2004 ASME International Mechanical Engineering Congress and Exposition
November 13-20, 2004, Anaheim, California USA

IMECE2004-60451

THERMAL ANALYZING OF DISC TYPE WINDINGS WITH DIRECTED OIL FLOW

Seyed Mehdi Pesteei Ph. D.
Department of Mechanical Engineering
Khoy Azad University
Urmia, Western Azerbaijan, 57169-33111, Iran
Tel: 98-441-2777040, Fax: 2777022, e-Mail:
sm.pesteei@mail.urmia.ac.ir

Mehdi Abloo M. Sc.[1]
Engineering and Design Department
Iran Transfo Company
Zandjan, 45195-118, Iran
Tel: 98-241-7271001, Fax: 7290542, e-Mail:
meh_abloo@yahoo.com

ABSTRACT

In this paper, a model of a disc type winding for a transformer with directed oil flow is presented which utilizes a network of oil flow paths. Along each path segment, oil velocities and temperature rises are computed. The model includes temperature dependent oil density, resistivity, and oil viscosity and temperature and velocity dependent heat transfer and friction coefficients. Because of the non- linearity, an iterative solution is necessary. Temperature and oil velocities are computed and compared with experiment values. From this, the average winding temperature rise and the hot spot temperature can be determined.

The model assumes the oil flows in definite paths and ignores local circulation, since we assume the oil flow is guided by means of oil flow washers, this assumption should be fairly accurate. The disc type winding is assumed to be subdivided into directed oil flow cooling paths.

Keywords: oil directed, disc type winding

1. INTRODUCTION

As known, the role of the oil in oil-immersed transformers, in addition to serving as insulation, is to act as a heat-transmitting cooling medium. The heat, of which the major part is generated in the active parts of the transformer, is taken up by the oil and carried into some kind of heat exchanger. The windings of large power transformers are generally cooled by pumping oil through a network of ducts in the windings. Usually the oil enters at the bottom of the winding and exhausts at the top. This results in an overall increase in temperatures up the windings. However, the hottest conductor temperature does not occur at the top of the winding. This usually considered to be due to the combined effect of maldistribution of oil flow and losses. The useful life of a transformer is determined, in part, by the ability of the transformer to dissipate the internally generated heat to the surroundings. Knowledge of the temperature and position of the hot spot is important for the design and operation of the transformer. For example, the rate of deterioration of the winding insulation increases as the conductor temperature increases. Therefore, it is necessary to know the hottest conductor temperature in order to ensure a reasonable life for the insulation. At present, design methods for a transformer give values for the rise of average winding temperature above the average bulk oil temperature. This value is then added to the oil temperature at the top of the winding to give the conductor temperature at the top. Then an arbitrary 10% of the average winding temperature rise is added to the top conductor temperature to get an estimate of the hot spot temperature. This is the standard method for estimating this temperature [2]. The position of the hot spot temperature is not known with any accuracy. The purpose of the work reported here is to improve on the standard method for estimating the hot spot temperature and to provide an estimate of the position of the hot spot. The mathematical model which has been developed to do this also provides estimates of the temperature, oil flow and oil pressure throughout the winding. The model has been developed into a computer program called TRFLOW.

NOMENCLATURE

$a_{i,j}$, $b_{i,j}$, $c_{i,j}$, d_i: coefficients defined in the appendix
A: duct cross sectional area
A_c: conductor cross sectional area

[1] - Corresponding Author. Submitted for review

A_s: duct surface area
C_P: specific heat of fluid
D: duct equivalent diameter
f: friction factor
g: gravitational acceleration
h: vertical height
h_f: convective heat transfer coefficient
I: current
K, K', K'': pressure-loss coefficients
k: thermal conductivity of fluid
L: duct length
M: total number of nodes
$\dot{m}$: Mass flow rate
Nu: Nusselt number
P: pressure
Pr: Prandtl number
$\dot{Q}$: Power input at a source of heat
$\dot{q}$: Wall heat flux
q_c: heat generated in unit time by unit length of conductor
R: electrical resistance for unit length of conductor
Re: Reynolds number based on equivalent diameter
S: change in pressure caused by a pressure source
t: temperature
t_b: bulk temperature of fluid
t_c: conductor temperature
t_I : temperature of any fluid injected at a node
t_0: reference temperature for electrical resistance at which $R=R_0$
t_w: average wall temperature
u: fluid velocity
α: temperature coefficient of electrical resistance
$\alpha_{i,j}$: connection matrix for the flow network, its value is: (+1 if nodes i, j are connected and i>j), (-1 if nodes i, j are connected and i<j), (0 if nodes i, j are not connected)
μ_b : Dynamic viscosity of fluid at its bulk temperature
μ_w : Dynamic viscosity of fluid at the wall temperature
Z: the fraction of the normal losses due to eddy currents for a disc
X: a dependent variable

