second, redundant, 3-phase-system uses the rest of the stator teeth).
The following calculation will be performed using analytical formulae to get a fast and clear comparison. The nonlinearity caused by the usual saturation of SRM is covered by a simple correction factor: As the relative comparison of the two redundancy concepts is of interest, this method leads to qualitative and quantitative good results.
In addition, a very good starting point for the detailed FEM-refinement of the most promising alternative is generated.

1. Introduction

For safety-critical applications, e.g. in the automotive or aerospace industry, electrical drives with redundancy are required. In the following two principally different SRM topologies, guaranteeing full redundancy, are investigated analytically to realize a fast and clear comparison:
Alternative 1 is a conventional 6/4-topology, with two identical motors on one axis. Therefore, even double end windings have to be installed into the envisaged overall volume. This reduces the stack length of the machines. It shall be assumed that the separation of the two machines can be realized with a very thin non-conductive separator, so that the thickness of this separator will be neglected in the following.
Alternative 2 is a 12/8-topology, where one 3-phase-system uses every other stator tooth. Herewith the specified torque has to be reached. The other six stator teeth are used for the redundancy ("second machine" in the same stator), the respective teeth carry the second 3-phase-system. This alternative incorporates just two end windings in the overall volume.
Figure 1 shows the two alternatives in a principle view; for a better understanding the rotating machines are cut and wound off.

2. Alternative 1 (6/4 configuration)

2.1. Torque calculation

As long as the reluctance machine is not saturated (i.e. as long as linearity is valid) the torque T generated by one coil can be calculated from the differentiation of the magnetic energy W_{mag} with respect to the angle γ [1,2]:

$$T = \frac{\partial}{\partial \gamma} W_{mag} = \mu_0 \frac{(N \cdot I)^2}{\delta} l_{Fe} \cdot r \tag{1}$$

Here μ_0 is the permeability of vacuum, N is the number of turns of a single coil, I is the current in this coil, δ is the air-gap width in the aligned

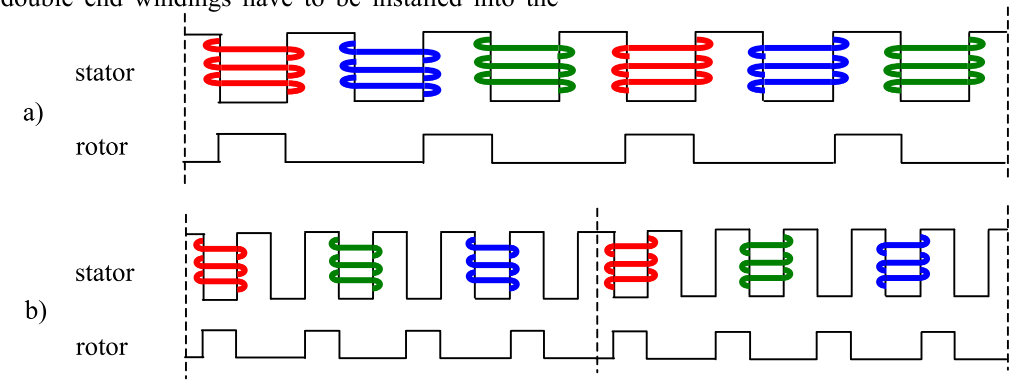

Fig.1: Principle sketch of two different switched reluctance machines with 3-phase-system (cut and wound off):
a) 6/4 configuration
b) 12/8 configuration (with every other stator tooth wound)

position, l_{Fe} is the iron stack length and r is the bore radius. Usually, for increasing the torque switched reluctance motors are operated far in saturation. To estimate this influence of the saturation the following simplified $\Psi - I -$diagram is regarded (it is assumed that the inductivity ratio between aligned and unaligned position is 4:1):

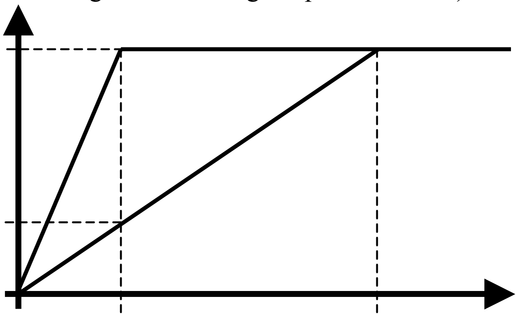

Fig. 2: Simplified $\Psi - I -$diagram (inductivity ratio 4:1).

