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ONE-DIMENSIONAL CUTTING STOCK PROBLEM (1D-CSP): A STUDY ON DATA DEPENDENT TRIM LOSS

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ABSTRACT

A generalized approach is introduced for getting minimal trim in one- dimensional cutting stock problem (1D-CSP) which occurs extensively while manufacturing of different engineering objects. Our study is especially focused on transmission tower manufacturing industry. The concept of computing total trim by using the idea of pre-defined sustainable trim has been explored in previous work of authors. Given m stock lengths $U_1, U_2, \dots, U_m$, the cutting plan consists of cutting of at most two order lengths at a time out of the demanded set of n order lengths $l_1, l_2, \dots, l_n$.

Considering the given data, the total trim has been computed corresponding to two different sustainable trims of order one (viz.t_s^1) and order two (viz.t_s^2). Introducing various values of sustainable trims as knots between t_s^1 and t_s^2, the total trim has been computed corresponding to each knot, the linear approximation has been constructed which predict the total trim loss at any arbitrary point t lying between t_s^1 and t_s^2.

KEYWORDS: First order sustainable trim, Second order sustainable trim, 1D-CSP, Non-negative integral valued (NIV) linear combination, Hat function, Linear approximation.

AMS (2000) SUBJECT CLASSIFICATION: 90C90; 90C27; 90C10.

1. INTRODUCTION

Various sizes of items are ordered in industries dealing with processing of materials such as steel, film, textiles etc. The ordered items are cut from bars, plates or sheets of raw materials that have a fixed size (cf. [10], [14], [15], [16]). During the process of cutting of these items, the problem for trying to minimize the waste of raw material is referred to as a cutting stock problem (CSP). The CSP plays an important role in the economic status of the processing industries and it has been entered into several fields of research like mathematical modeling, computer programming, ocean engineering, mechanical engineering etc. (cf. [6], [11], [12], [16]). In the clothing industry, the first phase of the stock cutting process is one-dimensional cutting of a smaller number of long pieces in the form of fabric rolls into a large number of short pieces named pattern shapes (cf. [10]).

The analytic approaches of optimization have been proposed by several researchers (see [1], [2], [4], [5], [7], [8], [9], [12], [13], [17], [18]), but it has been noticed that from implementation point of view, these methods have been discarded by almost all industries due to their complexity. The analytic approach consists of following consecutive steps:

- Mathematical modeling
- Algorithm
- Cutting plan
- Computation of trim loss

These steps have been proposed and verified theoretically and then the relevant industries are supposed to adopt the method practically. But, it has been observed that there is a big-gap between analytic solution and its practical implementation. Our work in this paper is mainly focused on the transmission tower manufacturing industry and in accordance with the practical approach, mathematical model has been designed. Powar et al in [12] (cf. [11] also) have proposed the more functional cutting plan which can be conveniently acceptable by any industry dealing with 1D-CSP.

The cutting plan described in [12] (see also [11]) consists of cutting of at most two order lengths at a time out of the required n order lengths $l_1, l_2, \ldots l_n$, in from a given set of m stock lengths $U_1, U_2, \ldots U_m$. At each stage, the cutting of at most two order lengths at a time will be continued till the required number of pieces are cut totally. The concept of sustainable trim of order one t_s^1 and of order two t_s^2 has been introduced in [12] and [11] respectively. This idea of pre-defined sustainable trims plays a crucial role in controlling the total trim.

Spline functions (piece-wise polynomial functions) are well-known best approximating functions which are extensively use in all branches of science and technology. For our analysis, we have considered a clan of data and computed the weighted average of order lengths. Correspond to each weighted average in the domain, the trim losses with respect to t_s^1 and t_s^2 have been computed. Using the basis functions (Hat functions or Chapeau functions) for linear approximation, linear splines (cf. [3]) are generated given any arbitrary set of data and the value of approximate trim loss may be predicted immediately in accordance with our proposed cutting plan.

2. PRE-REQUISITES

All stock lengths and order lengths, we consider as integers throughout our analysis. According to the requirement, the lengths can be converted into integers by multiplying them with $10^n (n \geq 1, \text{integer})$. We use the following notations:

$B(n) -$ Block of integers $0, 1, \ldots, n$ (index set), $j \in B(n)$ means j can be any number from the set $\{0, 1, 2, \ldots, n\}$.

$l_i -$ Order lengths $i = 0, 1, 2, \ldots, n$ arranged in ascending order with respect to length and $l_0 = 0$ by convention.

$d_i -$ Required number of pieces of order length l_i, $d_0 = 0$.

$U_j -$ Stock length $(j = 1, 2, \ldots, m)$ arranged in ascending order with respect to length.

It has been noticed that in particular, in the transmission tower designing industry that most of the required number of order lengths i.e. d_i's are integral multiple of each other. In view of this observation, we classify the order lengths in the following two categories in accordance with their required number of pieces:

Category I: (C-I) We collect all those order lengths whose required number of pieces are integral multiple of each others.

Category II: (C-II) It is the collection of all those order lengths whose required number of pieces are prime numbers (their common multiple is 1).