2. REPRESENTATION OF THE COOLING IN A LARGE POWER TRANSFORMER

The cooling circuit of a large power transformer with forced-oil circulation and with oil coolers is shown in Fig. 1. Piping with heated oil and piping with cooled oil connect pumps and coolers to transformer. All the pumps are arranged in parallel.

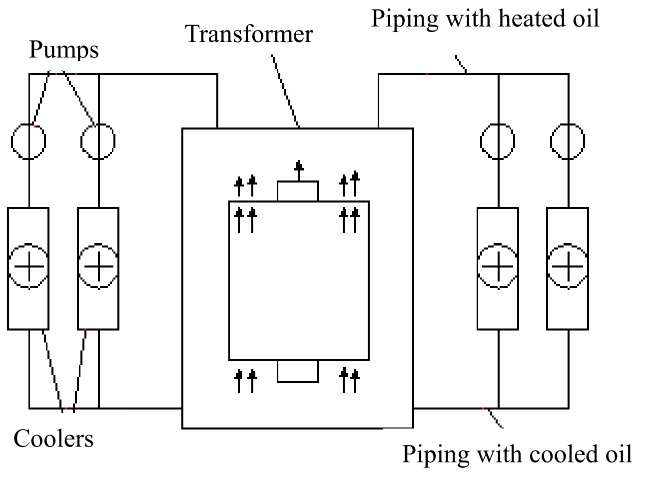

Fig.1. Schematic diagram of the cooling circuit

The case considered is a disc type of winding which is cooled by the directed forced flow of the oil. The schematic diagram of winding cooling of a transformer is shown in Fig. 2. The winding consists of an insulated conductor, which is wound spirally into discs. Each disc is separated from its neighbors so that there are gaps (ducts) between the adjacent conductor discs through which cooling oil can flow. Oil is pumped into the bottom of the transformer. From there it flows in the winding regions.

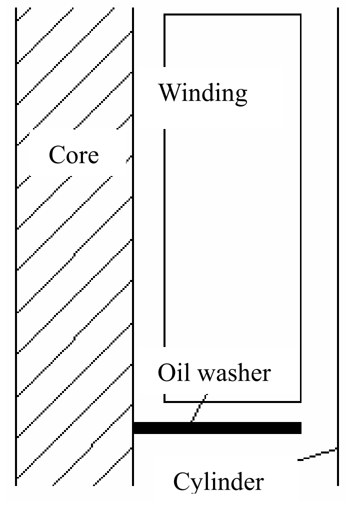

Fig. 2. Schematic diagram of a winding cooling

In every winding, there are several baffle plates, which completely block the oil duct on one side of winding on one side, see Fig. 3.

The baffle plates, which are approximately equispaced up the winding, alternate from one side of the winding to the other.

