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Groundwater Analysis and Numerical Simulation Based on Grey Theory

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Abstract: In view of the deficiency of the traditional methods, based on the grey theory, GM (1, 1) model is established to predict the groundwater level. The proposed model was applied to predict the groundwater level in Xiaonanhai Spring. The prediction result was compared with that of the traditional method and the reported results in the Xiaonanhai Spring. It is indicated that the performance of the proposed model is practically feasible in the application of prediction of groundwater level and its application is simple.

Keywords: Grey theory, groundwater level, prediction

INTRODUCTION

As everyone knows, groundwater is one of the foundations that all lives depend on for existence on the earth. Groundwater trends have the close relations with atmosphere precipitation, while there is obvious seasonality in variation of atmosphere precipitation, so the groundwater level is changed by the seasonality and periodicity. In this case, when the groundwater level is predicted, the original data of groundwater need to be dealt with in general (Yeh *et al*., 1995; Yeh and Chen, 2004). In this course, the cycle one, trend one and random one need be calculated separately after the three items isolated, we can get the final results. The traditional analysis and prediction method of groundwater is mainly the mathematics model of the determinacy and random statistical method, for example finite element, finite difference, analyze, harmony wave analysis, time series analysis, probability statistic (Qin *et al*., 2010; Chen *et al*., 2006), etc. These methods are mainly based on linear theory. Because of simplification of the models, precision is not high (Zhang *et al*., 2002; Luo *et al*., 2003). So in this study, on the basis of analyzing in depth the predicting method, adopting gray dynamic groups combined with neural network to predict the groundwater level, the better results have got.

At present, many scholars in the United States, the United Kingdom, Japan, Germany and other countries have engaged in the research and application of grey system. Grey system theory focuses on the problems which are difficult to solve using fuzzy mathematics and statistics. The characteristics of modeling features are “less data”. And the study focused on the “clear epitaxial and not clear connotation”. The main content of the grey system theory includes the thought system based on grey philosophy, theoretical system based on grey algebra system, grey equation and grey matrix.

In this study, the grey degree of groundwater is analyzed. According to the analysis, the GM (1, 1) is adapted to predict the groundwater level in Xiaonanhai Spring.

ANALYSIS OF GREY DEGREE

Grey degree: Assuming the mean value, $\bar{x}$ and variance, σ^2 of the groundwater level in every month can be found. Accorinf to the large numbers principles, when the number of the data is enough, the mean of samples $\bar{x}$ can be expressed as a normal distribution, N (μ, $\frac{\sigma}{\sqrt{n}}$) and $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ satisfies normal distribution, where μ is the mean of population. If the confidence level, (1-α), of μ is given, then the confidence interval of μ is $[\bar{x} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}]$, where $Z_{\frac{\alpha}{2}}$ is the accumulated probability corresponding to standard normal value at $\frac{\alpha}{2}$ (Yeh and Chen, 2004).

In this study, the confidence interval of groundwater level can be as the grey interval the grey degree can be defined as Eq. (1):

$$G_d = \frac{n_{out}}{n_{all}} \tag{1}$$

Basic model: The essence of GM (1, 1) (Li *et al*., 2002; Wang *et al*., 2002) is to accumulate the original data in order to obtain regular data. By setting up the differential equation model, we obtain the fitted curve in order to predict the system. Assuming the observed original water quality is as follows:

$$\left\{x^{(0)}(t) = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$$

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The data were treated with an Accumulated Generating Operation (AGO):

$$\left\{x^{(1)}(t) = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\} \qquad (2)$$

where: $x^{(1)}$ (k) = $\sum_{t=1}^{k} x^{(0)}$(i), k = 1, 2,…n; Mean value of $x^{(1)}$ x = (x (2), x (3), …x (n))
where,

$$x(t) = \frac{1}{2}(x^{(1)}(t-1)) \quad t = 2, 3, \ldots, n$$

Corresponding differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \qquad (3)$$

Utilizing least square method to solve parameters α, u. where t is the time, a and u are the parameters to be determined:

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y_N \qquad (4)$$

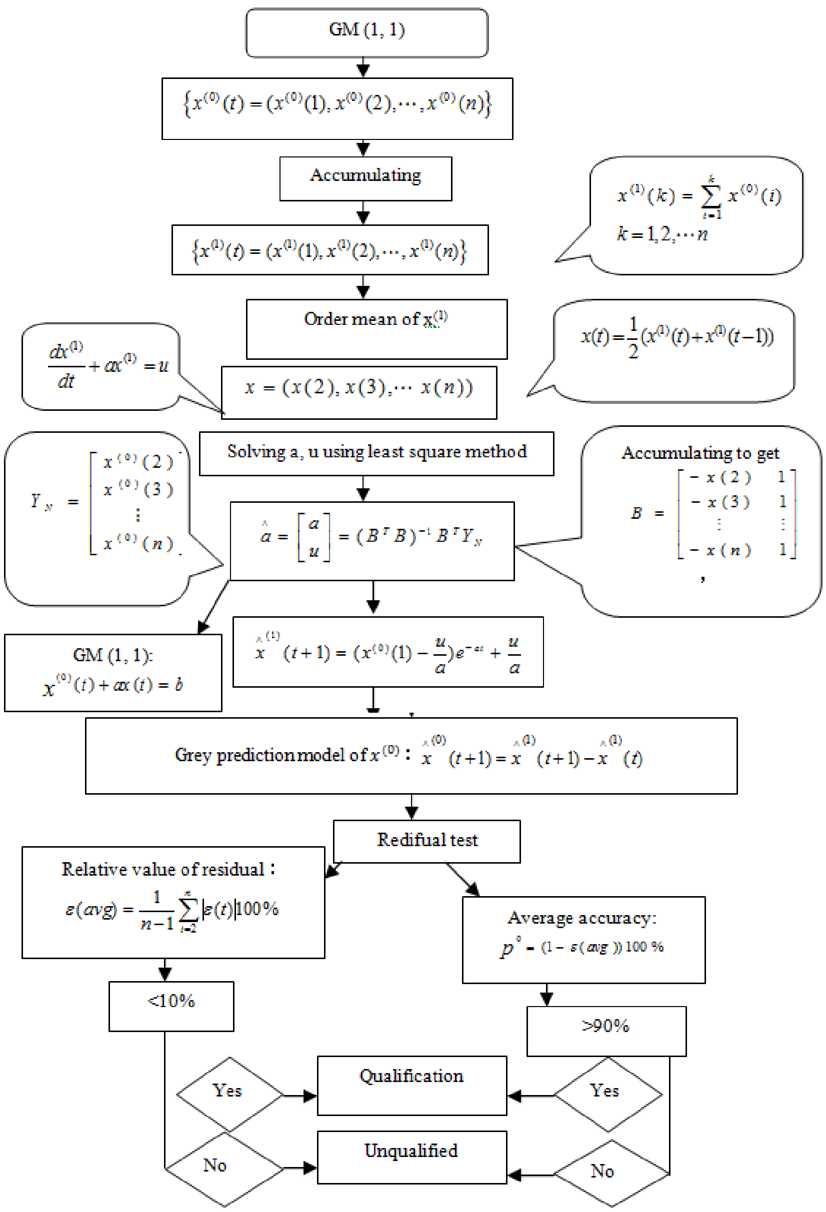

Fig. 1: Calculation chart of GM (1, 1)

where,

$$B = \begin{bmatrix} -x(2) & 1 \\ -x(3) & 1 \\ \vdots & \vdots \\ -x(n) & 1 \end{bmatrix} Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Grey predicting model of $x^{(1)}$:

$$\hat{x}^{(1)}(t+1) = (x^{(0)}(1) - \frac{u}{a})e^{-at} + \frac{u}{a} \quad (5)$$

Grey predicting model of $x^{(0)}$:

$$\hat{x}^{(0)}(t+1) = (1 - e^{a})(x^{(0)}(1) - \frac{u}{a})e^{-at} \quad (6)$$

In order to differentiate models good or bad, the methods are adopted as follows:

$$e_k = x^{(0)}(k) - \hat{x}^{(0)}(k) \;\; k = 1, 2, \cdots, n)$$

$$\bar{e} = \frac{1}{n}\sum_{k=1}^{n} e_k \quad \bar{x}^{(0)} = \frac{1}{n}\sum_{t=1}^{n} x^{(0)}(t)$$