3. SUSTAINABLE TRIMS

3.1 Order One

In order to cut the linear combination a_{ij} (say) of the two order lengths l_i and l_j from the given stock lengths $U_1, U_2, \dots, U_m$, we have to decide upto what extent, we allow the raw material to convert into the scrape. Throughout our cutting process (excluding the last step where it is possible only that few piece of some order length are left to cut), we follow the restriction that $0 \leq U_k - a_{ij} \leq t_s^1, \quad k = 1,2, \dots, m$ and t_s^1 is the sustainable trim of order one and defined as follows:

$$L = \frac{l_0 d_0 + l_1 d_1 + \cdots + l_n d_n}{d_0 + d_1 + \cdots + d_n} = \frac{\sum_{i=0}^{n} l_i d_i}{\sum_{j=0}^{n} d_j}$$

We next define $L_k = |U_k - iL|$ $\quad (k = 1, 2, \dots, m$ and i is an appropriate positive integer ≥ 1, for which L_k is minimum)

where $U_1, U_2, \dots, U_m$ are the stock lengths. We finally define

$$t_s^1 = \frac{\sum_{k=1}^{m} L_k}{m} \tag{3.1}$$

which is the desired sustainable trim of order one.

3.1 Remark

Analytically, it has been noticed that the average value covers the acceptable, over all original values. Hence, we have taken the weighted mean of total required lengths.

3.2 Order Two

Following the same restriction as for t_s^1 and using the notations from section 2, we define

$$S_n = \frac{\mathrm{l_0 d_0} + \cdots + \mathrm{l_n d_n}}{\mathrm{d_0} + \mathrm{d_1} + \cdots + \mathrm{d_n}} = \frac{\sum_{i=0}^{n} l_i d_i}{\sum_{j=0}^{n} d_j}$$

By convention, $S_0 = 0, \; S_1 = l_1, \quad S_2 = \frac{l_0 d_0 + l_1 d_1 + l_2 d_2}{d_0 + d_1 + d_2}, \dots, S_n = \frac{\sum_{i=0}^{n} l_i d_i}{\sum_{j=0}^{n} d_j}$

We next define the second order weighted means

$$S_n^1 = \frac{S_0 + S_1 + \cdots + S_n}{n+1}.$$

Consider $\quad L_k = |U_k - i\, S_n^1|, \; k = 1,2, \dots, m.$

i is an appropriate positive integers ≥ 1, for which L_k is minimum. $U_1, U_2, \dots, U_m$ are stock lengths.

3.2 Remark

S_n^1 is the average order length which is assumed to be cut from the given stock length $U_j (j = 1, \dots, m)$. The integer i denotes the number of pieces of average stock length to be cut from the stock length U_k. We finally define

$$t_s^2 = \frac{\sum_{k=1}^{n} L_k}{m} \tag{3.2}$$

which is the sustainable trim of order two viz. t_s^2.

4. MATHEMATICAL MODELING

We first consider C-I and define the following ratios:

$$\frac{d_i}{d_j} = \frac{\alpha_i \times r_{ij}}{\beta_j \times r_{ij}} \tag{4.1}$$

(where r_{ij} is a positive integer , $j \in B(n), i > j$)

Note: It is not necessary to consider always the largest common factor between d_i and d_j. Any other factor r_i (if exists) may be selected according to the length of stock to minimize the trim.

In view of (4.1), define the following set:

$$A = \{a_{ij} = \alpha_i l_i + \beta_j l_j : a_{ij} \le U_m,\ \alpha_i,\ \beta_j \ge 0, \text{integer } \big(i < j, i, j \in B(n)\big)\} \tag{4.2}$$

We are now in a position to define the sets $A_k \subseteq A$ $(k = 1, 2, \dots, m)$ as follows:

$$A_k = \{a_{ij}: 0 \le U_k - a_{ij} \le t_s^\lambda,\ k \in B(m),\ i, j \in B(n),\ a_{ij} \in A, \lambda = 1,2\} \tag{4.3}$$

where t_s^λ is defined by (3.1) and (3.2) respectively for $\lambda = 1$ and $\lambda = 2$.

At this stage, we may come across with the following situations:

- $A_k = \phi,\ \forall\ k \in B(m)$, in this case, all the order lengths have to shift in C-II.
- In view of the definition of t_s^λ, the sets A_k $(k \in B(m))$ may or may not cover all order lengths belonging to Category-I.

In view of above observations and the definition of the sets A_k, we redefine our categories I and II as follows:

Category-I (C-I) Let l_{α_1}, $l_{\alpha_2}, \dots, l_{\alpha_p}$ order lengths have been covered by the sets A_k $(k = 1, 2, .., m)$. For convenience, we denote these order lengths by $l_1, l_2, \dots, l_p$ arranged in ascending order with respect to the length.

4.1 Remark

There may exist some order lengths l_i and l_j (say) such that d_i and d_j ofcourse are multiple of each others but the length of combination a_{ij} exceeds the largest stock length U_m or $(U_j - a_{ij})$ exceeds the sustainable trim loss. We shift all such order lengths to Category-II and finally, we assume that the order lengths $l_1, l_2, \dots, l_p$ have been covered by Category-I.

Category-II (C-II) The remaining all order lengths $N = n - p$ denoted by $l_1, l_2, \dots, l_N$ arranged in ascending order with respect to the length.

4.2 Remark

(i) The real numbers t_s^{λ} $(\lambda = 1, 2)$ defined by (3.1) and (3.2) play a crucial role in the computation of total trim loss. It is natural to expect that the trim loss can be minimized by considering the minimum value lying between 0 and t_s^{λ}, but it has been experienced practically in the industries that by increasing the value of t_s^{λ}, the impact on the total trim loss results in a significantly acceptable range in some particular cases. But we are strict to t_s^{λ} only.

(ii) In order to implement the algorithm smoothly, the data of more than one tower (preferably of same pattern) may be clubbed.