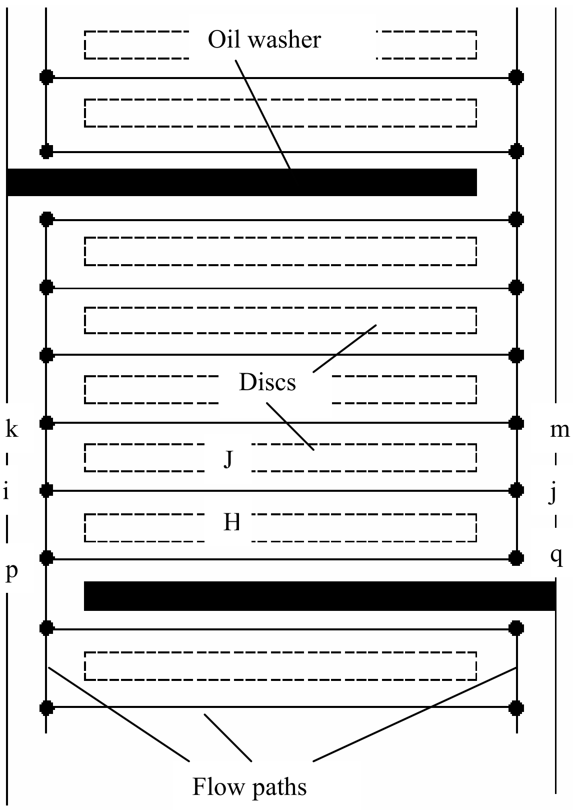

Fig.3. Flow path in the winding and network model

The group of cooling ducts between adjacent baffle plates is known as a pass. Thus the winding is made up of several similar passes in series. On reaching the top of the winding the oil passes into the header and then onto the coolers.

3. THE MATHEMATICAL MODEL

A collection of interconnecting flow paths or ducts can be represented on the network diagram of the type shown in Fig. 3. Each element of the network represents a single path that the nodes usually being placed at the junctions. In the model, values of the pressure and bulk temperature are determined for each node. Values of fluid velocity and average wall temperature are determined for each path.

The required values of pressure, bulk temperature, fluid velocity and wall temperature are obtained by solving the set of equations which can be obtained from the following:

(a) Conservation of mass applied to each node
(b) Conservation of thermal energy applied to each node
(c) Pressure-drop equation applied to each path
(d) Heat-transfer equation applied to each path

The set of equation which are obtained from (a) to (d) are given below.

Application of the conservation of mass to a node (i), see Fig. 3 gives:

$$\sum_{\substack{j=1 \\ j\neq i}}^{M} \alpha_{i,j}\rho_{i,j}A_{i,j} = \dot{m}_i \tag{1}$$

The pressure drop equation of path (i, j) joining nodes i and j can be expressed as:

$$P_i - P_j = -\alpha_{i,j}\tfrac{1}{2}\rho_{i,j}\left|u_{i,j}\right|u_{i,j} \times \left[K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right] - \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K'_{i,j}\tfrac{1}{2}\rho_{k,m}u_{k,m}^2 - \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K''_{i,j}\tfrac{1}{2}\rho_{p,q}u_{p,q}^2 - \rho_{i,j}g\left(h_i - h_j\right) + \alpha_{i,j}S_{i,j} \tag{2}$$

Where the first term represents the losses, which are related to the velocity head, for example friction; the second and third terms are the losses, which are proportional to the velocity head in another pipe, these could occur at junctions; the fourth term represents the gravitational head; the fifth term allows for any pressure sources (pumps) or sinks. The second and third terms in Eq. (2) are negligible [3, 4].

Conservation of thermal energy applied to each node gives:

$$\dot{Q}_i = \sum_j \left\{\left|\alpha_{i,j}\right|\tfrac{1}{2}\left(1 - \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}\right)\right\} \times \rho_{i,j}A_{i,j}C_p\left|u_{i,j}\right|\left(t_b\right)_i - \sum_j \left\{\left|\alpha_{i,j}\right|\tfrac{1}{2}\left(1 + \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}\right)\right\} \times \left[\rho_{i,j}A_{i,j}C_p u_{i,j}\left(t_b\right)_j + \dot{q}_{i,j}\left(A_s\right)_{i,j}\right] + \tfrac{1}{2}\left(1 - \frac{\dot{m}_i}{\left|\dot{m}_i\right|}\right)\dot{m}_i C_p\left(t_b\right)_i + \tfrac{1}{2}\left(1 + \frac{\dot{m}_i}{\left|\dot{m}_i\right|}\right) \times \dot{m}_i C_p\left(t_I\right)_i \tag{3}$$

The first term in curly brackets is +1 if the flow is from node i to node j and is zero otherwise. The second term in curly brackets is +1 if the flow is from node j to node i and is zero otherwise. The last two terms of Eq. (3) allow for the injection or extraction of mass at node i.

The equation for the transfer of heat from the wall to the fluid for the path (i, j) can be written as:

$$\dot{q}_{i,j} = Nu_{i,j}\frac{k}{D_{i,j}} \times \Big\{ (t_w)_{i,j} - \Big[\tfrac{1}{2}\Big(1 - \frac{\alpha_{i,j}u_{i,j}}{|u_{i,j}|} \Big)(t_b)_i + \tfrac{1}{2}\Big(1 + \frac{\alpha_{i,j}u_{i,j}}{|u_{i,j}|} \Big)(t_b)_j + \frac{\tfrac{1}{2}\dot{q}_{i,j}(A_s)_{i,j}}{\rho_{i,j}C_p A_{i,j}|u_{i,j}|} \Big] \Big\} \tag{4}$$

Where the term in square brackets is the mean bulk temperature of the fluid in the path (i, j).