From this diagram the torque can be calculated from the area between the characteristics for „aligned" and „unaligned".

Considering the above assumptions and simplification, the saturated operation of the SRM gives up to four times the torque of the unsaturated operation (depending on the current level).

To cope with this, in the following a correction factor $F_1 = 3.5$ will be applied to calculate the torque in saturated operation.

Assuming that the stator and rotor teeth have equal width b_{tooth}, the mean torque during one rotation is:

$$T_{mean} = F_1 \frac{6 \cdot \mu_0}{\pi \cdot \delta} \left(N \cdot I \right)^2 \cdot l_{Fe} \cdot b_{tooth} \tag{2}$$

The torque equation gives:

$$T_{mean} = T_{stat} + 2 \cdot J_{rotor} \frac{d\omega}{dt} \tag{3}$$

T_{stat} is the stationary part of the load torque (e.g. friction) including the acceleration of the coupled masses (as the acceleration $a = d\omega / dt$ is constant for a certain operating point and the coupled masses do not change with different motor designs, this part of the acceleration torque can be considered in T_{stat}). J_{rotor} is the inertia of the rotor, in this special case it has to be considered twice, because even the rotor of the redundant motor has to be accelerated. It can be calculated approximately like follows:

$$J_{rotor} \approx \gamma_{Fe} \cdot l_{Fe} \cdot r^3 \cdot b_{tooth} \tag{4}$$

The variable γ_{Fe} is the specific weight of iron. From the equations (2) to (4) and the equation

$$B = \mu_0 \cdot N \cdot I / \delta \qquad \Rightarrow \qquad N \cdot I = B \cdot \delta / \mu_0 \tag{5}$$

(which is true under the assumption of linearity [1]) the following equation can be deduced:

$$l_{Fe} \cdot b_{tooth} = \frac{T_{stat}}{F_1 \frac{6 \cdot \delta}{\pi \cdot \mu_0} B^2 - 2 \cdot \gamma_{Fe} \cdot r^3 \cdot a} \tag{6}$$

Right to the equation sign there are

- constants (numbers, π, μ_0, γ_{Fe}),
- values determined by the application (T_{stat}, a),
- chosen (but well motivated) data ($F_1 = 3.5$, $\delta = 0.25m$, $B = 1.8T$)
- and the variable r.

With $l_{Fe} \cdot b_{tooth}$ the pole area dependent on the bore radius r is determined.

2.2. Loss calculation

Beside the torque generation it is decisive for the machine design, that the losses of the machine can be dissipated. In the following a loss dissipation over the machine surface is assumed.

The losses of an energized coil $P_{loss,coil}$ can be calculated as follows:

$$P_{loss,coil} = R_{coil} \cdot I^2 \tag{7}$$

with R_{coil} as coil resistance. This coil resistance is:

$$R_{coil} = \frac{\rho_{Cu}}{k_{Cu}} N^2 \frac{l_{coil,mean}}{b_{coil,mean} \cdot h_{slot}} \tag{8}$$

Here is ρ_{Cu} the specific resistance of copper, $l_{coil,mean}$ the mean length of one turn, k_{Cu} the copper filling factor (in the following $k_{Cu} = 0.5$ is assumed), $b_{coil,mean}$ the mean width of the coil and h_{slot} the slot height. The mean length of one turn can be calculated as follows (assuming the distance end winding to iron stack to be $0.5 \cdot b_{coil,mean}$):

$$l_{coil,mean} = 2 l_{Fe} + \left(2 + \pi \right) b_{coil,mean} + 2 b_{tooth} \tag{9}$$

Further is true:

$$b_{coil,mean} = \pi \cdot \left(r + \frac{1}{2} h_{slot} \right) \cdot \frac{1}{6} - \frac{b_{tooth}}{2} \tag{10}$$

$$h_{slot} = r_{tot} - r - h_{yoke} \tag{11}$$

with r_{tot} as outer radius of the machine and h_{yoke} as stator yoke height. As the flux of one tooth flows in two directions in the yoke, one gets:

$$h_{yoke} \approx 0.5 \cdot b_{tooth} \tag{12}$$

Introducing eq. (10) to (12) into eq. (9) gives:

$$\begin{aligned} l_{coil,mean} &= 2 \cdot l_{Fe} + \left(2 + \pi \right) \cdot b_{coil,mean} + 2 \cdot b_{tooth} \\ &= 2 \cdot l_{Fe} + \left[1 - \frac{\pi}{2} - \frac{\left(2 + \pi \right) \cdot \pi}{24} \right] \cdot b_{tooth} + \\ &\quad + \frac{\left(2 + \pi \right) \cdot \pi}{12} \left(r + r_{tot} \right) \end{aligned} \tag{13}$$