Now consider Category-II and order lengths $l_1, l_2, \dots, l_N$ with the required number of pieces $d_1, d_2, \dots, d_N$ respectively. For $i \neq j$, $i, j \in B(N)$, define:

$$d_i = n_1 \alpha_{i1} + d_{i1} \tag{4.4}$$

$$d_j = n_1 \beta_{j1} + d_{j1} \tag{4.5}$$

$$0 \leq U_k - \left(l_i \alpha_{i1} + l_j \beta_{j1}\right) = (w_k \text{ say}) \leq t_s^{\lambda} \;\; (\lambda = 1, 2)$$

for at least one value of k (k=1,2,…,m). The number n_1 has been chosen in such a way that w_k attains a minimum value lying between 0 and t_s^{λ}.

Similarly, choose a number n_2 satisfying the following condition:

$$d_{i1} = n_2 \alpha_{i2} + d_{i2} \tag{4.6}$$

$$d_{j1} = n_2 \beta_{j2} + d_{j2} \tag{4.7}$$

Proceeding this way, we finally define

$$d_{i,s-1} = n_s \alpha_{is} + d_{is} \tag{4.8}$$

$$d_{j,s-1} = n_s \beta_{js} + d_{js} \tag{4.9}$$

The process would be continued till either $d_{is} = 0$ or $d_{js} = 0$ and in view of (4.4) - (4.9), we have

$$d_i = \sum_{k=1}^{s} n_k \alpha_{ik} + d_{is} \qquad \alpha_{ik} > \alpha_{i,k+1} \tag{4.10}$$

$$d_j = \sum_{k=1}^{s} n_k \beta_{jk} + d_{js} \qquad \beta_{jk} > \beta_{i,k+1} \tag{4.11}$$

$$d_{us} = n_{s+1} \delta_{u,s+1} + d_{u,s+1} \qquad (d_{u,s+1} < \delta_{u,s+1}) \tag{4.12}$$

where $u = i$ or j, $\delta = \alpha$ or β for i or j respectively. Also $n_k, \alpha_{ik}, \beta_{jk}$ are positive integers, may be selected according to the length of stock in order to minimize the trim.

Referring relation (4.10)-(4.12), we now define the set

$$B = \{b_{ij}^{r}, b_u^{s+1}, b_u^{s+2} : b_{ij}^{r} = \alpha_{ir} l_i + \beta_{jr} l_j, b_u^{s+1} = \delta_{u,s+1} l_u, b_u^{s+2} = d_{u,s+1} l_u, \text{where } b_{ij}^{r}, b_u^{s+1}, b_u^{s+2} \leq U_m, u = i \text{ or } j, \delta = \alpha \text{ or } \beta \text{ according as } u = i \text{ or } j \text{ repectively}, r = 1, 2, \dots, s.\; i, j = 0, 1, \dots, N\} \tag{4.13}$$

Define $|c_i| = \max c_{ij}$, where $c = a$ or b for fixed i and arbitrary j

$$c_{ij} \leq U_m \tag{4.14}$$

In view of relation (4.13), we now define

$$B_k^r = \{b_{ij}^r, b_u^{s+1}, b_u^{s+2}: 0 < \left|U_k - \left(b_{ij}^r | b_u^{s+1} | b_u^{s+2}\right)\right| < t_s^\lambda \tag{4.15}$$

$$(\lambda = 1, 2), r = 1, 2, \ldots, s$$

5. CUTTING PLAN

It has been noticed practically that with the preference of starting from the largest order lengths to the smaller ones, the cutting process has been executed in general as the smaller order lengths left behind can be adjusted easily amongst them and results in less trim loss (see Figure 8.1).

- **Cutting of the largest order length l_p from category-I**

Referring relation (4.14), we consider $|a_p|$. In view of A_k [cf. relation (4.10)], there exist sets $A_q, A_r, A_s, \ldots$ $(1 \leq q, r, s, \ldots \leq m)$ containing $|a_p|$ along with some other a_{ij}'s. Corresponding to each set $A_q, A_r, A_s, \ldots$ respective fixed stock lengths $U_q, U_r, U_s, \ldots$ have been assigned. We select the combination a_{pq} corresponding to the smallest stock length U_p and focus our attention on it for the first step of cutting.

Let $\left|a_p\right| = a_{pq}$(say) for $q \in B(p)_{\sim p}$ where $a_{pq} = \alpha_p l_p + \beta_q l_q$ satisfying the condition:

$$\frac{d_p}{d_q} = \frac{\alpha_p \times r_{pq}}{\beta_q \times r_{pq}} \tag{5.1}$$

In view of (5.1), it may be noted that by cutting r_{pq} bars of stock length U_p, total number of required pieces of order lengths l_p and l_q are cut.

Define

$$t_p^1 = r_{pq}\left(U_p - \left|a_p\right|\right) \leq r_{pq} t_s^\lambda \ \ (\lambda = 1, 2) \tag{5.2}$$

- **Cutting of other stock lengths from the set A_r**

For $i, j \neq p, q$, we next consider the largest order length l_u (say) contained in A_r and consider $|a_u|$ for $a_{uv} \in A_k$ corresponding to the stock length U_u satisfying the condition:

$$\frac{d_u}{d_v} = \frac{\alpha_u \times r_{uv}}{\beta_v \times r_{sv}} \qquad \text{for some } v \in B(p)_{\sim\{p,q,u\}}$$

Referring relation (5.1), it is clear that by cutting r_{uv} bars of the stock length U_u, total number of required pieces of order lengths l_u and l_v have been cut. Define

$$t_u^1 = r_u \left(U_u - |a_u|\right) \leq r_u t_s^\lambda \ \ (\lambda = 1, 2) \tag{5.3}$$

Proceeding this way, for $i, j \neq p, q, u, v$, we consider the next largest order length out of the remaining once and applying the same technique as before, the trim loss with respect to corresponding stock lengths U_j's has been computed. The process is continued till all order lengths belonging to category I are totally exhausted.

$$T_1 = \sum_{p \in B(n)} t_p^1 \tag{5.4}$$

If this cutting process covers all the order lengths $l_1, l_2, \ldots, l_n$, then STOP.

