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A Dynamic Absorber for Gear Systems Operating in Resonance and Instability Regions

There are many practical situations where resonances and instabilities in pinion-gear systems are difficult to predict in the design stage due to the unreliability of estimating the mesh stiffness and damping parameters. This paper presents a procedure for the design of an optimal dynamic absorber system which can be used in conditions where preliminary analysis shows that high dynamic tooth loads are likely to occur. The optimal parameters for the absorber are given in a generalized form in order to simplify its design for a particular gear system.

Introduction

In the previous paper (reference [1]) it has been shown that it is rather difficult, in the design stage, to insure the stability and safe performance of pinion-gear systems. This paper investigates the design of a vibration absorber for gear drives which can provide an appropriate solution for such conditions.

The dynamic equations of motion for the absorber system are developed in non-dimensional form to determine the pertinent variables necessary for general description of the absorber system. The conditions for uncoupling these equations are specified. The decision parameters to be considered in the optimization of the design are specified. A pattern search is employed to evaluate the optimal parameters for different gear system conditions and the results are given as design charts. The response of the optimal absorber shows its effectiveness even when the theoretical uncoupling conditions are not met.

Model of the System With Absorber

A schematic diagram of the envisioned system is shown in Fig. 1(*a*) and an elastic-mass system representation of it is shown in Fig. 1(*b*). The original single pinion is replaced with a two part system that is internally preloaded using a relatively soft spring. The preload causes the absorber to be in contact on the back side of the gear tooth as in the case of antibacklash gears. The design includes a frictional damper for energy dissipation whenever relative motion occurs between the pinion and absorber. This allows for limiting the vibratory motion without affecting the rotational efficiency of the gear drive.

A model of the system is shown in Fig. 2 where all parameters are referred to the high speed (*H.S.*). The main system parameters are:

$T_1(t)$ = pinion load torque
$T_2(t)$ = gear load torque
I_p = pinion inertia
I_a = absorber inertia
I_g = gear inertia
T_p = preload torque
T_f = frictional torque
$K_1(t), K_2(t)$ = mesh stiffness functions
C_1, C_2 = viscous damping constants in each mesh
$\theta_p, \theta_g, \theta_a$ = coordinates of motion relative to a moving reference at nominal operating speed

Since the preload spring, which supplies the preload torque, T_p, is very soft relative to the stiffness of a gear mesh, it will be included in the dynamic model as a constant torque applied to the pinion and absorber.

The dynamic equations of motion for the system are

$$I_p\ddot{\theta}_p + C_1(\dot{\theta}_p - \dot{\theta}_g) + T_f sgn(\dot{\theta}_p - \dot{\theta}_a) + K_1(t)(\theta_p - \theta_g) = T_1(t) - T_p \quad (1)$$

$$I_g\ddot{\theta}_g + C_1(\dot{\theta}_g - \dot{\theta}_p) + C_2(\dot{\theta}_g - \dot{\theta}_a) + K_1(t)(\theta_g - \theta_p) + K_2(t)(\theta_g - \theta_a) = T_2(t) \quad (2)$$

$$I_a\ddot{\theta}_a + C_2(\dot{\theta}_a - \dot{\theta}_p) + T_f sgn(\dot{\theta}_a - \dot{\theta}_p) + K_2(t)(\theta_a - \theta_g) = T_p \quad (3)$$

Assuming a steady transmitted load with a sinusoidal component is aplied to the pinion, the torque load can be expressed as:

$$T_1(t) = -T_o + T_e \sin \omega_e t \quad (4)$$

$$T_2(t) = T_o \quad (5)$$

The gear is assumed to be driving in this case. A change of sign on the T_o and T_p terms if the direction of rotation changes produces no change in the final result.

Conditions for Uncoupling the Equations of Motion

In order to simplify the analysis of the dynamic response of

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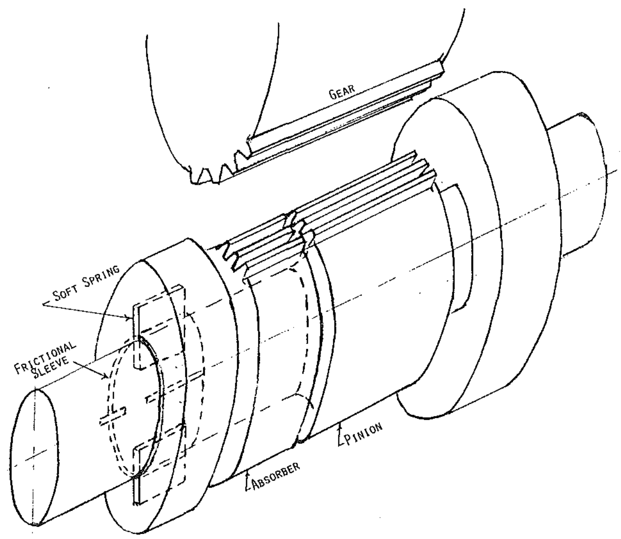