Equations. (1) to (4) are the required equations. Before these equations can be solved information must be supplied which enables the coefficients of the equations to be derived. This information consists of: the physical properties of the fluid; the geometry of the networks; pressure loss coefficients; Nusselt numbers or heat transfer coefficients; source/sinks of pressure, heat and mass.

These equations form a complete set for the network considered. Their solution provides values of pressure, P_i, and bulk temperature $(t_b)_i$ at each node and values of velocity, $u_{i,j}$, and mean wall temperature $(t_w)_{i,j}$ for each duct. In order to solve the equations, they are rearranged to give:

(a) A set of simultaneous linear equations for pressure with variable coefficients depending on velocity (and temperature if the properties are temperature dependent)

(b) A set of simultaneous linear equations for bulk temperature with variable coefficients

(c) Nonlinear equations for velocity and wall temperature.

These equations are solved by an iterative technique.

3.1. The network model

A network such as that shown in Fig. 3 can represent the group of cooling duct, which constitute a pass.

The model of the complete winding is constructed by joining together several networks so that there is one for every pass in the winding.

It is necessary to define the winding geometry, the cooling fluid properties, the heat generation, the boundary values and the following coefficients for the ducts: friction factors, pressure-loss coefficients and Nusselt numbers. The source term, $S_{i,j}$, in the pressure equation is zero for the network considered.

3.2. Coefficient values

The Reynolds numbers for the flows in the cooling ducts of the winding will generally be considerably less than 2000, so the flow is assumed to be laminar.

For the friction factor, the solution procedure assumes:

$$f = 24\,Re^{-1}\left(\frac{\mu_w}{\mu_b}\right)^{0.58} \tag{5}$$

Rosenhow and Hartnet [5] represent Eq. (5) for a parallel plate duct, i.e. a rectangular duct with a very high aspect ratio.

For Nusselt number, the formula that is assumed:

$$Nu = 2.44\left(\frac{L/D}{Re\,Pr}\right)^{-1/3} \times \left(\frac{\mu_w}{\mu_b}\right)^{-0.14} \tag{6}$$

In which it must be that:

$$\frac{L/D}{Re\,Pr} < 0.026$$

Rosenhow and Hartnet [5] represent Eq. (6) for Nusselt number.

For transformer oil the Pr number is approximately 75. [2]

However, the value Nu required by the formula is an average value over the length L of the duct, therefore

$$Nu = \frac{\int_0^L Nu_{loc}\,dx}{L} \tag{7}$$

It is assumed that the pressure-loss due to combining and joining fluid at nodes can be neglected. Also the heat source at a node which is represented by Q was zero for all ducts.

The heat removed by the cooling oil consists of the ohmic losses of the winding conductor plus losses, which are caused by the induced eddy currents in the winding. Thus, the losses for a disc J can be expressed as: Eq. (8)

$$(\dot{q}_c)_J = R_0 I^2 (1 + \alpha\Delta T)\left(1 + \frac{Z}{100}\right) \tag{8}$$

3.3. An illustrative example

In order to illustrate the method, the following example was chosen. The high voltage winding of a 200 MVA transformer operating at 400 KV has been considered. A winding with 13 passes and 20 ducts per pass was considered. Property values for transformer oil are given in ref [2]. These values were taken as constant except for the density and viscosity. The density was calculated from the expansion coefficients and the viscosity of oil was calculated using following formula that based on a table of transformer oil viscosities verses temperature given in [6].

$$\mu = \frac{6900}{(T+50)^3} \tag{9}$$

In Eq. (9) T is in °C and μ.is in Ns/m^2.

