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LAYUP OPTIMIZATION FOR MAXIMUM BUCKLING LOAD CONSIDERING BOUNDED UNCERTAINTY

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ABSTRACT

Optimal design of composite laminates with uncertain in-plane loadings and material properties is considered. The stacking sequence is designed to have maximum buckling load based on anti-optimization approach. To consider the above-mentioned uncertain properties, the convex modeling, interval analysis and Monte Carlo simulation techniques are used in calculating objective function. For the stacking sequence optimization, it is used the modified genetic algorithm which handles the discrete ply angles and the constraints easily.

Numerical results are given for rectangular laminates of various aspect ratios. The optimal solutions from the deterministic and the stochastic cases are obtained and it is demonstrated the importance of considering uncertainty. The buckling load carried by a deterministic design is much less than the one carried by a design uncertainty considered when both are subjected to uncertain loads. Also, it is examined the effects of the method for considering uncertainty on the optimization process in the light of computational efficiency and reliability of solutions obtained.

INTRODUCTION

Composite materials are widely used for structural components because of their superior stiffness-to-weight and strength-to-weight ratios. Also, configurations of a laminate such as stacking sequence and ply thickness can be tailored to meet various design requirements. Thus extensive research efforts have been devoted to the design optimization of composite laminates in connection with various objectives and constraints. Park [1] considered the optimal design of laminated plates under in-plane loading. Kim et al. [2] studied the optimal stacking sequence design of symmetrically laminated plates under in-plane loading to maximize load-bearing, using Tsai-Wu failure criterion as an objective function. Tauchert and Adibhalta [3] investigated the arrangement of laminated plates with respect to maximizing bending stiffness. A multilevel optimization scheme [4-6], in which ply angle and thickness were designed separately at each level of optimization, has been used to meet the various design requirements such as stiffness, natural frequency, buckling load, and weight. The branch and bound algorithm was used to handle the discrete ply angles in stacking sequence design of composite laminate [7, 8].

In practical applications, external loadings and material properties are always subject to a certain amount of scatter. Such situations occur due to a lack of a priori knowledge about the exact operational conditions and the imperfections in manufacturing processes. For the structural reliability and safety, unavoidable uncertainties should not be ignored at the design and analysis stage.

In this paper, it is developed an optimization scheme that can consider the uncertainty in external loading and material properties. The algorithm is applied to a problem of buckling load maximization. Considering the uncertainty in optimization process needs an effective tool for calculating the objective function. Two different methods, which are the convex modeling and Monte Carlo simulation (MCS), are used fro this purpose.

In convex modeling, only the information for the bounds of uncertain properties is needed. The convex sets covering the uncertain properties are constructed and the objective function is linearized with respect to these properties. The extreme values exist on the boundary of the convex set because the objective function is a linear function of this set. Thus the extremum can easily be found with computational efficiency. In interval analysis, the uncertain parameters are modeled by interval numbers and the linearized objective function is calculated by interval arithmetic. The scheme is slightly simpler than that of convex analysis, and gives more accurate results comparing the width of extremum values.

In Monte Carlo simulation, the probabilistic characteristics of uncertain properties are assumed and the random deviates are generated. Some well-known functions such as normal, beta and uniform distributions are used to model the uncertainty. The objective function is calculated using the generated random sets and the extremum is found among those values.

For the stacking sequence optimization, it is used the modified genetic algorithm (GA) which can easily handle discrete ply angle and produce alternative optima in repeated runs. The application of genetic algorithm is initially reported by Hajela [9] for composite structures. Riche and Haftka [10] proposed genetic algorithm to optimize the stacking sequence of composite laminate for buckling load maximization. For the same problem, Liu at al. [11] has provided permutation genetic algorithm. A recessive gene repair strategy was introduced by Todoroki and Haftka [12] for implementing the given constraints.