- **Cutting of the largest order length l_N from Category-II**

Referring definition of B_k^r [cf. relation (4.15)], we first set $r = 1$ and consider ${b_N}_j^1$ for fixed N and arbitrary j and select ${b_N}_j^1$ as follows:

$$|b_N| = \max_{j \in B(n)_{\sim N}} {b_N}_j^1$$

Such that $|b_N| \leq U_j$ for some $j \in B(m)$.

Now, corresponding to $|b_N|$, there exists sets $B_q^1, B_r^1, B_s^1, \ldots$ associated with the stock lengths $U_q, U_r, U_s, \ldots$ respectively containing $|b_N|$. We select the set B_q^1 corresponding to the smallest stock length U_q. In view of the relation (4.15), we have

$${b_N}_j^1 = \alpha_{N1} l_N + \beta_{j1} l_j \quad \text{for } j \in B(N)_{\sim N}.$$

It is clear from relations (4.10) and (4.11), that by cutting n_1 bars of stock length U_q, we cut $n_1 . \alpha_{N1}$ pieces of order length l_N and $n_1 . \beta_{j1}$ pieces of order length l_j. Our aim is to finish cutting of only two order lengths first l_N and l_j (fixed) at a time. Following cases may arise:

Case1. Either $d_N - n_1 . \alpha_{N1} < \alpha_{N1}$ or $d_j - n_1 \beta_{j1} < \beta_{j1}$ or both the inequalities hold together.

Case 2. Either $d_N = n_1 . \alpha_{N1}$ or $d_j = n_1 \beta_{j1}$.

5.1 Remark

Here two cases will not hold together because in that case l_N and l_j will belong to Category-I.

We first deal with the case 1. In view of the relation (4.13), we next consider

$$b_{N_j}^2 = \alpha_{N2} l_N + \beta_{j2} l_2 \quad (j \text{ fixed as given by (4.13)})$$

Now, corresponding to $b_{N_j}^2$, there exist sets $B_q^2, B_r^2, B_s^2, \ldots$ containing it. The sets $B_q^2, B_r^2, B_s^2, \ldots$ are associated with the stock length $U_q, U_r, U_s, \ldots$ respectively. We select the set B_r^2(say) corresponding to the smallest stock length U_r. It is clear from relations (4.10) and (4.11) that by cutting n_2 bars of stock length U_r, we cut $n_2 . \alpha_{N2}$ more pieces of order length l_N and $n_2 . \beta_{j2}$ more pieces of order length l_j. We continue the process till either $d_N = \sum_{k=1}^{s} n_k \alpha_{ik}$ or $d_j = \sum_{k=1}^{s} n_k \beta_{jk}$.

Let if possible $d_N = \sum_{k=1}^{s} n_k \alpha_{ik}$ holds, then d_j would be of the form

$$d_j = \sum_{k=1}^{s} n_k \beta_{jk} + d_{js}$$

where we express $d_{js} = n_{s+1} \beta_{j,s+1} + d_{j,s+1}$ $(d_{j,s+1} < \beta_{j,s+1})$.

Referring the relation (4.13), we now consider

$$b_j^{s+1} = \beta_{j,s+1} l_j$$

Now, corresponding to b_j^{s+1}, there exists sets $B_q^{s+1}, B_r^{s+1}, B_s^{s+1}, \ldots$ containing it. The sets $B_q^{s+1}, B_r^{s+1}, B_s^{s+1}, \ldots$ associated with the stock lengths $U_q, U_r, U_s, \ldots$ respectively. We select the set B_s^{s+1} corresponding to the smallest stock length U_s(say). It is clear from relations (4.10) and (4.11) that by cutting of stock length U_s, we cut $n_{s+1}.d_{j,s+1}$ pieces of order length l_j. Now $d_{j,s+1}$ pieces of order length l_j are left to cut out of d_j. We now consider $U_k - d_{j,s+1} l_j$ for all $k = 1, 2, \ldots, m$ and select the minimum difference corresponding to the stock length U_t(say), all pieces of order length l_j have been cut.

5.2 Remark

At this last step of cutting $U_t - d_{j,s+1}.l_j$ may exceed the sustainable trim t_s.

We now compute the trim loss corresponding to the order lengths l_N and l_j belonging to the Category-II.

$$t_{r_1}^1 = \left(U_q - b_{N_j}^1\right) n_1 + \left(U_r - b_{N_j}^2\right) n_2 + \cdots \qquad = \sum_{r=1}^{s} \left(U_l - b_{N_j}^r\right) n_r + (U_t - d_{j,s+1} l_j)$$

Order lengths l_N and l_j belonging to category-II have been cut completely. Remaining order lengths we again arrange in increasing order $l_1, l_2, \ldots, l_M$ (say). We first consider

$$|b_M| = \max_{j \in B(M)_{\sim M}} b_{M_j}^1 \qquad j \in B(M)_{\sim M}$$

such that $|b_M| \leq U_j$ for some $j \in B(m)$.