4. RESULTS

Predictions of the oil-flow distribution in the first two passes are shown in Fig. 4. The results for the remaining passes are the same to within 1%. Predictions of mean and maximum conductor disc temperatures are shown in Fig. 5. Also shown are the predictions of oil temperature at the inlet and outlet of each pass. The variation in heat flux up the transformer winding owing to the thermal effects is not insignificant for the conditions investigated. These effects are due to the change in electrical resistance with temperature and the dependence of heat distribution from a disc on the conductor to oil temperature difference on each side of the disc. The variation was of the order of 10% through a pass owing to a combination of these effects. However, the variation up the winding for a given duct in each pass was only 4%; this explains why there is no noticeable deviation from a linear profile for the oil temperature predictions in Fig. 5. The peaks in conductor temperatures towards the center of each pass are a direct consequence of the low flows existing there.

Predictions were also obtained for constant oil properties to illustrate the importance of allowing for property variations. This resulted in the velocities of Fig. 4 being changed by a maximum of 5% and the conductor temperatures of Fig. 5 were increased. The maximum increases in temperature occurred towards the center of each pass where the mean values were increased by approximately 3°C and the maximum values by 5°C.

5. DISCUSSION

Clearly, the computer program TRFLOW that has been developed can be used to predict the conductor temperatures and the oil flow and pressure in a directed flow, forced oil-cooled transformer.

Qualitative experimental confirmation of the predicted oil and conductor temperature rises (Fig. 5) is provided by the measurements at power factory's laboratory in author's company. Also computational results were compared with the results given in ref. [7].

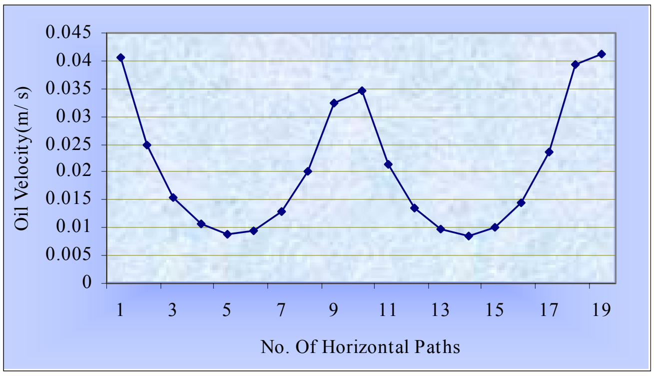

Fig. 4. Distribution of flow between cooling ducts

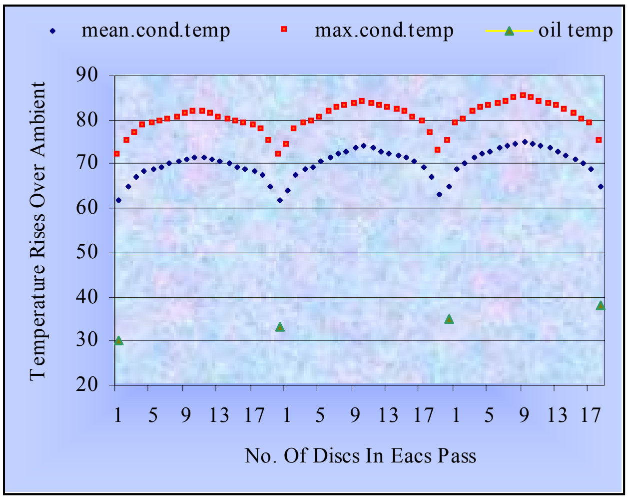

Fig. 5. Temperature rises of the conductor and the oil in the model

The predictions indicate that the hot spot for the representation used occurs in the middle-conductor disc of the top pass of the winding. It is worth noting from the predictions that the peak conductor temperature in each of the top few passes does not differ by more than a few degrees from the winding hot spot temperature.

6. CONCLUSIONS

A numerical procedure has been developed that predicts the flow and pressure of a fluid flowing in a network of interconnecting ducts. If heat is transferred to or from the fluid then the fluid and duct-wall temperatures can also be predicted.

Predictions of oil flow and temperature and conductor temperature have been obtained. This enables the winding hot-spot temperature and its position to be determined. For the particular represented transformer, the results indicate that the hot spot occurs in the top pass of the winding on the middle-conductor disc.

7. ACKNOWLEDGMENTS

This work was carried out at the engineering and design department of Iran Transfo company, also test results were obtained at the its electrical laboratory.