Fig. 1(a) **Schematic showing mechanical arrangement of absorber**

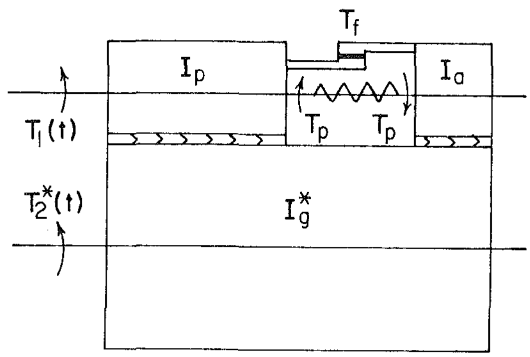

Fig. 1(b) **Elastic-mass system for absorber**

the absorber, the equations of motion will be uncoupled using the procedure described in reference [2]. The theoretical uncoupling conditions require that the stiffness variation functions are the same and the damping in the problem is of the Rayleigh type. Using the notation

$$K_1(t) = \bar{K}_1(\phi_1(t))$$
$$K_2(t) = \bar{K}_2(\phi_2(t))$$

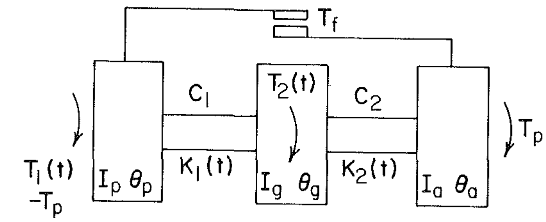

Fig. 2 **Dynamic model of absorber**

where

$\bar{K}_1$ = average torsional stiffness of pinion-gear mesh over a cycle.
$\bar{K}_2$ = average torsional stiffness of absorber-gear mesh
$\phi_i(t)$ = periodic stiffness variation functions $i = 1,2$

The theoretical uncoupling conditions therefore require that $\phi_1(t) = \phi_2(t)$. Since both the absorber and pinion have the same number of teeth, the mesh frequency, ω_m, will be the same for both meshes. In order for the stiffness variations to be in phase, a new pair of teeth should enter the zone of action in the absorber-gear mesh at the same instant as a pair enters the zone of action in the pinion-gear mesh. Since the pinion-absorber system acts like an antibacklash gear, the absorber-gear mesh is generally expected to be out of phase with the pinion-gear mesh for a standard gear. By advancing the position of the teeth in the absorber section of the gear relative to the pinion section, the two stiffness variations can be adjusted to be in phase at least on a geometric basis. The required shift is given by

$$\text{Toothspace} + \Theta_a - \Theta_r \qquad (6)$$

where

Θ_a is angle of approach with gear driving

Θ_r is angle of recess with gear driving

References [2] and [3] show that stiffness variations in phase lead to narrower instability regions in a two-stage gear drive.

The other uncoupling requirement is that the stiffness should vary between the same limit. For spur gear drives, this is automatically satisfied since the contact ratios in each mesh will be the same and the numbers of pairs of teeth in contact will be the same at all times. In helical gear drives, however, the relationships are more complex and it will be assumed that the upper and lower limits of each mesh stiffness are designed to be as close as possible. Accordingly, it will be assumed in

Nomenclature

I_{po} = original system pinion inertia
R_K = Rayleigh spring damping constant for damping in the mesh
I_g = gear inertia
T_o = steady load on system
T_e = sinusoidal fluctuation load amplitude applied to pinion
$\bar{K}$ = average torsional stiffness of mesh of face width FW (stiffness of original system) = K_{min} $(c(V_{max}-1)+(2-V_{max}))$
FW = total face width of original system
$\phi(t)$ = stiffness variation function
T_{max} = maximum allowable torque per inch of face
T_{pmax}, T_{pmin} = maximum and minimum torques per inch of face of pinion (face width FW_p) = $K_1(t)$ $(\theta_g - \theta_p)/FW_p$
T_{amax}, T_{amin} = maximum and minimum torques per inch of face of absorber (face width FW_a) = $K_2(t)$ $(\theta_a - \theta_g)/FW_a$
ω_{n2}, ω_{n3} = absorber system natural frequencies
W_1, W_2 = weights on objective function
T_o^* = nominal load per inch of face in original system = T_o/FW
I_e = effective inertia of original system = $(I_{po}I_g)/(I_{po}+I_g)$
ω_n = original system natural frequency = $\sqrt{\bar{K}/I_e}$
ξ_{mesh} = damping ratio of original system = $1/2\,R_K\omega_n$
K_{min} = minimum mesh stiffness of

the analysis that $\phi_1(t) = \phi_2(t) = \phi(t)$. It should be noted here that deviations from this theoretical conditions to accommodate practical design situations do not appreciably affect the performance of the absorber.

The other required condition for uncoupling the equations of motion is that the damping should be of the Rayleigh type. The damping coefficients attributed to the mesh can be reasonably assumed to be proportional to the average stiffnesses of each mesh since the dissipation is primarily due to sliding friction. The frictional force is proportional to nominal load on the mesh. Both nominal load and stiffness are proportional to face width and the low damping ratios expected in gear meshes (lower than .05), makes Rayleigh damping a reasonable assumption. Accordingly

$$C_1 = R_K \bar{K}_1 \tag{7}$$

$$C_2 = R_K \bar{K}_2 \tag{8}$$

R_K is defined based on the damping ratio of the original system which the absorber is designed to replace. Thus, if the original pinion-gear system had a damping ratio ξ and natural frequency ω_n, R_K is found from

$$R_K = \frac{2\xi}{\omega_n} \tag{9}$$

Modelling the effect of the frictional torque T_f as Rayleigh damping requires the evaluation of an equivalent R_M constant. Thus, it will be modelled as Rayleigh mass damping acting on the pinion and absorber. Since mass damping theoretically requires that a damper is applied to each mass in the system, corrections will be necessary in the resulting uncoupled equations of motion. Therefore, in the equations of motion equations (1-3) $T_f \; sgn(\dot{\theta}_p - \dot{\theta}_a)$ is replaced by $C_p \dot{\theta}_p$ and $T_f \; sgn(\dot{\theta}_a - \dot{\theta}_p)$ is replaced by $C_a \dot{\theta}_a$. The relationship between R_M and T_f is developed in the following section.