Proceeding in a similar manner, we get

$$t'_{r_2} = \sum_{r=1}^{t} \left(U_l - b_{M_j}^r\right) n_r + \left(U_t - d_{j,s+1} l_j\right) \qquad l \in B(m)$$

We continue the process till all order lengths are exhausted and get

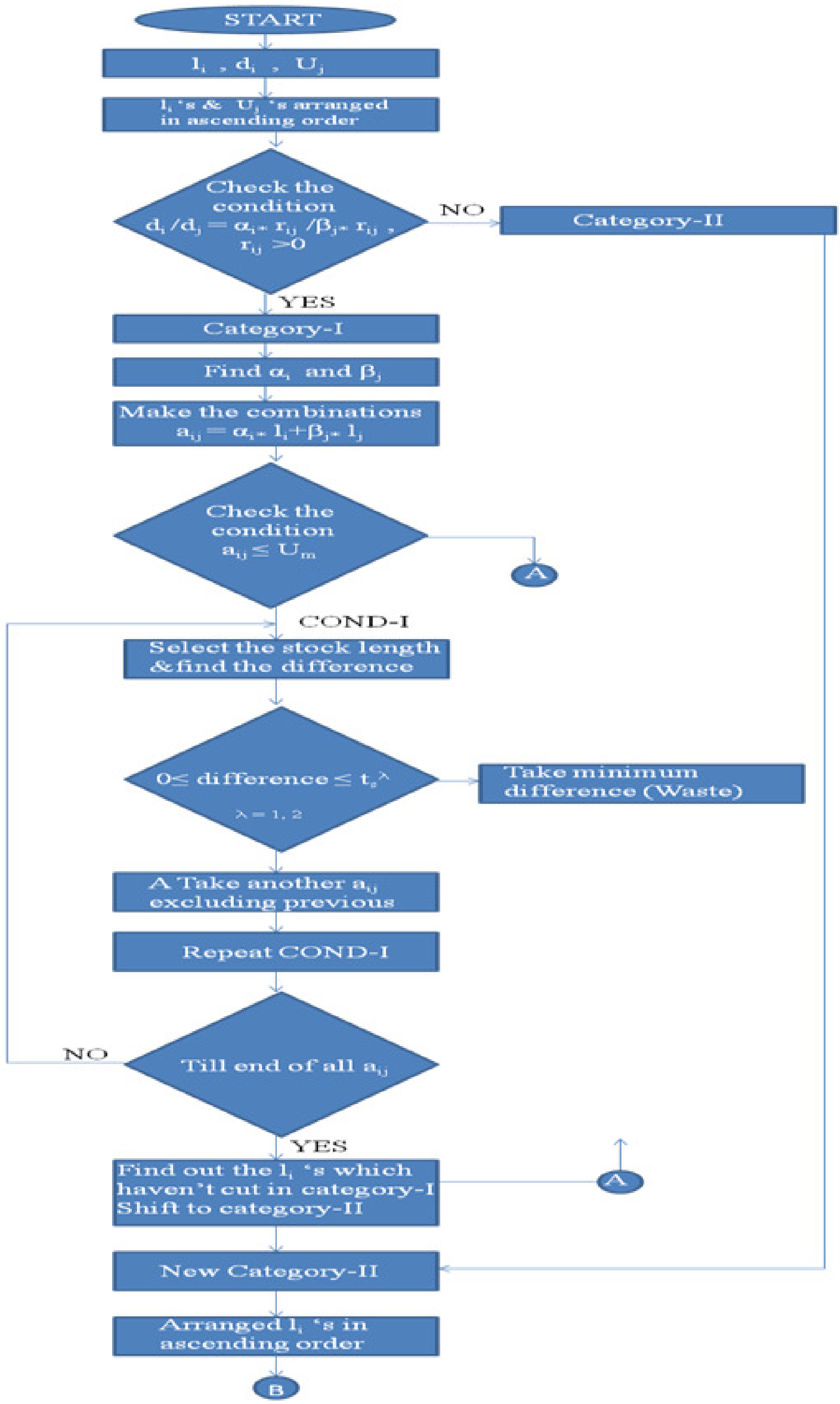
$$T_2 = t'_{r_1} + t'_{r_2} + \cdots$$

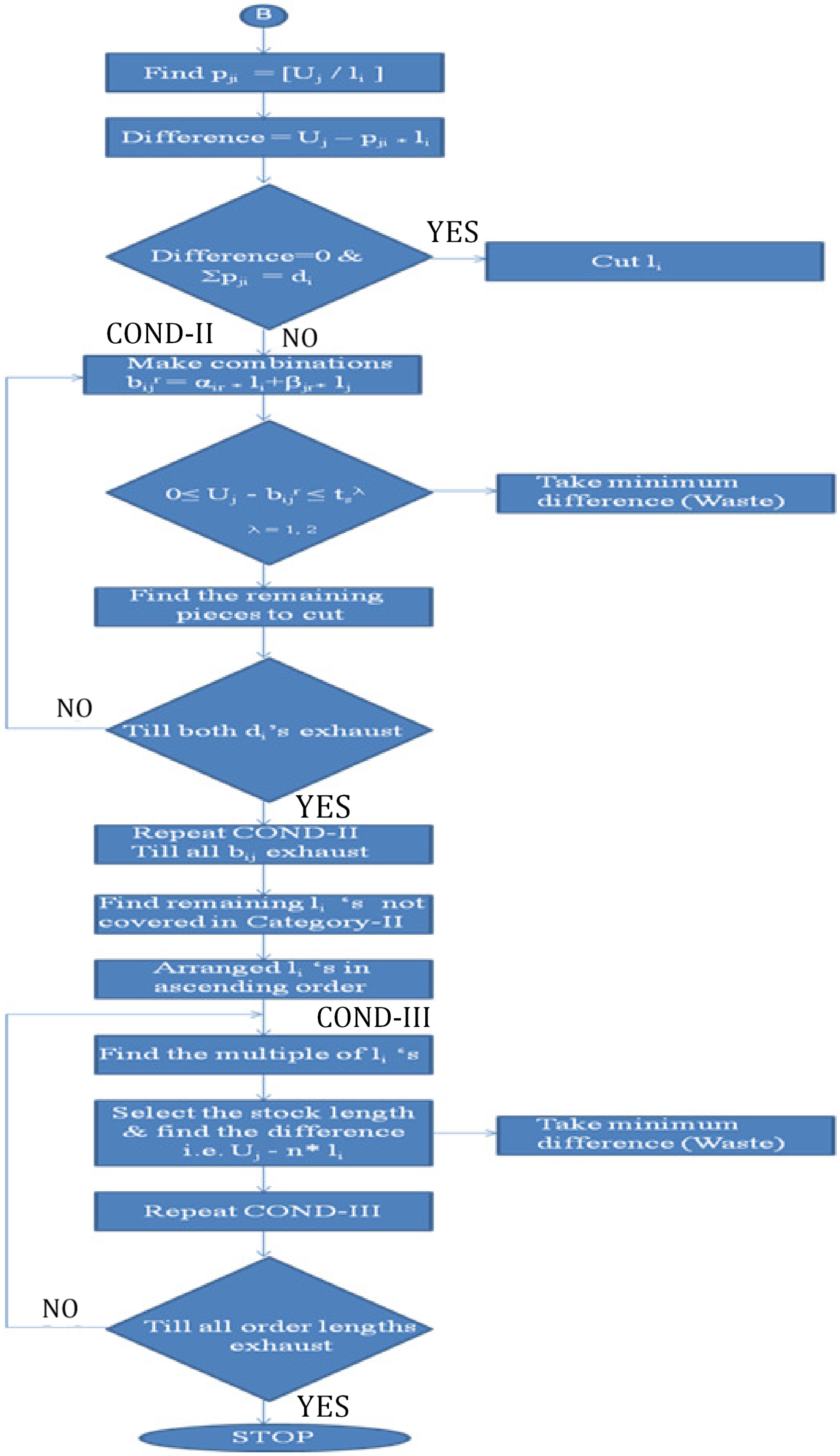
Finally, we get total trim

$$T = T_1 + T_2.$$

The percentages of trim lose with respect to t_s^1 and t_s^2 have been computed in accordance with the total stock length used.

6. ALGORITHM


START
l_i , d_i , U_j
l_i 's & U_j 's arranged in ascending order
Check the condition $d_i / d_j = \alpha_i * r_{ij} / \beta_j * r_{ij}$, $r_{ij} > 0$
NO
Category-II
YES
Category-I
Find α_i and β_j
Make the combinations $a_{ij} = \alpha_i * l_i + \beta_j * l_j$
Check the condition $a_{ij} \leq U_m$
A
COND-I
Select the stock length & find the difference
$0 \leq$ difference $\leq t_s^\lambda$
$\lambda = 1, 2$
Take minimum difference (Waste)
A Take another a_{ij} excluding previous
Repeat COND-I
NO
Till end of all a_{ij}
YES
Find out the l_i 's which haven't cut in category-I Shift to category-II
A
New Category-II
Arranged l_i 's in ascending order
B


B
Find $p_{ji} = [U_j / l_i]$
Difference $= U_j - p_{ji} * l_i$
Difference=0 & $\Sigma p_{ji} = d_i$
YES
Cut l_i
COND-II
NO
Make combinations $b_{ij}^r = \alpha_{ir} * l_i + \beta_{jr} * l_j$