8. REFERENCES

[1] Godec. Z and Sarunac. R, 1992, "Steady-State Temperature Rise of ONAN/ONAF/OFAF Transformers", IEE Proceedings, Vol. 139, No. 5, PP. 448-454

[2] Karsai. K, Kerenyi. D and Kiss. L, 1987,"*Large Power Transformers*", Elsevier, Amsterdam-Oxford-NY-Tokyo, PP. 351-535, Chap. 6

[3] Pivrnec. M, Allen. P. H. G and Havlicek, 1987, "Calculation of the Forced, Directed Oil-Circulation Rate through a Transformer Cooling System", IEE Proceedings, Vol. 134, No. 4, PP. 306-312

[4] Childs. E. P, 1987, "Flow Maldistribution in Disc-Type Power Transformer Winding", ASME Proceedings, PP. 137-143

[5] Rosenhow. W. M and Hartnet. J. P, 1973, "*Handbook of Heat Transfer*", Mc Graw-Hill

[6] Kreith. F and Black. W. Z, 1980, "*Basic Heat Transfer"* Harper and Row Publishers, New York

[7] Pierce. L. W, 1992, "An Investigation of the Thermal Performance of an Oil Filled Transformer Winding", IEEE Transaction on Power Delivery, Vol. 3, PP. 1347-1357

[8] Bejan. A, Kraus. A. D, 2003,"*Heat Transfer Handbook*", John Wiley & Sons, New Jersey, Chaps. 5, 6

9. APPENDIX

The first step of solution is to derive a set of simultaneous equations for pressure. From Eq. (2):

$$u_{i,j} = \frac{\left(P_i - P_j\right)}{\left[-\alpha_{i,j}\frac{1}{2}\rho_{i,j}\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]} + \frac{\left[\alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K'_{i,j}\frac{1}{2}\rho_{k,m}u_{k,m}^2 + \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K''_{i,j}\frac{1}{2}\rho_{p,q}u_{p,q}^2\right]}{\left[-\alpha_{i,j}\frac{1}{2}\rho_{i,j}\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]} + \frac{\rho_{i,j}g\left(h_i - h_j\right) - \alpha_{i,j}S_{i,j}}{\left[-\alpha_{i,j}\frac{1}{2}\rho_{i,j}\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]} \quad (10)$$

Substituting in Eq. (2) and applying it to each node i=1…N:

$$\sum_{j=1}^{M} a_{i,j}P_j = b_i \text{.......} i = 1 \rightarrow M \quad (11)$$

Where

$$a_{i,i} = \sum_{\substack{j=i \\ j\neq i}}^{M} \alpha_{i,j}\frac{\rho_{i,j}A_{i,j}}{\left\{-\alpha_{i,j}\frac{1}{2}\rho_{i,j}\left|u_{i,j}\right|\left[K_{i,j} + 4\left(\frac{fL}{D}\right)_{i,j}\right]\right\}}$$

$$\underset{j\neq i}{a_{i,j}} = -\alpha_{i,j}\frac{\rho_{i,j}A_{i,j}}{\left\{-\alpha_{i,j}\frac{1}{2}\rho_{i,j}\left|u_{i,j}\right|\left[K_{i,j} + 4\left(\frac{fL}{D}\right)_{i,j}\right]\right\}}$$

$$b_i = -\dot{m}_i - \sum_{\substack{j=1\\ j\neq i}} \alpha_{i,j}\rho_{i,j}A_{i,j} \times \frac{\left[\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\left(K'_{i,j}\tfrac{1}{2}\rho_{k,m}u_{k,m}^2 + K''_{i,j}\tfrac{1}{2}\rho_{p,q}u_{p,q}^2\right)\right]}{\left\{-\alpha_{i,j}\tfrac{1}{2}\rho_{i,j}|u_{i,j}|\left[K_{i,j} + 4\left(\frac{fL}{D}\right)_{i,j}\right]\right\}}$$

$$- \sum_{\substack{j=1\\ j\neq i}} \alpha_{i,j}\rho_{i,j}A_{i,j} \times \frac{\rho_{i,j}\left(h_i - h_j\right) - \alpha_{i,j}S_{i,j}}{\left\{-\alpha_{i,j}\tfrac{1}{2}\rho_{i,j}|u_{i,j}|\left[K_{i,j} + 4\left(\frac{fL}{D}\right)_{i,j}\right]\right\}}$$

Eq. (3) applied to each node can be expressed as:

$$\sum_{j=1}^{M} c_{i,j}\left(t_b\right)_j = d_i\,,\ \ i = 1 \rightarrow M \qquad (12)$$