The equations of motion incorporating all the above assumptions can be written in matrix form as:

$$[I_N]\{\ddot{\theta}\} + [R_K[\bar{K}] + R_M[I_N]]\{\dot{\theta}\} + \phi(t)[\bar{K}]\{\theta\} = \{f(t)\} \tag{10}$$

where

$$[I_N] = \begin{bmatrix} I_p & 0 & 0 \\ 0 & I_g & 0 \\ 0 & 0 & I_a \end{bmatrix}$$

$$[\bar{K}] = \begin{bmatrix} \bar{K}_1 & -\bar{K}_1 & 0 \\ -\bar{K}_1 & \bar{K}_1 + \bar{K}_2 & -\bar{K}_2 \\ 0 & -\bar{K}_2 & \bar{K}_2 \end{bmatrix}$$

$$\{f(t)\} = \begin{Bmatrix} -T_o - T_p + T_e \sin \omega_e t \\ T_o \\ T_p \end{Bmatrix}$$

$$\{\theta\} = \begin{Bmatrix} \theta_p \\ \theta_g \\ \theta_a \end{Bmatrix}$$

The system natural frequencies can therefore be found as:

$$\omega_{n1}^2 = 0$$

$$\omega_{n2}^2 = A - [A^2 - B]^{1/2} \tag{11}$$

$$\omega_{n3}^2 = A + [A^2 - B]^{1/2} \tag{12}$$

where

$$A = \frac{1}{2}\left[\frac{\bar{K}_1}{I_p} + \frac{\bar{K}_1 + \bar{K}_2}{I_g} + \frac{\bar{K}_2}{I_a}\right] B = \frac{\bar{K}_1 \bar{K}_2 (I_p + I_g + I_a)}{I_p I_g I_a}$$

and the corresponding normal mode vectors are

$$\{T\}_i = \begin{Bmatrix} 1 \\ (\bar{K}_1 - I_p \omega_{ni}^2)/\bar{K}_1 \\ [(\bar{K}_1 - I_p \omega^2_{ni})\bar{K}_1]/[(\bar{K}_2 - I_a \omega_{ni}^2)\bar{K}_1] \end{Bmatrix} i = 1,2,3 \tag{13}$$

Placing each mode vector into a matrix $[T]$, the coordinate transformation which uncouples the equations of motion is therefore

$$\{\theta\} = [T]\{\eta\} \tag{14}$$

where $\{\eta\}$ are the normal mode coordinates

$$[T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \epsilon_{22} & \epsilon_{23} \\ 1 & \epsilon_{32} & \epsilon_{33} \end{bmatrix}$$

Nomenclature (cont.)

original pinion-gear system

V_{max} = K_{max}/K_{min} = ratio of maximum to minimum stiffness

c = generalized contact ratio = 1 + $\frac{\text{period of maximum stiffness}}{\text{mesh period}}$ $1 \leq c \leq 2$

F_1 = face width of absorber/face width of original system and average torsional stiffness of absorber mesh/average torsional stiffness of original system mesh $= \frac{FW_a}{FW} = \frac{\bar{K}_2}{\bar{K}}$

μ_1 = absorber inertia/original pinion inertia = I_a/I_{po}

μ_2 = pinion inertia/original pinion inertia = I_p/I_{po}

C_{TP} = preload torque per inch face of absorber/nominal torque per inch face of original system = $(T_p/FW_a(/(T_o^*)$

F_2 = face width of pinion/face width of original system and average torsional stiffness of pinion mesh/average torsional stiffness of original system mesh $= \frac{FW_p}{FW} = \frac{\bar{K}_1}{\bar{K}}$

$\{DV\}$ = Design Vector = $\{F_1, \mu_1, \mu_2, C_{TP}, F_2\}$

$\{DV\}_{max}, \{DV\}_{min}$ = upper and lower limits on $\{DV\}$

$\left(\frac{\omega_e}{\omega_n}, \frac{\omega_e}{\omega_m}\right)$ = general point in frequency domain of original system

The resulting uncoupled equations of motion can be obtained as described in [4] as:

$$\{\ddot{\eta}\}+[R_K[\Omega_n^2]+R_M[I]]\{\dot{\eta}\}+\phi(t)[\Omega^2]\{\eta\}=\{f^*(t)\} \tag{15}$$

where

$$[\Omega^2]=\begin{bmatrix}0 & 0 & 0\\ 0 & \omega_{n2}^2 & 0\\ 0 & 0 & \omega_{n3}^2\end{bmatrix}$$

$$[I]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{ identity matrix}$$

$$\{f^*(t)\}=\begin{Bmatrix} T_e \sin \omega_e t/M_1 \\ (-T_o(1-\epsilon_{22})-T_p(1-\epsilon_{32})+T_e \sin \omega_e t)/M_2 \\ (-T_o(1-\epsilon_{23})-T_p(1-\epsilon_{33})+T_e \sin \omega_e t)/M_3 \end{Bmatrix}$$

and

$$M_1=I_p+I_a+I_g$$
$$M_2=I_p+\epsilon_{22}^2 I_g+\epsilon_{32}^2 I_a$$
$$M_3=I_p+\epsilon_{23}^2 I_g+\epsilon_{33}^2 I_a$$