$0 \le U_j - b_{ij}^r \le t_s^\lambda$
$\lambda = 1, 2$
Take minimum difference (Waste)
Find the remaining pieces to cut
NO
Till both d_i's exhaust
YES
Repeat COND-II Till all b_{ij} exhaust
Find remaining l_i 's not covered in Category-II
Arranged l_i 's in ascending order
COND-III
Find the multiple of l_i 's
Select the stock length & find the difference i.e. $U_j - n * l_i$
Take minimum difference (Waste)
Repeat COND-III
NO
Till all order lengths exhaust
YES
STOP

7. HAT FUNCTIONS AND THEIR APPLICATIONS IN 1D-CSP

Piecewise linear approximation may not have the practical significance in comparison to cubic spline or even higher order approximation. But it shows most of the essential features of piecewise polynomial approximation in a simple and easily understandable setting.

Consider the closed interval [a, b] on the real line and define a partition P:

$$P: a = \tau_1 < \tau_2 \ldots, < \tau_n = b$$

Let $\$_2$denote the collection or linear space of all continous broken lines on [a, b] with breaks at $\tau_2, \ldots, \tau_{n-1}$ (see figure 7.1). We define the basis for $\$_2$ (cf. [3])

Let $\tau_0 := \tau_1$, $\tau_{n+1} := \tau_n$ and set

$$H_i(x) = \begin{cases} (x - \tau_{i-1}) / (\tau_i - \tau_{i-1}), & \tau_{i-1} < x \leq \tau_i \\ (\tau_{i+1} - x)/(\tau_{i+1} - \tau_i), & \tau_i \leq x < \tau_{i+1} \\ 0 \quad , & \text{otherwise} \end{cases} \tag{7.1}$$