In the present study, optimal designs of composite laminates are given to maximize the buckling load when uncertainty in biaxial loading and material properties exist. Numerical results are given for various aspect rations of laminated plates. First, optimal solutions from deterministic and probabilistic cases are obtained and the difference is investigated. Then, convex modeling and Monte Carlo simulation approach are compared by analyzing the computation time and the reliability of solution with respect to the degree of uncertainty.

PROBLEM DESCRIPTION

A rectangular composite laminate of length *a*, width *b*, and thickness *h* is subjected to in-plane compressive loads N_x and N_y in the *x* and *y* directions, respectively (Fig 1). The laminate is symmetric, balanced about the mid-plane and made of layers each of thickness *t*. The values of external loads N_x and N_y are not fixed ones and have a bounded uncertainty.

The laminate buckles into *m* and *n* half-waves in the *x* and *y* directions when the amplitude parameter reaches a value λ_b. In case of simply supported plate, λ_b is given by classical lamination theory as

$$\lambda_b = \pi^2 \frac{\left[D_{11}(m/a)^4 + 2(D_{12}+2D_{66})(m/a)^2(n/b)^2 + D_{22}(n/b)^4\right]}{(m/a)^2 N_x + (n/b)^2 N_y} \qquad (1)$$

where D_{ij} are the flexural stiffness. The smallest value of λ_b, function of (*m*, *n*), is the critical buckling load λ_{cb}.

The optimization problem is to maximize the critical buckling load by changing the ply orientations. The ply orientation angles are limited to 0°, ±45° and 90°, thus discrete optimization methodology is required. In order to consider the uncertainties, the modeling of uncertain design parameters and the efficient tool for calculation of objective function are needed. The convex modeling, interval analysis and probabilistic approach are adopted in optimization process to calculate the objective function.

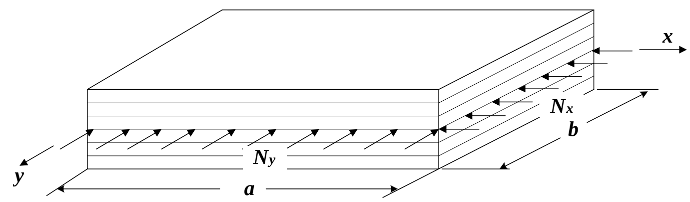


Fig 1: A laminate under uncertain compressive loads.

MODELING OF UNCERTAINTY

Convex modeling

In order to consider the uncertainty via probabilistic approach, the function of probability distributions should be informed. However, the probability function of scattering distribution requires sufficient data measuring and sometimes this information may not be available. If the uncertainties under consideration are bounded with respect to the nominal reference values, convex set with scattering bound of design parameters can be easily constructed.

Convex modeling can be utilized in the constraint equations or objective function of the optimization problem to consider uncertain parameters [13-15]. In our case, convex modeling is applied to the critical buckling load calculation and the procedure for analysis is outlined as follows.

As a first step, Eq. (1) is linearized with respect to the uncertain parameters. It is assumed that N_x, N_y, E_L, E_T, G_{LT} and ν_{LT} vary arbitrary around their nominal values with the condition that these variations are small and bounded. Equation (1) can be written as a function of these parameters.

$$\lambda_b = \lambda_b(X_1, X_2, X_3, X_4, X_5, X_6) \qquad (2)$$

This can be expanded up to linear terms as follows,

$$\lambda_b(X_i^0 + \delta_i) = \lambda_b(X_i^0) + \sum_{i=1}^{6} \frac{\partial \lambda_b(X_i^0)}{\partial X_i} \delta_i \qquad (3)$$

The vectors $\{f\}$, $\{\delta\}$ are defined as follows,

$$\{f\}^T = \left[\frac{\partial\lambda_b(X_i^0)}{\partial X_1}, \frac{\partial\lambda_b(X_i^0)}{\partial X_2}, \frac{\partial\lambda_b(X_i^0)}{\partial X_3}, \frac{\partial\lambda_b(X_i^0)}{\partial X_4}, \frac{\partial\lambda_b(X_i^0)}{\partial X_5}, \frac{\partial\lambda_b(X_i^0)}{\partial X_6}\right] \quad (4)$$

$$\{\delta\}^T = [\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6] \quad (5)$$