Since the relative motions $\theta_p-\theta_g$, $\theta_g-\theta_a$ are of interest in finding the tooth loads, they can be expressed as:

$$(\theta_p-\theta_g)=(1-\epsilon_{22})\eta_2+(1-\epsilon_{23})\eta_3 \tag{16}$$

$$(\theta_g-\theta_a)=(\epsilon_{22}-\epsilon_{32})\eta_2+(\epsilon_{23}-\epsilon_{33})\eta_3 \tag{17}$$

The damping ratios in the second and third mode equations are found from

$$\xi_i=\frac{1}{2}[R_K\omega_{ni}+R_M/\omega_{ni}] \quad i=2,3 \tag{18}$$

Equivalent R_M Value for the Coulomb Friction Torque T_f

An approximate effective R_M value can be developed based on equivalent energy dissipation per cycle for any particular value of T_f as:

$$R_M=\frac{10\left(\frac{I_{po}}{I_g}\right)^{1.5} 8T_f}{\left[1-\frac{I_{po}}{4I_g}\right]I_{po}\omega_n\pi\left(\frac{2T_o}{\bar{K}_{\min}}\right)} \tag{19}$$

where I_{po} is the moment of inertia of the original pinion.

The above equation is used to define R_M for the approximate normal mode analysis and is found to give approximately equal response amplitude to those obtained by numerical integration with Coulomb damping.

Selection of Design Variables

The design variables which will influence the response of the system can be written as:

$$\frac{\bar{K}_1}{\bar{K}}=\frac{FW_p}{FW}=F_2 \tag{20}$$

$$\frac{\bar{K}_2}{\bar{K}}=\frac{FW_a}{FW}=F_1 \tag{21}$$

$$\frac{T_p/FW_a}{T_o^*}=C_{TP} \tag{22}$$

$$\frac{I_p}{I_{po}}=\mu_2 \tag{23}$$

$$\frac{I_a}{I_{po}}=\mu_1 \tag{24}$$

where FW_a, FW_p are face widths of the absorber and pinion, respectively.

It is reasonable to assume that the stiffness is proportional to the face width in gears. Accordingly, specifying the face width also specifies stiffness. T_p is chosen to be equal to the nominal load per inch of fact (T_o^*) times face width of the absorber (FW_a) and a non-dimensional constant C_{TP}.

T_f is selected to be $.8T_p$, so that slip between the absorber and pinion is always possible.

Non-Dimensional Equations of Motion for Absorber

Writing equation (1) in terms of the design variables and the parameters of the original system gives

$$\left(\frac{I_{po}}{I_e}\right)\mu_2\ddot{\theta}_p+R_K F_2\omega_n^2(\dot{\theta}_p-\dot{\theta}_g)$$
$$+.8C_{TP}F_1\omega_n^2\frac{T_o}{\bar{K}}sgn(\dot{\theta}_p-\dot{\theta}_a)$$
$$+F_2\omega_n^2\phi(t)(\theta_p-\theta_g)=\omega_n^2\frac{T_o}{\bar{K}}\left(-1-C_{TP}F_1+\frac{T_e}{T_o}\sin\omega_e t\right) \tag{25}$$

Defining

$$\beta=\frac{\theta}{\theta_{st}}$$

where

$$\theta_{st}=\frac{T_o}{\bar{K}}$$
$$R_K\omega_n=2\xi_{\text{mesh}}$$
$$\tau=\omega_n t$$

yields

$$\beta'=\frac{\dot{\theta}}{\theta_{st}\omega_n}$$
$$\beta''=\frac{\ddot{\theta}}{\theta_{st}\omega_n^2}$$

where (′) denotes $d/d\tau$. Substituting for θ, $\dot{\theta}$, and $\ddot{\theta}$ in equation (25) and dividing by $\theta_{st}\omega_n^2$ gives

$$\frac{I_{po}}{I_e}\mu_2\beta_p''+2\xi_{\text{mesh}}F_2(\beta_p'-\beta_g')+.8C_{TP}F_1 sgn(\beta_p'-\beta_a')$$
$$+F_2\phi(\tau)(\beta_p-\beta_g)=\left[-1-C_{TP}F_1+\frac{T_e}{T_o}\sin\left(\frac{\omega_e}{\omega_n}\tau\right)\right] \tag{26}$$

Similarly, equation (2) becomes

$$\frac{I_g}{I_e}\beta_g''+2\xi_{\text{mesh}}F_2(\beta_g'-\beta_p')+2\xi_{\text{mesh}}F_1(\beta_g'-\beta_a')$$
$$+F_2\phi(\tau)(\beta_g-\beta_p)+F_1\phi(\tau)(\beta_g-\beta_a)=1 \tag{27}$$