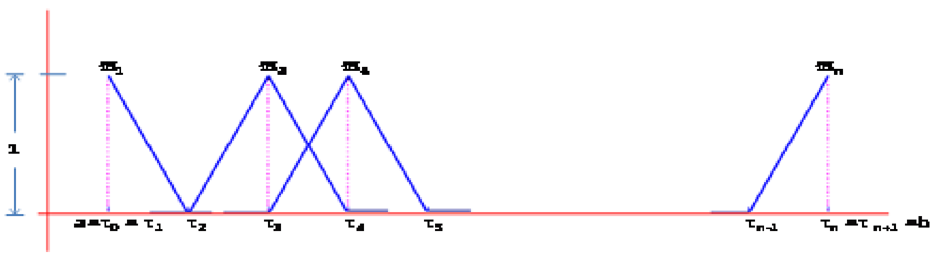


Figure 7.1 Hat Functions

It may be checked easily that the set S=$\{H_i(x)\}_{i=1}^{n}$ is linearly independent. If we denote the broken line interpolation to g by $I_2 g$, then it is given by

$$I_2 g = \sum_{i=1}^{n} g(\tau_i) H_i \tag{7.2}$$

which satisfy the interpolatory conditions

$$I_2 g(\tau_i) = g(\tau_i) \quad i = 1, \ldots\ldots, n$$

7.1 Applications

For the given data, the average order lengths τ_j have to be computed say τ_0 is the shortest and τ_n is the longest average order length. Consider the closed interval $[\tau_0, \tau_n]$ on R and define the partition P as follows:

$$P: a = \tau_0 < \tau_1 \ldots\ldots < \tau_n = b \tag{7.3}$$

Corresponding to each τ_i $(i = 0, \ldots.n)$, the total trim loss $(t_i \; say)$ $t_i = g(\tau_i)$ by using the algorithm described in section 6 has to be computed.

Similarly, corresponding to the sustainable trims of order one t_s^1 and two t_s^2, we define a partition and compute the broken line interpolation $I_2g\ \tau\epsilon\ [\tau_0, \tau_n]$.

Since $t_k's$ are known, the basis functions (Hat functions) $H_i's$ as described in (7.1) can be computed for the partition P in (7.3). Referring relation (7.2), we can write the broken line interpolant I_2g for the approximate trim loss as follows:

$$I_2g = \sum_{i=1}^{n} g(\tau_i)H_i$$

which gives the information of approximate trim loss at any arbitrary point $\tau\epsilon\ [\tau_0, \tau_n]$.

7.1 Remark

Broken lines are neither very smooth nor very efficient approximation. Both for a smoother and more efficient approximation one has to go to piecewise polynomial approximation with higher order pieces. The most popular choice continues to be a piecewise cubic approximating function. Since, it is the first step of splines to enter into a field of cutting-stock problem (CSP), authors have started with the simplest formulation of approximate function for the trim loss. Higher order approximation would be constructed in accordance with the demand of this method in the practical field.

8. ILLUSTRATIVE EXAMPLE

Tables of data

Order lengths and required number of pieces

S.No.	Order lengths (in cm.)	Required no. of pieces	S.No.	Order lengths (in cm.)	Required no. of pieces
1.	801	03	6.	498	16
2.	748	24	7.	492	39
3.	733	46	8.	471	21
4.	641	23	9.	327	40
5.	548	39	10.	303	32

Table 8.1

Available stock lengths

S.No.	Stock lengths (in cm.)	S.No.	Stock lengths (in cm.)
1.	2110	4.	3883
2.	2210	5.	4177
3.	3120	6.	4239