Further is true:

$$b_{coil,mean} = \pi \cdot \left(r + \frac{1}{2} h_{slot} \right) \cdot \frac{1}{6} - \frac{b_{tooth}}{2}$$
$$= \left(-\frac{1}{2} - \frac{\pi}{24} \right) \cdot b_{tooth} + \frac{\pi}{12} \cdot \left(r + r_{tot} \right) \quad (14)$$

$$h_{slot} = r_{tot} - r - h_{yoke} = r_{tot} - r - \frac{1}{2} b_{tooth} \quad (15)$$

Introducing the eq. (13) to (15) and (8) into eq. (7) gives:

$$P_{loss,coil} = \frac{\rho_{Cu}}{k_{Cu}} \left(NI \right)^2 \cdot$$
$$\cdot \frac{2 \cdot l_{Fe} + A \cdot b_{tooth} + B \cdot \left(r + r_{tot} \right)}{\left[C \cdot b_{tooth} + \frac{\pi}{12} \left(r + r_{tot} \right) \right] \left(r_{tot} - r - \frac{b_{tooth}}{2} \right)} \quad (16)$$
$$A = 1 - \frac{\pi}{2} - \frac{\left(2 + \pi \right) \pi}{24},$$
$$B = \frac{\left(2 + \pi \right) \pi}{12}, \qquad C = -\frac{1}{2} - \frac{\pi}{24}$$

Substitution of NI with eq. (5), regarding that every coil is only energized if the respective stator tooth and a rotor tooth are at least partly overlapping, regarding that each phase consists of two coils and the machine of three phases gives for the mean losses of the machine:

$$P_{loss,mean} = \frac{b_{tooth}}{r} \cdot \frac{\rho_{Cu}}{k_{Cu}} \left(\frac{B \cdot \delta}{\mu_0} \right)^2 \cdot$$
$$\cdot \frac{\frac{24}{\pi} l_{Fe} + \frac{12}{\pi} A \cdot b_{tooth} + \left(2 + \pi \right) \left(r + r_{tot} \right)}{\left[C \cdot b_{tooth} + \frac{\pi}{12} \left(r + r_{tot} \right) \right] \left(r_{tot} - r - \frac{b_{tooth}}{2} \right)} \quad (17)$$

Using some fundamental equations and transformations the specific losses (losses per surface area) can be computed as follows (if only one machine is energized, the losses shall be dissipated over the surface of this machine):

$$P_{spec} = \frac{\rho_{Cu}}{2 \cdot \pi \cdot k_{Cu}} \left(\frac{B \cdot \delta}{\mu_0} \right)^2 \cdot$$
$$\cdot \frac{\dfrac{\frac{24}{\pi} l_{Fe} + \frac{12}{\pi} A \cdot b_{tooth} + \left(2 + \pi \right) \left(r + r_{tot} \right)}{\left[C \cdot b_{tooth} + \frac{\pi}{12} \left(r + r_{tot} \right) \right] \left(r_{tot} - r - \frac{b_{tooth}}{2} \right)}}{\dfrac{r_{tot} \cdot r}{b_{tooth}} \left(l_{Fe} - D \cdot b_{tooth} + \frac{\pi}{4} \left(r + r_{tot} \right) + r_{tot} \right)} \quad (18)$$
$$D = \frac{3}{2} + \frac{\pi}{8}$$

The specific losses, which are determined by the cooling conditions of the application, are dependent on both unknowns l_{Fe} and b_{tooth} (please refer to eq. (6)) and on:

- constants (numbers, π, μ_0, ρ_{Cu}),
- chosen (but well motivated) data (δ, B, k_{Cu})
- and the variables r, r_{tot}.