Equation (3) becomes

$$\frac{I_p}{I_e}\mu_1\beta_a''+2\xi_{\text{mesh}}F_1(\beta_a'-\beta_g')+F_1\phi(\tau)(\beta_a-\beta_g)$$
$$+.8C_{TP}F_1 sgn(\beta_a'-\beta_p')=C_{TP}F_1 \tag{28}$$

Since

$$\frac{I_{po}}{I_e} = \frac{(I_{po}+I_g)}{I_g} = \left(\frac{I_{po}}{I_g}\right)+1 \tag{29}$$

and

$$\frac{I_g}{I_e} = \frac{(I_{po}/I_g)+1}{I_{po}/I_g} \tag{30}$$

the parameters needed to describe the system non-dimensionally are therefore:

$$\frac{T_e}{T_o}, \frac{I_{po}}{I_g}, \xi_{\text{mesh}}, \frac{\omega_e}{\omega_n}, \frac{\omega_e}{\omega_m},$$

plus the design variables

$$\frac{I_p}{I_{po}}, \frac{I_a}{I_{po}}, \frac{FW_a}{FW}, \frac{T_p/FW_a}{T_o^{\bullet}} \text{ and } \frac{FW_p}{FW}$$

The Optimal Absorber

The design of the absorber is formulated as an optimization problem as follows:

Given: $I_{po}, R_K, I_g, T_o, T_e, \left(\frac{\omega_e}{\omega_n}, \frac{\omega_e}{\omega_m}\right), \bar{K}, \phi(t), FW,$

$T_{\max}, W_1, W_2$ (31)

from which

$$\frac{I_{po}}{I_g}, \frac{T_e}{T_o}, \xi_{\text{mesh}}, \omega_n$$

can be calculated for the original system

Find $\{DV\}$

to minimize

$$U=\left[\left(\left(\max\left(\frac{T_{a\max}}{T_o^{\bullet}}, \frac{T_{p\max}}{T_o^{\bullet}}\right)-\min\left(\frac{T_{p\min}}{T_o^{\bullet}}, \frac{T_{a\min}}{T_o^{\bullet}}\right)\right)\right.\right.$$
$$+W_2\max\left(\frac{T_{p\max}}{T_o^{\bullet}}-\frac{T_{\max}}{T_o^{\bullet}}, \frac{T_{a\max}}{T_o^{\bullet}}\right.$$
$$\left.\left.\left.-\frac{T_{\max}}{T_o^{\bullet}}, -W_1\frac{T_{p\min}}{T_o^{\bullet}}, -W_1\frac{T_{p\min}}{T_o^{\bullet}}\right)\right)\right] \tag{32}$$

Subject to:

$$\{DV\}_{\min} \leqq \{DV\} \leqq \{DV\}_{\max} \tag{33}$$

$$F_1+F_2=1 \tag{34}$$

$$\frac{T_{a\min}}{T_o^{\bullet}}, \frac{T_{p\min}}{T_o^{\bullet}} > 0 \tag{35}$$

$$\frac{T_{p\max}}{T_o^{\bullet}}, \frac{T_{a\max}}{T_o^{\bullet}} < \frac{T_{\max}}{T_o^{\bullet}} \tag{36}$$

The objective function U implies that the overall load fluctuations in the pinion and absorber be minimized. It also includes a function which limits the maximum violation of the constraints on allowable loading in the mesh (equations (35) and (36)). These constraints are not explicitly enforced, but only appear in the objective function. The first term in the objective function represents the overall amplitude of mesh torque variation. The second term represents either the maximum loading constraint violation (a positive value), or the constraint which is closest to being violated (a negative value) if equation (35) and equation (36) are satisfied. It then selects the worst case of the two points

$$\left(\frac{\omega_e}{\omega_{n2}}, \frac{\omega_e}{\omega_m}\right) \text{and} \left(\frac{\omega_e}{\omega_{n3}}, \frac{\omega_e}{\omega_m}\right)$$

in the frequency domain analyzed for each design. The two points are defined by first assigning ω_e/ω_{n2} the generalized ω_e/ω_n value and then assigning ω_e/ω_{n3} the generalized ω_e/ω_n value for the given ω_e/ω_m. By careful selection of the generalized $(\omega_e/\omega_n, \omega_e/\omega_m)$ point used, it is found that this objective function, besides improving the response at this point, will also improve the response of the system at all other frequencies and eliminate the instabilities and superharmonic resonance conditions. It is possible to include more than one set of points in the optimization if desired.