Where

$$c_{i,j} = -|\alpha_{i,j}|\ \tfrac{1}{2}\left(1+\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)\rho_{i,j}A_{i,j}C_p|u_{i,j}|$$

$$d_i = \sum_{\substack{j=\\ j\neq 1}}^{M} |\alpha_{i,j}|\tfrac{1}{2}\left(1+\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)\dot{q}_{i,j}\left(A_S\right)_{i,j} +$$

$$\dot{Q}_i + \tfrac{1}{2}\left(1+\frac{\dot{m}_i}{|\dot{m}_i|}\right)\dot{m}_iC_p\left(t_I\right)_i$$

The wall temperature equation can be obtained by rearranging Eq. (4) to give:

$$\left(t_w\right)_{i,j} = \frac{\dot{q}_{i,j}D_{i,j}}{kNu_{i,j}} + \tfrac{1}{2}\left(1-\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)\left(t_b\right)_i$$

$$+ \tfrac{1}{2}\left(1+\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)\left(t_b\right)_j + \frac{\tfrac{1}{2}\dot{q}_{i,j}\left(A_s\right)_{i,j}}{\rho_{i,j}C_PA_{i,j}|u_{i,j}|} \qquad (13)$$

The set of equations represented by Eq. (10) to Eq. (13) were solved by an iterative procedure. For the nth iteration the Steps in the solution were:

Step 1: the linear simultaneous equations

$$\sum_{j=1}^{M} a_{i,j}^{(n-1)}p_j^{(n)} = b_i^{(n-1)} \qquad i = 1 \rightarrow M$$

Were solved for $P_i^{(n)}$.

Step 2: the linear simultaneous equations

$$\sum_{j=1}^{M} c_{i,j}^{(n-1)}\left(t_b\right)_j^{(n)} = d_i^{(n-1)} \qquad i = 1 \rightarrow M$$

Were solved for $(t_b)_i^{(n)}$

Step 3: the equation for wall temperature

$$\left(t_w\right)_{i,j}^{(n)} = \left(\frac{\dot{q}_{i,j}D_{i,j}}{kNu_{i,j}}\right)^{(n-1)} + \left[\tfrac{1}{2}\left(1-\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)^{(n-1)}\right]\left(t_b\right)_i^{(n)} + \left[\tfrac{1}{2}\left(1+\alpha_{i,j}\frac{u_{i,j}}{|u_{i,j}|}\right)^{(n-1)}\right]\left(t_b\right)_j^{(n)} + \left(\frac{\tfrac{1}{2}\dot{q}_{i,j}\left(A_s\right)_{i,j}}{\rho_{i,j}C_pA_{i,j}|u_{i,j}|}\right)^{(n-1)}$$

Step 4: the equation for velocity was expressed as

$$\left(u_{i,j}\right)^{(n-1/2)} = \frac{\left(P_i - P_j\right)^{(n)}}{\left[-\alpha_{i,j}\frac{1}{2}\rho\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]}$$

$$\frac{\alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K'_{i,j}\frac{1}{2}\rho u_{k,m}^{2} + \alpha_{i,j}\frac{u_{i,j}}{\left|u_{i,j}\right|}K''_{i,j}\frac{1}{2}\rho u_{p,q}^{2}}{\left[-\alpha_{i,j}\frac{1}{2}\rho\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]^{(n-1)}} +$$

$$\frac{\rho g\left(h_i - h_j\right) - \alpha_{i,j}S_{i,j}}{\left[-\alpha_{i,j}\frac{1}{2}\rho\left|u_{i,j}\right|\left(K_{i,j} + \frac{4f_{i,j}L_{i,j}}{D_{i,j}}\right)\right]^{(n-1)}}$$

$$u_{i,j}^{(n)} = u_{i,j}^{(n-1)} + 0.4\left(u_{i,j}^{(n-1/2)} - u_{i,j}^{(n-1)}\right)$$

This denotes an iteration equation with underrelaxation. The value of the relaxation coefficient is 0.4. [8]

Step 5:

Go to step 1 and repeat steps 1 to 5 for the (n+1) th iteration. The iterative loop was repeated until the following convergence criterion was satisfied for each dependent variable:

$$\frac{X^{(n)} - X^{(n-1)}}{X^{n}} \leq 0.001 - 0.003 \qquad (15)$$

Where the value chosen depends on the particular problem.