Then the perturbed buckling load can be symbolically given as,

$$\lambda_b(X_i^0 + \delta_i) = \lambda_b(X_i^0) + \{f\}^T\{\delta\} \quad (6)$$

If it is assumed that δ_i construct convex set, then from the linearity of Eq. (6), extreme values are on the boundary of convex set. The constructed convex set of ellipsoid shape is derived as follows,

$$Z(e) = \left\{\delta : \sum_{i=1}^{6} \frac{\delta_i^2}{e_i^2} \le 1\right\} \quad (7)$$

In order to obtain e_i, following lagrangian should be minimized.

$$L = Ce_1e_2e_3e_4e_5e_6 + \lambda\left(\frac{\Delta_1^2}{e_1^2} + \frac{\Delta_2^2}{e_2^2} + \frac{\Delta_3^2}{e_3^2} + \frac{\Delta_4^2}{e_4^2} + \frac{\Delta_5^2}{e_5^2} + \frac{\Delta_6^2}{e_6^2} - 1\right) \quad (8)$$

where Δ_i is the maximum deviation of parameter X_i. Through the variational process, e_i are obtained as

$$e_i = \sqrt{n}\Delta_i \quad (9)$$

where, n is the number of parameters which has uncertainty.

The problem of finding extremum buckling load with the uncertain parameters having the deviation δ_i is constructed as the following form.

$$\lambda_b\big|_{extremum} = \underset{\{\delta\}\in C(e)}{extremum}\left(\lambda(X_i^0) + \{f\}^T\{\delta\}\right) \quad (10)$$

$$C(e) = \left\{\delta : \sum_{i=1}^{6} \frac{\delta_i^2}{e_i^2} = 1\right\} \quad (11)$$

The problem can be expressed by the following Lagrangian,

$$L(\delta) = \{f\}^T\{\delta\} + \lambda(\{\delta\}^T\{\varepsilon\}\{\delta\} - 1) \quad (12)$$

where $\{\varepsilon\}$ is a diagonal matrix whose diagonal elements are $\varepsilon_{ii} = 1/e_i^2$. After obtaining Lagrange multiplier, $\{\delta\}$ for extremum buckling loads are obtained as

$$\{\delta\} = \pm\frac{1}{\sqrt{\{f\}^T\{\varepsilon\}^{-1}\{f\}}}\{\varepsilon\}^{-1}\{f\} \quad (13)$$

Extremum buckling loads considering bounded scattered design parameters can be finally obtained as

$$\left.\begin{matrix}\lambda_{b,\max}\\ \lambda_{b,\min}\end{matrix}\right\} = \lambda_b\left(X_i^0\right) \pm \sqrt{\{f\}^T\{\varepsilon\}^{-1}\{f\}} = \lambda_b\left(X_i^0\right) \pm \sqrt{\sum_{i=1}^{6}\left[e_i\frac{\partial\lambda_b\left(X_i^0\right)}{\partial X_i}\right]^2} \quad (14)$$

Interval Analysis

In interval analysis, the uncertain parameters are modeled by an interval number as

$$X_i^I = \left[\underline{X}_i, \overline{X}_i\right] \quad (15)$$

Thus, the buckling problem with uncertain parameters can be stated as

$$\lambda_b\big|_{extremum} = \underset{X\in X^I}{extremum}\left(\lambda(X_i^0) + \{f\}^T\{\delta\}\right) \quad (16)$$

$$X^I = \left[\underline{X}, \overline{X}\right] = \{X : \underline{X} \le X \le \overline{X},\ X = (X_i)_m,\ \underline{X} = (\underline{X}_i)_m,\ \overline{X} = (\overline{X}_i)_m \in R^m\} \quad (17)$$

where m is the number of uncertain parameters. The central values and the radius of interval variables can be defined as

$$X_i^0 = \frac{\overline{X}_i + \underline{X}_i}{2}, \quad \Delta_i = \frac{\overline{X}_i - \underline{X}_i}{2} \quad (18)$$