Table 8.2

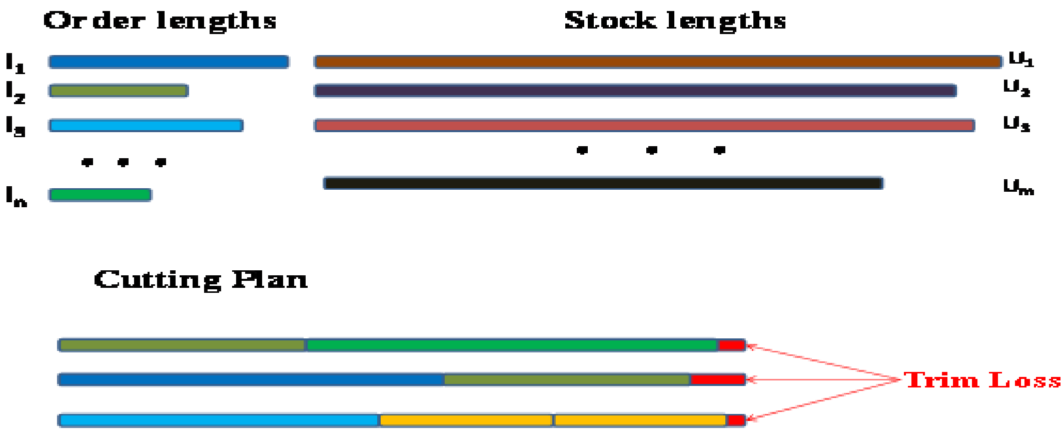


Figure 8.1 Cutting Plan

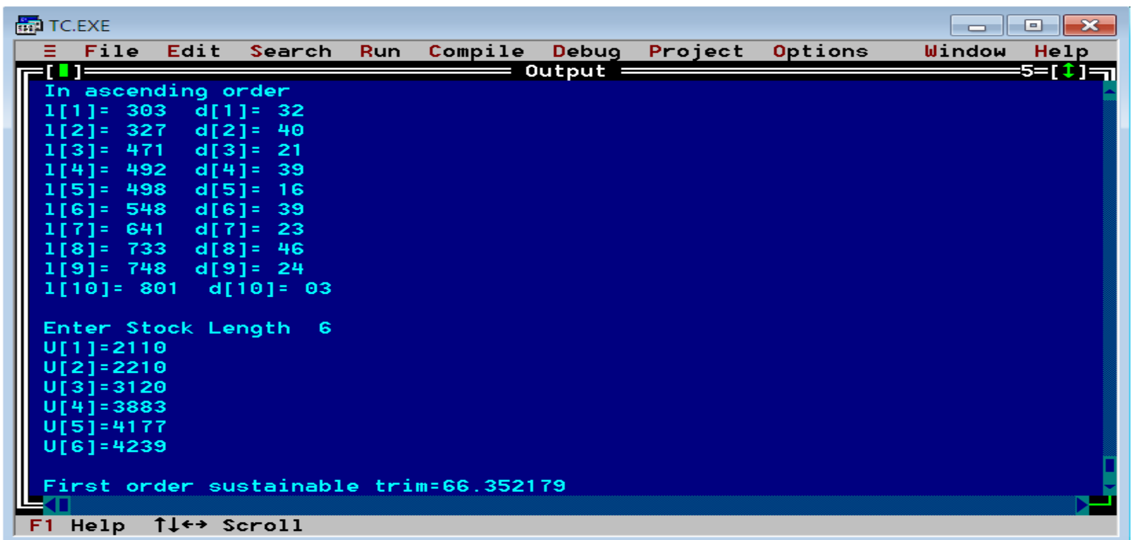


Figure 8.2 Screen shot of the programming

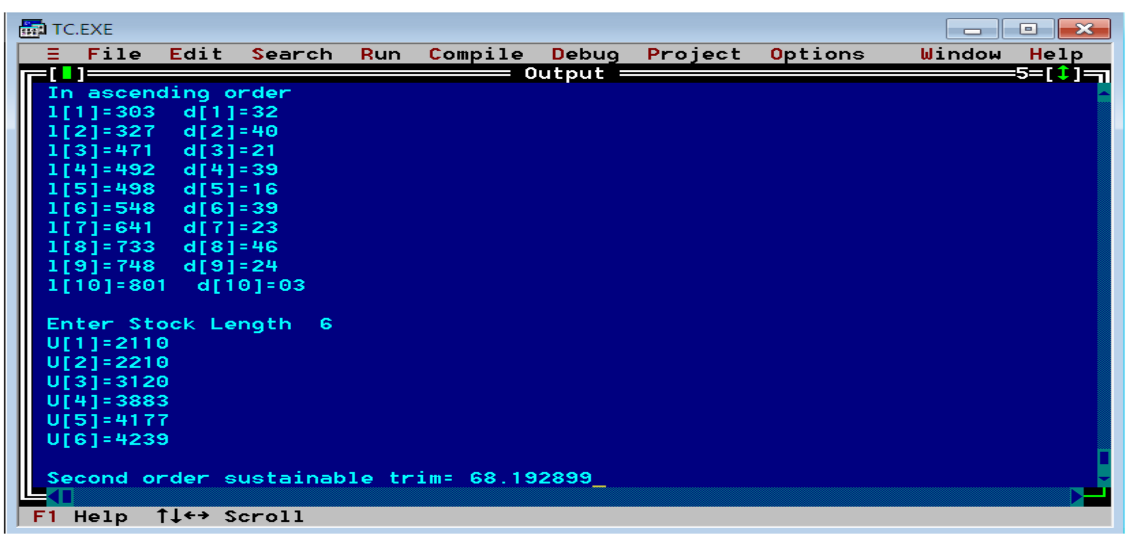


Figure 8.3 Screen shot of the programming

Computations as discussed in sections 3 and 5

First order sustainable trim (t_s^1) = 66.3591,

Second order sustainable trim (t_s^2) = 68.1928

S. No.	Sustainable Trims	Total Trim in %
1.	66.3591	0.3461
2.	66.8176	0.3461
3.	67.2756	0.1225
4.	67.7342	0.1225
5.	68.1928	0.1225

Table 8.15

Partition $P = \{\tau_1(= 66.3591) < \tau_2(= 66.8176) < \tau_3(= 67.2756) < \tau_4(= 67.7342) < \tau_5(= 68.1928)\}$

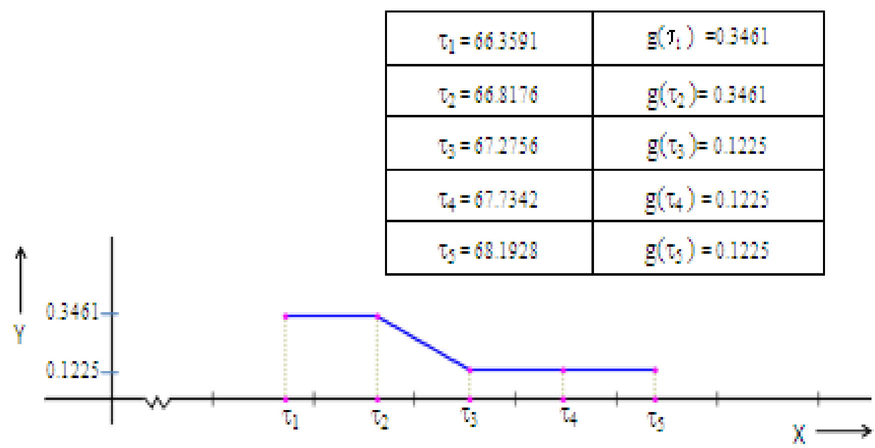


Figure 8.4 Linear splines for total trim loss corresponding to various sustainable trims

9. CONCLUSION

It is interesting to note that the impact of sustainable trim on the total trim plays key role in 1D-CSP. Referring Figure 8.4, it is apparent that corresponding to the second order sustainable trim (t_s^2) which is greater than the sustainable trim of order one (t_s^1), the total trim loss is minimal corresponding to (t_s^2) in the interval $[t_s^1, t_s^2]$. It is suggested that for a given data, and given stock lengths, after computing (t_s^1) and (t_s^2), the minimal trim loss may be predicated locally as per the proposed cutting plan.

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