2.3. Parametric optimization

To come to an optimum machine design, a parametric optimization procedure (similar to [3]) will be applied. This procedure consists of the following steps:

- In the interesting region a combination (r_{tot}, r) is chosen.
- From eq. (6) and (18) the variables l_{Fe} and b_{tooth} can be determined so that the desired torque is generated and the loss dissipation is guaranteed (two equations with two unknown variables).
- Herewith for each combination (r_{tot}, r) all other data of interest (e.g. length of the machine, volume, etc.) can be calculated using the equations shown above.
- The optimum machine design can then be calculated using a predefined target function or taking results from a graphical representation.

The task is now to solve the two equations (6) and (18) for the two unknown variables l_{Fe} and b_{tooth}. After some extensive transformations this leads to an equation of fourth order in b_{tooth}. Such an equation may have four zeros. The solutions can be found using numerical methods, e.g. like included in the software package MathCad [4]. The interesting region of solutions is

$$0 < b_{tooth} < \frac{2 \cdot \pi \cdot r}{6} = \frac{\pi \cdot r}{3} \quad (19)$$

because only in this region there are meaningful results for the tooth width. For practical reasons it may be useful to further limit the interesting region of solutions, e.g. if a zero of the equation is positive but very small. The following limitation of possible solutions could be advantageous:

$$0.05 \frac{\pi \cdot r}{3} < b_{tooth} < 0.95 \frac{\pi \cdot r}{3} \quad (20)$$

If there is more than one solution in this region (which may be possible), then the smallest value of b_{tooth} has to be taken that leads to a positive value of l_{Fe}. The reasons are:

- Solutions, fulfilling eq. (20), but not leading to a positive iron stack length l_{Fe}, may exist. Such solutions have to be eliminated. From eq. (6) can be deduced that a positive iron stack length l_{Fe} is synonymous to the following condition:

$$r^3 < F_1 \frac{3}{\pi \cdot \mu_0 \cdot \gamma_{Fe}} \cdot \frac{B^2 \cdot \delta}{a} \quad (21)$$

- Choosing the smallest possible value for b_{tooth} (if more than one meaningful solution exists) ensures that (with the given bore radius r) the smallest inertia is chosen.[1]

2.4. Voltage equation

The terminal voltage, that has to be the same for the different alternatives of one application, was not disposed of so far. Just the value of $N \cdot I$ is determined by eq. (5). Assuming that the number of turns N can be adapted to the terminal voltage in such a way that the desired value of $N \cdot I$ is achieved, the terminal voltage does not have to be regarded in detail.

2.5. Results

As an example, a machine design with the following data will be presented:

- input data:
  - specific losses: $P_{spec} = 500\, W/m^2$
  - acceleration: $a = 5000\, rad/s^2$
  - inertia of the load: $J_{load} = 0.7 \cdot 10^{-4}\, kgm^2$
  - static torque: $T_{stat} = 1.56 Nm + J_{load} \cdot a$
- Parameter:
  - outer stator radius: $r_{tot} = 16mm \ldots 36mm$ (20 points in this interval)
  - bore radius: $r = 0.2 \cdot r_{tot} \ldots 0.8 \cdot r_{tot}$ (10 points in this interval)

The following figures show the tooth width b_{tooth} and the total length (two machines) l_{tot} as a function of outer stator radius and bore radius. Values equal to zero mean that there is no solution for the respective combination (r_{tot}, r). On the horizontal axes the number of calculated points are given (i.e. for the outer stator radius the values 1 to 20, for the relative bore radius the values 1 to 10), the vertical axis contains the respective value in SI-units.

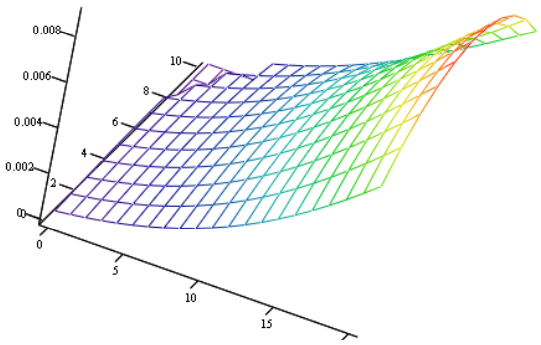

Fig. 3: Stator tooth width b_{tooth} in m as a function of outer stator radius and relative bore radius for the 6/4 configuration.