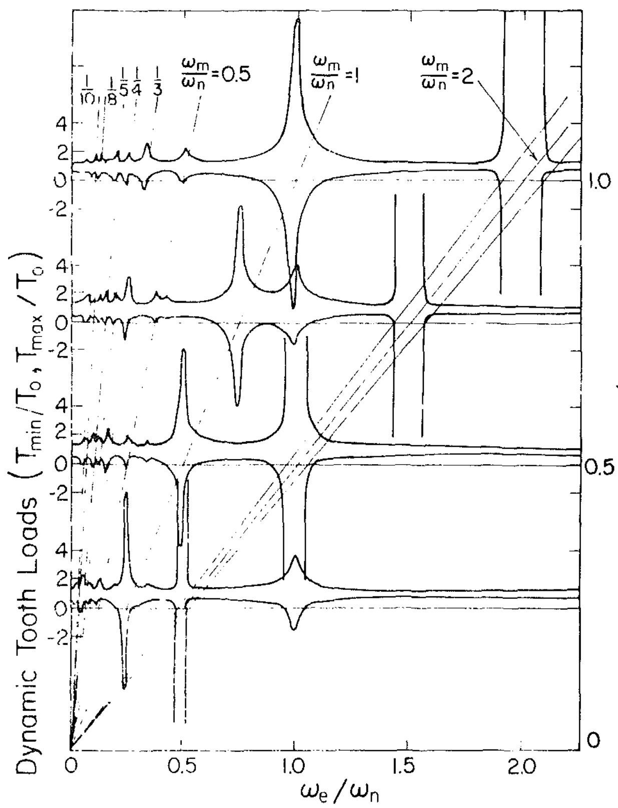

Fig. 3 Response spectrum for the original pinion-gear system

Search Algorithm

The search algorithm used is a pattern search technique similar to the one used by Baxa and Seireg [5]. The technique requires a starting point satisfying constraint equation (33); a maximum number of iterations, $N_{\max}$; and a relative stepsize, δ which is used to calculate scaled stepsizes for each variable. An option is provided for enforcement of constraint equation (34). If it is enforced only the first four variables are used; if not, all five variables are optimized.

Given the starting point, each element of $\{DV\}$ is sequentially changed by the appropriate increment (stepped) in the positive direction. If improvement of the objective occurs, the tested point becomes the design vector, and the next variable is tested from the new point. If not, the element of $\{DV\}$ is tested with a negative step from the starting point, with the tested point becoming the design vector if improvement of the objective occurs. If neither step results in improvement, the next element of the design vector $\{DV\}$ is tested in a similar manner. One iteration is completed when a pass through all the design variables in $\{DV\}$ is completed. A logic variable tests for a change in any of the variables during each iteration. The program terminates when either the maximum number of iterations is exceeded, or all the variables are tested in both directions and no changes occur.

Example Problem

An example problem is presented for illustration using a

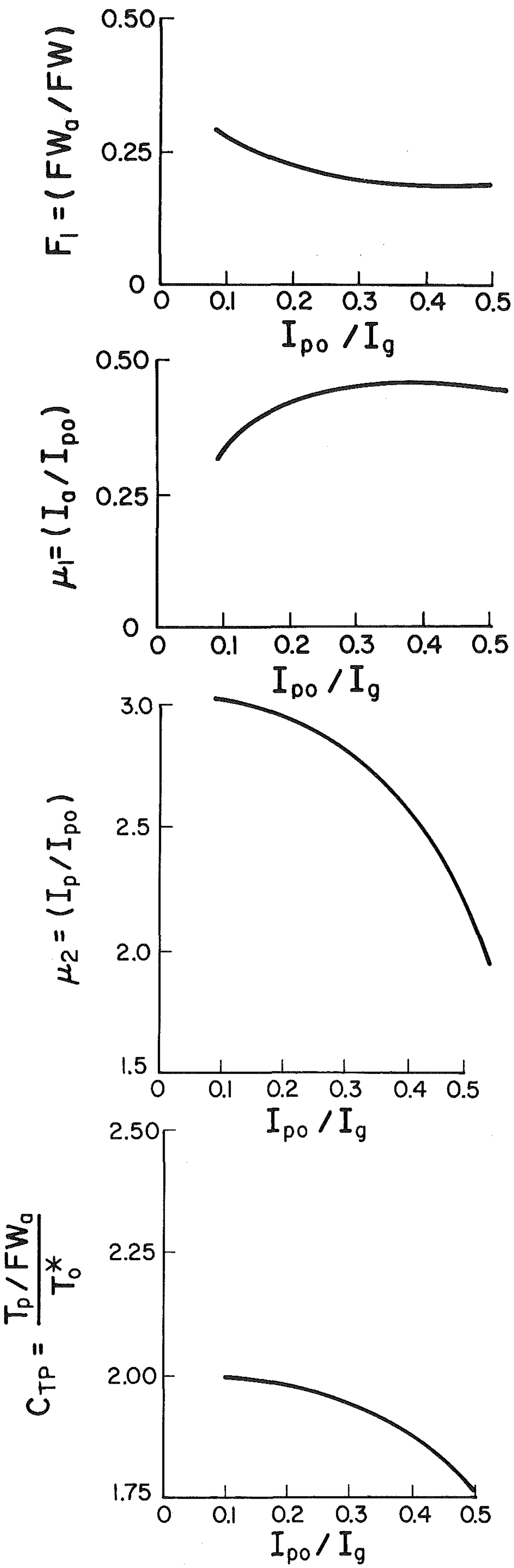

Fig. 4 Optimum parameters for absorber system

case with a rectangular stiffness variation. The parameters given are: $I_{po} = .2190$ lb_f-in.-sec^2; $I_g = .438$ lb_f-in.-sec^2; $\bar{K} = 5.84144 \times 10^7$ in.-lb_f/rad; $c = 1.5$ (contact ratio); $V_{max} = 1.4$; $R_K = 1.9997 \times 10^{-6}$ sec; $FW = 5.24$ in.; $T_o = 3948.85$ in.-lb_f; $T_e = 592.33$ in.-lb_f; W_1, $W_2 = 1.0$; $T_{max} = 2T_o^*$; and $(\omega_e/\omega_n,\ \omega_e/\omega_m) = (1, 1)$. From which: $I_e = .1460$ lb_f-in.-sec^2; $T_o^* = 753.60$ in.-lb_f/in. face; $\omega_n = 20002.47$ rad/sec; $I_{po}/I_g = .5$; $\xi_{mesh} = .02$; and $T_e/T_o = .15$; which corresponds to a $(T_e/T_o\ I_e/I_{po}) = .1$ or 10% effective load fluctuation on the mesh of the original pinion-gear system. The optimization will operate on the peak resonant point which is the $\omega_e/\omega_n = \omega_{ni}/\omega_n$, $\omega_e/\omega_m = 1.0$ point for each absorber system natural frequency. It will select the worst of the two points for the merit value.