Thus, the extremal value problem (16) subject to the constraint $\underline{X} \le X \le \overline{X}$ can be transformed into the following form.

$$\lambda_b\big|_{extremum} = \underset{\{\delta\}\in\Delta^I}{extremum}\left(\lambda(X_i^0) + \{f\}^T\{\delta\}\right) \quad (19)$$

$$\Delta^I = \left\{\delta : \delta \in R^m, -\Delta \le \delta \le \Delta\right\} \quad (20)$$

Using the interval arithmetic, the lower and upper bounds of buckling load can be obtained.

$$\lambda_b^I = \left[\underline{\lambda_b}, \overline{\lambda_b}\right] = \lambda^0 + \left[-|f|^T\Delta, |f|^T\Delta\right] \quad (21)$$

Finally, we have the buckling load with uncertain parameters from interval analysis.

$$\lambda_b = \lambda^0 - \left| f \right|^T \Delta \tag{22}$$

The lower bounds of buckling loads obtained by Eq. (22) are always lager than those from convex analysis of Eq. (14). It can be proved using Chaucy-Schwartz inequality [16]. Also, the inequality can be easily deduced by comparison of the uncertain parameter sets constructed from each method. The ellipsoidal set of convex modeling envelops the box set of interval analysis. Because the ellipsoid is obtained to have a minimum volume while the corners of the box are on the surface of it. For illustration, Fig 2 depicts two uncertain parameter sets in 3-Dimensional space.

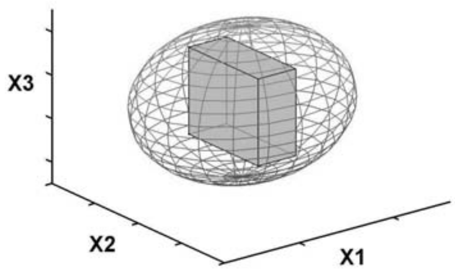


Fig 2: Comparison of uncertain parameter sets.

Probabilistic function

To evaluate the critical buckling load using Monte Carlo simulation, the probabilistic characteristics of uncertain parameters should be defined. Namely, the probabilistic distribution functions and the corresponding probabilistic data are assumed to model the random variables.

In this study, well-known functions such as normal, beta and uniform distributions are used to model the uncertain parameters. The density of normal distribution of $N(\mu_X, \sigma_X^2)$ is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \exp\left[-\frac{1}{2} \left(\frac{x-\mu_x}{\sigma_X} \right)^2 \right] \tag{23}$$

The beta distribution ($0 < x < 1$) is given by

$$f(x) = \begin{cases} \dfrac{x^{\alpha_1 - 1}(1-x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)} & if\ 0 < x < 1 \\ 0 & \end{cases} \tag{24}$$

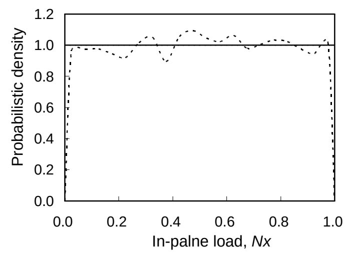
Where $B(\alpha_1, \alpha_2)$ is the *beta function*, defined by

$$B(\alpha_1, \alpha_2) = \int_0^1 t^{\alpha_1 - 1}(1-t)^{\alpha_2 - 1} dt \tag{25}$$

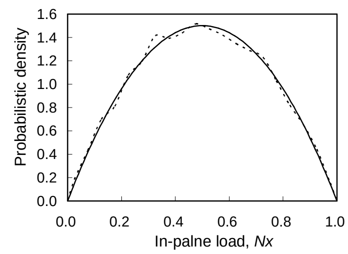
Then the random deviates are generated according to a pre-determined distribution function. After generating the values of all the uncertain parameters, buckling load is evaluated deterministically for each set of random variables. Thus the min-max values of buckling load can be determined from the results of numerical enumeration.