[1] Another meaningful alternative is to choose the tooth width as close as possible to the slot width to ensure low torque ripple and high starting torque independent of the rotor position, see [5].

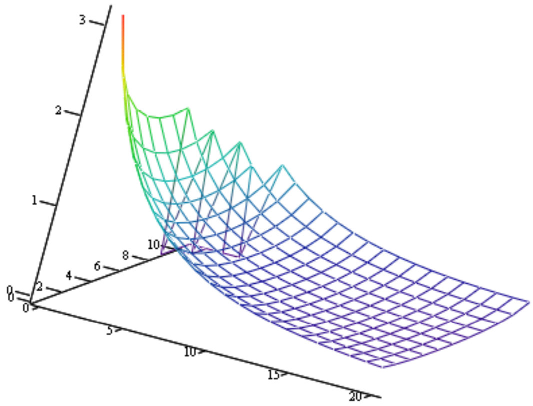

Fig. 4: Total length (two machines) l_{tot} in m as a function of outer stator radius and relative bore radius for the 6/4 configuration.

3. Alternative 2 (12/8 configuration)

3.1. Calculation procedure

The calculation procedure for this alternative is very similar to the procedure in chapter 2, only some equations have to be adapted to the special requirements of this drive. The mean torque is (please refer to eq. (2) for alternative 1):

$$T_{mean} = F_1 \frac{12 \cdot \delta}{\pi \cdot \mu_0} B^2 \cdot l_{Fe} \cdot b_{tooth} \qquad (22)$$

In a simplified model it can be assumed that the 12/8 configuration (alternative 2) has half the tooth width and double the tooth length compared to the 6/4 configuration (alternative 1). This means that the inductivities of both alternatives are similar.
Moreover, the 12/8 configuration can generate double the torque of the 6/4 configuration (please compare eq. (22) and eq. (2)). As the stator coils of the 12/8 configuration are energized twice as often, especially in high speed operation this high frequency may have the drawback that the current in the coil can not rise and fall fast enough. This results in a torque drop, as the torque is proportional to the squared current (in non-saturated operation).
This effect will be analyzed in the following:

- As a simplification, it will be assumed that the current rises and falls linearly in the same time period, in between the current is assumed being constant.
- The most advantegeous case is present, if the time periods for current rise and fall are neglegible against the time period of constant current. Therefore, the mean value of the current during the on-time is equal to the maximum value for both alternatives (this means equal torque per on-time intervall for both alternatives, in the following this will be described with the additional factor $F_2 = 1$).
- The most disadvantegeous case is present, if the time period for constant current is zero (i.e. the current characteristic is a triangle). Because of the double number of teeth, the current of the 12/8 configuration reaches just half the

value of the 6/4 configuration. Therefore, the mean value of the current (based on the respective on-time) for the 12/8 configuration is half the value of the 6/4 configuration (this means per on-time intervall a torque reduction of the 12/8 configuration to one quarter compared to the 6/4 configuration, in the following this is described by the additional factor $F_2 = 0.25$)

- Usually the switched reluctance motor is operated far in saturation resulting in a torque that is less than proportional to the squared current. Consequently, reducing the current to one half reduces the torque to a value higher than a quarter. Therefore, the possible values of the factor F_2 usually are further limited as against $0.25 \le F_2 \le 1$.

The mean torque during one rotation is now:

$$T_{mean} = F_1 \cdot F_2 \frac{12 \cdot \delta}{\pi \cdot \mu_0} B^2 \cdot l_{Fe} \cdot b_{tooth} \tag{23}$$

The torque equation is:

$$T_{mean} = T_{stat} + J_{rotor} \frac{d\omega}{dt} \tag{24}$$

Consequently, this leads to (please refer to eq. (6) for the 6/4 configuration):

$$l_{Fe} \cdot b_{tooth} = \frac{T_{stat}}{F_1 \cdot F_2 \frac{12 \cdot \delta}{\pi \cdot \mu_0} B^2 - 2 \cdot \gamma_{Fe} \cdot r^3 \cdot a} \tag{25}$$