Since the out of phase loading condition is worse for $\omega_e/\omega_m = 1.0$, as shown in [1], it is this case that will be solved. The analysis of the absorber will automatically solve the worst case (out of phase) for a positive T_e value.

The original pinion-gear mesh torque magnifications are shown in Fig. 3 which is used for comparison with the response of the optimum absorber.

In order to begin the optimization, starting points are selected and the upper and lower constraints of equation (33) are specified. Constraint equation (34) is enforced in this example, leaving only the first four variables to be optimized. For the cases presented:

$$\{DV\}_{max} = \{.5,.5,3.0,2.0,N/A\}$$
$$\{DV\}_{min} = \{.05,.1,.75,.5,N/A\}$$
$$\delta = .05$$
$$N_{max} = 40 \qquad (35)$$

The relative stepsizes using δ are found from

$$\Delta DV_i = (DV_{max_i} - DV_{min_i})\,\delta \qquad (36)$$

which yields the stepsize vector, in this case, to be

$$\{\Delta DV\} = \{.0225,.02,.1125,.075,N/A\} \qquad (37)$$

Several starting points are used to increase probability of finding a suitable optimum design since the design region is poorly structured and many local optimums can be expected. It has generally been found that the starting point selected in the middle of the allowable region usually yields the solution with the highest merit.

Comprehensive Design Charts

Using the optimal design parameters calculated for both rectangular and sinusoidal stiffness variations, curves which represent reasonable relationships between each of the design variables and the ratio I_{po}/I_g are developed for the case of a 10% effective load fluctuation acting on the mesh of the original pinion-gear system. These are shown in Fig. 4. These constructed curves are generalized approximation, but have the advantage of eliminating the need for specific knowledge of ω_e/ω_m ratio and the actual stiffness variation function $\phi(t)$ which are generally difficult to predict. Constraint equation (34) is enforced for these curves.

Comparison of results using the parameters in Fig. 4 with those obtained by optimizing for each ω_e/ω_m value shows that no significant change in the absorber system response occurred from using the same design parameters for the entire frequency domain encountered at a given I_{po}/I_g value. Thus, eliminating ω_e/ω_m as a variable in developing the design charts in Figs. 4 had no effect on the performance of the absorber for all practical purposes.

Figure 5 shows the response of the absorber system which can be compared to the original system with the same rectangular stiffness given in Fig. 3. As can be seen all the in-

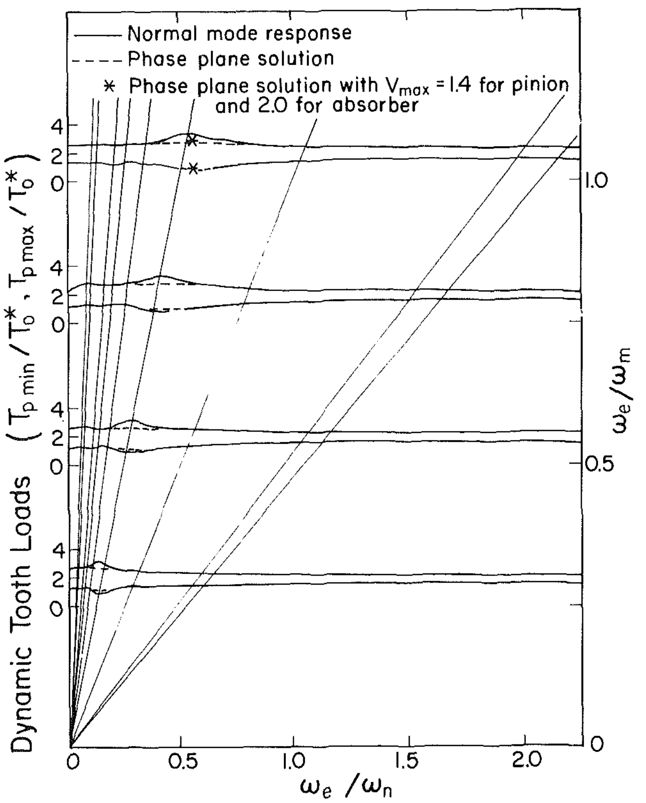

Fig. 5(a) Response spectrum for the pinion tooth load in the system with optimal absorber. (I_{po}/I_g = .2, ξ_{mesh} = .02).