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n}[x^{(0)}(k) - \bar{x}]^2 \quad S_2^2 = \frac{1}{n}\sum_{k=1}^{n}[e(k) - \bar{e}]^2$$

while c $= \frac{S_2}{S_1}$<0.35, P $= \{|e_k - \bar{e}| < 06745 S_1\}$ >0.95, the precision of the models is very accurate indeed:

$$\Delta(t) = x^{(0)}(t) - \hat{x}^{(0)}(t) \quad (7)$$

$$\varepsilon(t) = \frac{x^{(0)}(t) - \hat{x}^{(0)}(t)}{\hat{x}^{(0)}(t)} \% \quad (8)$$

$$\varepsilon(avg) = \frac{1}{n-1}\sum_{t=2}^{n}|\varepsilon(t)| 100\% \quad (9)$$

$$p^{0} = (1 - \varepsilon(avg)) 100 \;\% \quad (10)$$

After testing, ε (avg) <10%, p^0>90%, the prediction model can be recognized as eligilibility.

In the case of relatively small fluctuations in the raw data series, the prediction accuracy is high using grey forecasting model. The method is simple, so the model can be practical. But if the original data sequence has large fluctuations, the model should be optimized using three point smoothing or the residual model of predicted value and the actual value to obtain a better prediction. In GM (1, 1) prediction model, parameters a, b are fixed once determined, regardless of the numbers of values, parameters will not change with time. The feature limits the GM (1, 1) is only suitable for short-term forecasts because a lot of factors will go into the system with the development of system with time. The accuracy of prediction model will become increasing weak with the time away from the origin, the predictable significance will diminish. The process of GM (1, 1) can be seen in Fig. 1.

Analysis of grey degree: Spring area of small south China sea is located in linzhou city and anyang county in Henan province. River water system belongs to Wei river water system in haihe basin, the main river is Huan River and there are a Xiaonanhai revervior and "artificial Tianhe" -red flag canal. The vents is in Huan river valley, the exposed elevation is between 131-135 m. Overall look in spring, western mountains and Lin basin aer recharge are and the central low mountains is runoff area and eastern Xinan sea is discharge area, whose total area is 934.6 Km^2. karst groundwater is buried deeply, the minimum depth is more than 10m, the influence of evaporation of groundwater on level is very small and the type of karst water discharge is mainly water drainage, mine drainage and artificial mining. The precipitation and evaporation measured in 2003, 2004, 2005 at 1# station is shown in Fig. 2.

Spring is located in transition between taihang uplift and north China plain settlement, whose west is Linzhou fault rupture, east to Tangxi fault rupture.

Seen from the figure, the rainfall in xiaonanhai spring is relatively abundant during June and September and evaporation in this period, is also the peak for the whole year.

Precipitation and evaporation can affect the groundwater level directly or indirectly. Infiltration of rainfall is the important recharge source of groundwater. The relationship between groundwater level and net precipitation (precipitation-evaporation) can be shown in Fig. 2.

From Fig. 2, net precipitation is consistent with groundwater level. When net precipitation is high, groundwater level is high. Recharge of precipitation on groundwater has synchronization with three months lag.

If grey degree is too large, that is to say that uncertainty is too much, the gray study has little meaning. Therefore, to study the groundwater level in xiaonanhai Spring, grey degree firstly studied.