For the specific losses, the following equation can be deduced:

$$P_{spec} = \frac{\rho_{Cu}}{2 \cdot \pi \cdot k_{Cu}} \left(\frac{B \cdot \delta}{\mu_0} \right)^2 \cdot$$

$$\cdot \frac{\frac{48}{\pi} l_{Fe} + (2+\pi)(r + r_{tot}) + E \cdot b_{tooth}}{\left[\frac{\pi}{24}(r + r_{tot}) + F \cdot b_{tooth} \right] \left(r_{tot} - r - \frac{b_{tooth}}{2} \right) \frac{r_{tot} \cdot r}{b_{tooth}} \left(l_{Fe} - G \cdot b_{tooth} + \frac{\pi}{8}(r + r_{tot}) + r_{tot} \right)} \tag{26}$$

$$E = \frac{24}{\pi} - 12 - \frac{2+\pi}{2},$$

$$F = -\frac{1}{2} - \frac{\pi}{48}, \qquad G = \frac{3}{2} + \frac{\pi}{16}$$

Now the same parametric optimization like for the 6/4 configuration can be performed. Again the two unknowns l_{Fe} and b_{tooth} have to be calculated for each chosen combination (r_{tot}, r). Like for the 6/4 configuration this leads to an equation of fourth order in b_{tooth}. The calculation procedure is now the same like described in section 2.3. The interesting region of solutions for the 12/8 configuration is (according to eq. (20)):

$$0.05 \frac{\pi \cdot r}{6} < b_{tooth} < 0.95 \frac{\pi \cdot r}{6} \tag{27}$$

The condition of a positive iron stack length can be deduced from eq. (25) and leads to:

$$r^3 < F_1 \cdot F_2 \frac{6}{\pi \cdot \mu_0 \cdot \gamma_{Fe}} \cdot \frac{B^2 \cdot \delta}{a} \tag{28}$$

3.2. Results

The following results for the 12/8 configuration are calculated for the same assumptions like for the 6/4 configuration. The additional factor for this configuration has been assumed to $F_2 = 0.6$.

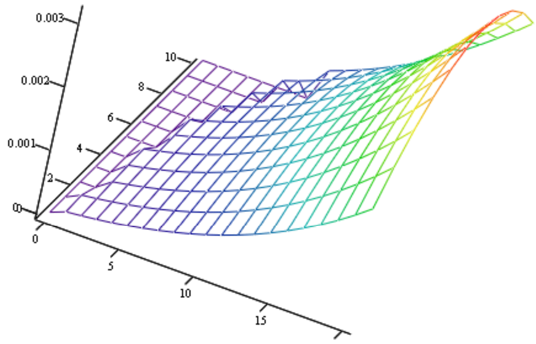

Fig. 5: Stator tooth width b_{tooth} in m as a function of outer stator radius and relative bore radius for the 12/8 configuration.

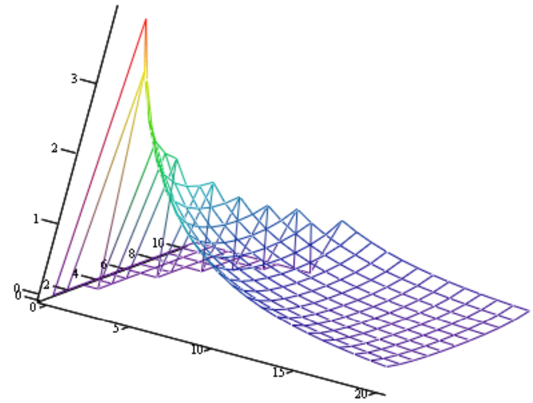

Fig. 6: Total length (one machine) l_{tot} in m as a function of outer stator radius and relative bore radius for the 12/8 configuration.

4. Comparison

The following table shows the results of machine designs according to two different optimization criteria: minimum total length and minimum volume. It is not claimed here that the numbers given in table 1 are absolute precise, but because of the correction factors F_1 and (only for the 12/8 configuration in addition) F_2 the values are at least quite near to reality. These analytical considerations even serve as basis to get an as good as possible starting point for a subsequent numerical refinement (e.g. applying FEM). In any case a qualitative comparison of both redundancy alternatives is possible.

It can be deduced from table 1 that alternative 2 (12/8 configuration) performs better than alternative 1. This result has been achieved using the assumption $F_2 = 0.6$. For different values of F_2 the result may be different, see below. Regarding the total length in table 1, the value for the 12/8 configuration is about 7% lower than for the 6/4 configuration. The same holds true for the total volume, as the outer stator diameter turns out to be the same for both alternatives.