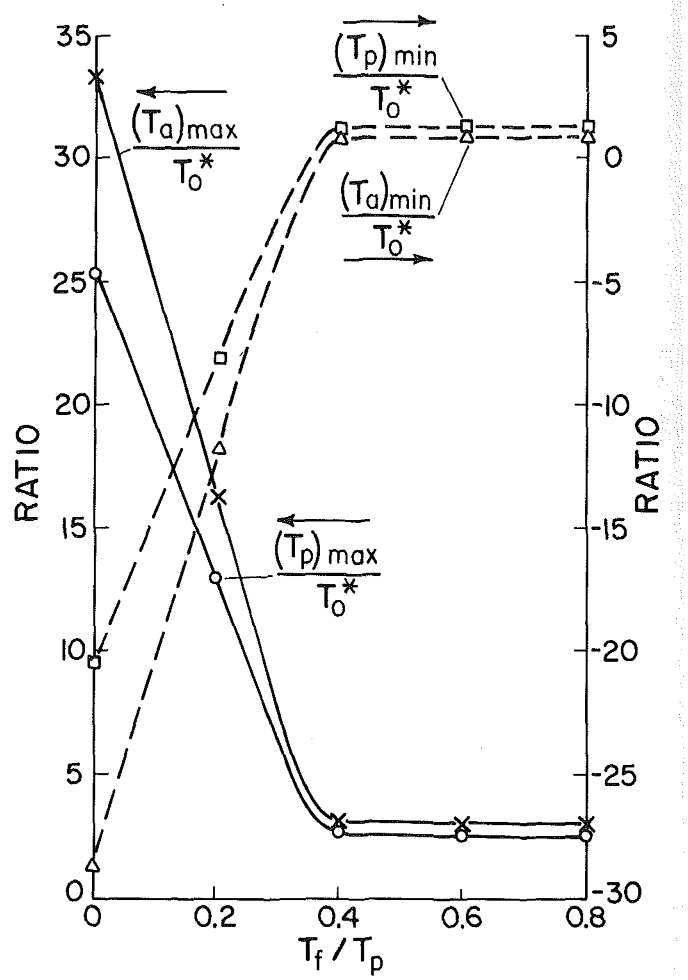

Fig. 6 Effect of T_f/T_p on response of absorber system

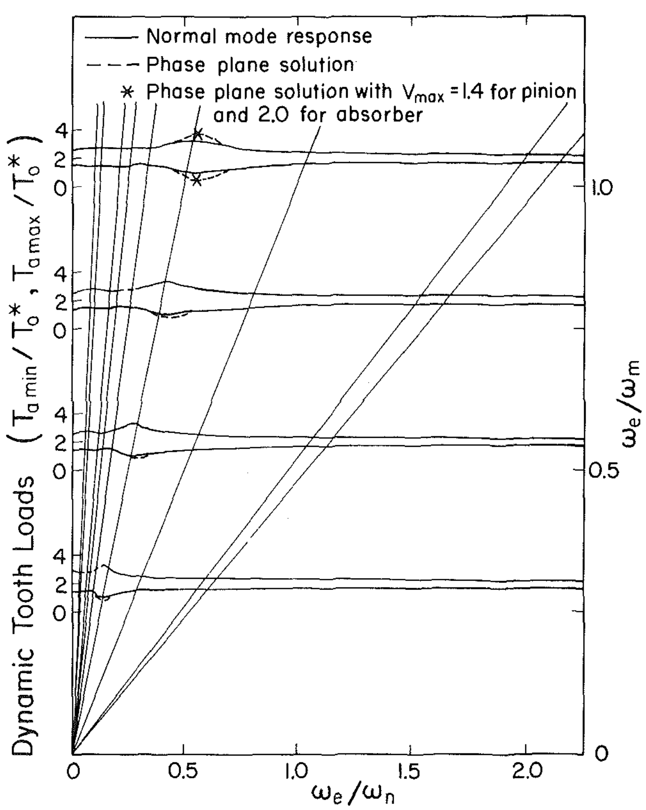

Fig. 5(b) Response spectrum for the absorber tooth load in the system with optimal absorber. (I_{po}/I_g = .2, ξ_{mesh} = .02).

stabilities have been eliminated and the peak resonant points practically flattened.

The figure also shows that the approximations utilized in the normal mode analysis give good results when checked by phase plane integration.

A sample check was performed to investigate the effect of mismatching the stiffness fluctuation in the absorber with that of the pinion. The cross points in Fig. 5 show the results for the extreme case of V_{max} = 2.0 for the absorber using the phase plane. No significant effect can be detected due to this discrepancy in the response.

In order to test the sensitivity of the absorber to the actual value of friction torque T_f used, the peak response point for ω_e/ω_m = 1.0 and I_{po}/I_g = .5 is checked for several values of T_f/T_p. Note that T_f/T_p = .8 is the value used in the developed approximate R_M relationship.

Figure 6 shows a plot of T_{pmax}/T_o^*, T_{pmin}/T_o^*, T_{amax}/T_o^*, T_{amin}/T_o^* versus T_f/T_p for the same design as calculated by the phase plane method. As can be seen from the figure the response of the absorber changes very little until T_f/T_p drops below the value of .4. Thus, the absorber would be expected to function properly even when the actual frictional torque is reduced to one-half of its design value.

Conclusion

It can be concluded from this study that an optimal dynamic absorber can be designed which is capable of providing safe gear system operation at all speeds including unexpected resonance and instability conditions for the pinion-gear system. The design charts which are given in this paper based on a computer optimization strategy can be used for the design of such absorber for any gear pair as a function of their moment of inertia. Such absorber systems would be invaluable in critical high speed operations where resonances or instabilities are suspected and cannot be exactly predicted beforehand.

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