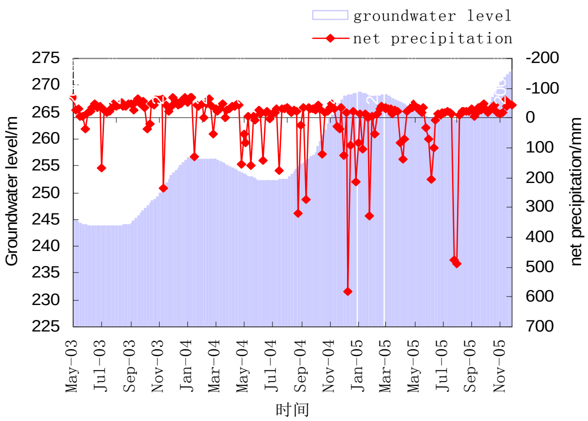

Fig. 2: Groundwater level in No. 1 well

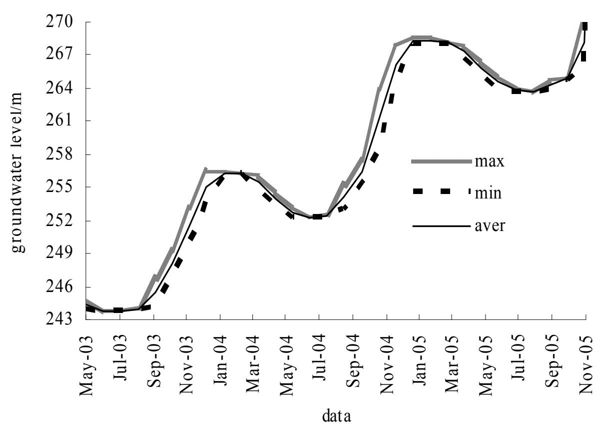

Fig. 3: Maximum, minimum, average groundwater level (1#)

The measured data is from 2003 to 2005 in 10 wells. There are 5 sets data measured monthly. No. 1 well is for the example which can be seen in figure.

From Fig. 3, the value is between 243 and 270 m. the groundwater level is lower in 2003, with the implementation of improvement measures, the groundwater level has increased.

In mathematical statistics, the confidence degree is refers to the probability of true value appears in a certain range. The confidence interval refers to the range of true value in a certain confidence degree. Confidence degree is from 0.95, 0.9, 0.8, 0.7 until to 0.1, each a different confidence level corresponds to a different confidence interval. According to the Eq. (2) to (1), the ration of the number of outside the confidence interval with the sample size is called grey value. If grey degree is 1, the systems become black system, indicating that the information has been completely unable to determine the true or false. If grey degree is zero, the system is a white system, indicating that the information is completely known. The specific calculation in May 2003, for the example as shown in Table 1

Table 1: No. 1 well in May 2003 confidence interval gray table

Confidence	Upper limitation	Lower limitation	Grey degree
0.95	244.812	244.0340	0.3333
0.90	244.728	244.1180	0.3333
0.80	244.646	244.1999	0.6667
0.70	244.658	244.1880	0.6667
0.60	244.562	244.2840	0.6667
0.50	244.533	244.3130	0.6667
0.40	244.508	244.3380	1
0.30	244.485	244.3610	1
0.20	244.463	244.3830	1
0.10	244.443	244.4030	1

Table 2: Variance and confidence table

Var	Confidence level
0.0000111≤Var≤0.00104	0.95
0.000104≤Var≤0.0108	0.90
0.0108≤Var≤0.107	0.80
0.107≤Var≤1.05	0.70
1.05≤Var≤68.6	0.60
68.6≤Var≤166	0.50

Table 3: Confidence and grey degree

Confidence level	Upper confidence level	Lower confidence level	Grey degree
0.95	244.812	244.034	0.3333
0.90	244.728	244.118	0.3333
0.80	244.646	244.199	0.6667
0.70	244.658	244.188	0.6667
0.60	244.562	244.284	0.6667
0.50	244.533	244.313	0.6667
0.40	244.508	244.338	1
0.30	244.485	244.361	1
0.20	244.463	244.383	1
0.10	244.443	244.403	1

Higher the confidence degree is, greater the confidence interval is, smaller the grey degree is. When confidence level is larger than 0.9, grey degree is 0.3333 which is relatively small. As confidence interval is reduced, confidence degree is lower and grey degree increased. When confidence drops below 0.4, grey degree is 1. This requires to select an appropriate confidence level to make grey degree and confidence interval are within a reasonable range.