Table 1: Data of both redundancy concepts due to different optimization criteria.

optimization to: (all values in mm)	minimum total length		minimum volume	
SR-machine alternative	6/4	12/8	6/4	12/8
outer stator radius r_{tot}	36	36	34	36
bore radius r	14	18	15	18
stator tooth width b_{tooth}	8.4	3.1	7.5	3.1
iron stack length l_{Fe}	55	129	63	129
total length l_{tot}	156	145	174	145

In the following, the influence of the factor F_2 will be analyzed in detail. The following figure shows the ratio of the total lengths of the 12/8 configuration and the 6/4 configuration $l_{tot,12/8} / l_{tot,6/4}$ as a function of the factor F_2 (all other conditions are unchanged), when optimized to the minimum total length. The factor F_2, which lies in the interval $0.25 \leq F_2 \leq 1$ only has an influence on the results of the 12/8 configuration.

The calculations resulted in the fact that always the maximum outer stator radius of $r_{tot} = 36mm$ has to be chosen to reach the minimum total length. Therefore, a similar characteristic may also be drawn for the ratio of the overall volumes (when optimized to the minimum total length).

It becomes obvious from figure 7 that (for the chosen conditions) both configurations are equal for $F_2 \approx 0.57$. Values larger than this ($F_2 > 0.57$) lead to an advantage for the 12/8 configuration, for smaller values the 6/4 configuration is advantageous. Similar dependencies can be expected even for different boundary conditions.

Regarding the numerical values of this example it can be deduced from figure 7 that, depending on the value of F_2, the minimum total length of the 12/8 configuration is between 54% and 252% of the minimum total length of the 6/4 configuration. The data of table 1 correspond to figure 7 for the factor $F_2 = 0.6$ (this factor had been assumed for the above calculations).

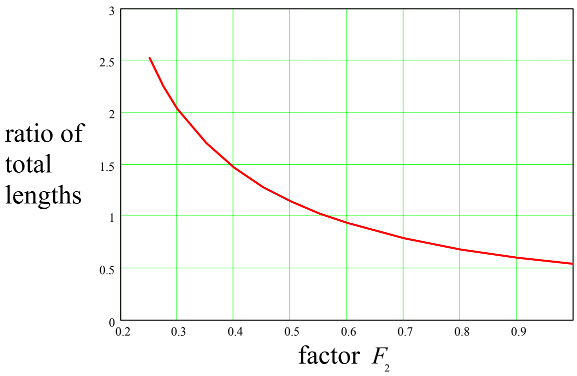

Fig. 7: Ratio of the total lengths of the 12/8 configuration and the 6/4 configuration $l_{tot,12/8} / l_{tot,6/4}$ as a function of the factor F_2.

5. Conclusions

In this paper two different, fully redundant SRM (switched reluctance machine) topologies were compared: a 6/4-topology with two identical motors on one axis, and a 12/8-topology, where one 3-phase-system uses every other stator tooth (and the second, redundant, 3-phase-system uses the rest of the stator teeth).

The analysis, based on analytical formulae only, uses the torque requirement and the loss dissipation as essential boundary conditions. The comparison is performed by applying a parametric optimization procedure.

It turned out during the analysis that the factor F_2 is critically decisive for the selection of the optimum redundant machine configuration. This factor reflects the current characteristics in the coils which are dependent on the speed of the specified operating point. Therefore, the speed of the specified operating point determines the selection of the optimum redundant machine configuration. Generally, at low speed the 12/8 configuration is advantageous, at high speed the 6/4 alternative.

References

[1] Gerling, D.: “Design of a Single-Phase Reluctance Actuator with Large Linear Stroke”, 8th European Conference on Power Electronics and Applications (EPE), 1999, Lausanne, Switzerland, CD-ROM proceedings

[2] Simonyi, K.: “Theoretische Elektrotechnik”, VEB Verlag der Wissenschaften, Berlin, 1980 (in German)

[3] Gerling, D.; Hofmann, H.: “Single-Phase Reluctance Actuator and Single-Phase Permanent Magnet Actuator Compared for Low-Cost Applications”, International Conference on Electrical Machines (ICEM), 2002, Brugge, Belgium, CD-ROM proceedings

[4] MathCad 2001: Online User-Handbook

[5] Miller, T.J.E.: „Switched Reluctance Motors and their Control“, Clarendon Press, 1993