Variance is to describe stability and volatility of random variable and centralized and decentralized. The variance is large, it indicates that the random variable is discrete. According to the variance of observed data monthly, confidence degree can be determine shown in Table 2. According to the var in Table 2, confidence level is determined to get confidence interval. Monthly groundwater level observed falls into the confidence interval, therefore, the confidence interval is relatively reliable. Grey degree corresponding to the confidence degree can be seen in Table 3.

APPLICATION OF GM (1, 1)

Groundwater in Xiaonanhai Spring can be forecasted using GM (1, 1). Mean value monthly are as the original sequence, each four data are as a group.
To the original sequence:

$$x^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(4)$$

$$= (244.423, 243.729, 243.742, 243.929)$$

Once accumulated 1-AGO:

$$x^{(1)}(k) = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(4)$$

$$x^{(1)}(k) = \sum_{1}^{k} x^{(0)}(i),\ i = 1,2,3,4$$

$$x^{(1)}(k) = (244.423, 488.152, 731.894, 975.823)$$

Mean value of $x^{(1)}$ one order $z^{(1)}$:

$$z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1))$$

$$z^{(1)} = (366.288, 610.023, 853.859)$$

$$C = \sum_{k=2}^{4} z^{(1)}(k) = 1830.17$$

$$D = \sum_{k=2}^{4} x^{(0)}(k) = 731.4$$

$$E = \sum_{k=2}^{4} z^{(1)}(k) x^{(0)}(k) = 446244.19$$

$$F = \sum_{k=2}^{4} (z^{(1)}(k))^2 = 1235370.1$$

$$\Delta_a = CD - (n-1)E = -146.2$$

$$\Delta = (n-1)F - C^2 = 356588.1$$

$$\Delta_b = DF - CE = 86846961.888$$

$$a = \frac{\Delta_a}{\Delta} = -0.0004 \qquad b = \frac{\Delta_b}{\Delta} = 243.55$$

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}$$

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

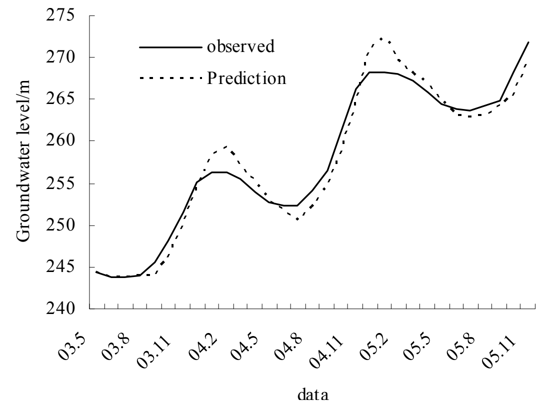

Fig. 4: Observation and prediction results

$$\hat{x}^{(0)}(5) = \hat{x}^{(1)}(12) - \hat{x}^{(1)}(4) = 244.0009$$

The results are the monthly value in Septembers, 2003. Then the data from June to September 2003, are as one group, which can be obtained the value in October, 2003. As shown in Fig. 4, the prediction value and obtained value are consistent, indicating the ideal fitting.

For the discriminative model, residual test can be used:

$$\Delta(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$$

where, Δ (k) is residual value:

$$\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{\hat{x}^{(0)}(k)}\%$$

ε (k) is relative value of residual ε (avg) = $\frac{1}{n-1}\sum_{k=2}^{n}|\varepsilon(k)|$100%, n = 4; = 0.6671% ε (avg) is average error p^0 = (1- ε (avg)) 100% = 9903329% ε (avg) <10% p^0>90%, So, the model is reasonable.

So, it is feasible to predict the groundwater in Xiaonanhai Spring using GM (1, 1) model. The results are reliable and feasible.

CONCLUSION

Application of grey theory to prediction of groundwater level is a novel research area. Based on the modeling results obtained in this study, the following conclusions can be drawn:

- The model is proposed by virtue of the dynamic characteristics of groundwater level, which increased the forecast precision. Therefore, the method is reliable and effective.

- This method can be used to deal with the nonlinear and periodical issues to improve the precision of the groundwater level prediction.
- This method is not only suitable for the dynamic prediction of the groundwater level, but also suitable for other respects (such as surface water quality, prediction of quality in the atmospheric environment and so on).

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