For example, the densities of three distribution functions are shown in

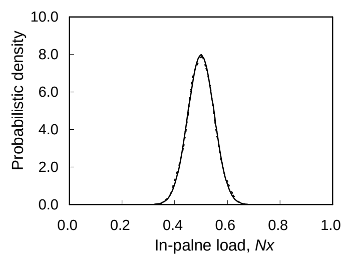
Fig **3**. The probabilistic model for $0 \le N_x \le 1$ is represented by uniform, beta distribution with $\alpha_1 = \alpha_2 = 2$, and $N(0.5, (0.05)^2)$. The solid lines are from analytic equations and the dotted lines are densities calculated from generated random number of 8,000 sampling points. The generated random deviates show a good agreement with analytic results.



(a) uniform distribution



(b) beta distribution



(c) normal distribution

Fig 3: Probability density functions.

OPTIMIZATION PROCEDURE

Anti-optimization problem

The optimization problem can be expressed as

$$\begin{aligned} &Maximize\ \lambda_b\left([\theta_1 / \theta_2 / \Lambda\ \theta_n]; X_1 \Lambda\ X_6\right) \\ &\quad \text{Subject to } \theta_i \in \{0^\circ, \pm 45^\circ, 90^\circ\} \text{ and } X_i^L \le X_i \le X_i^U \end{aligned} \tag{26}$$

where X_i^L and X_i^U are the lower and upper bounds of uncertain parameter X_i, respectively.

This problem becomes a kind of anti-optimization [15, 16]. Namely, the solution process involves the minimization of the objective function with respect to uncertain parameters if the design objective is to maximize it and this leads to a min-max problem. Thus the minimum buckling load is determined with uncertain quantities for any given stacking sequences. This is the worst case of in-plane loading on the laminate with uncertain material properties. The minimum buckling load is maximized by selecting the stacking sequence optimally. This procedure can be written as

$$\underset{\theta_k}{Max}\ \underset{X_i}{Min}\ \lambda_b(\theta_k ; X_i) \tag{27}$$

The solution of Eq. (27) produces the best stacking sequence to maximize the buckling load under the worst possible case with uncertain design parameters.

Genetic algorithm

The design objective of the present study is to obtain layups of laminate that has maximum buckling load under uncertain design parameters. The ply angles are selected as design variables and limited to a fixed set of angles such as 0°, ±45° and 90°. Thus layup design becomes a combinatorial optimization problem, and accordingly needs discrete optimization techniques.

The genetic algorithm is well suited for the layup optimization. It has previously been used for various optimization problems of composite laminates. Also, because of their random nature, they easily produce alternative optima in repeated runs. This property is particularly important in layup optimization, because widely different layups can have very similar performance.

Three constraints are applied to the present optimization problem. The first one is the symmetric layup constraint, but this is satisfied automatically by the coding rule that only half of the laminate is represented in a chromosome. The second constraint is a requirement of balanced laminate construction, which is intended to eliminate undesirable extensional-flexural coupling. The third constraint is a limit of four contiguous plies with the same fiber orientation, which reduces the problem of matrix cracking. It is not easy to enforce these constraints in genetic algorithm. In the present study, a recessive-gene-like repair strategy [12] is applied to implement the constraints.

Operation of genetic algorithm

The flowchart for the process of genetic algorithm is represented in Fig 4. The initial population of laminate is generated at random. In case of using Monte Carlo simulation, the random numbers for in-plane loads and material properties are generated to calculate fitness function. For each laminate, the minimum buckling load is found by convex analysis or Monte Carlo simulation.

The best laminate of each generation is always copied into the next generation, which is called an elitist strategy. Selection is executed by a linear search through a roulette wheel slots weighted in proportion to fitness value of each laminate. After selection, single-point crossover is conducted with a probability value of P_c. When crossover is not conducted, the first parent is copied into the next generation. Mutation is applied to the chromosome with a probability of P_m, except for the best chromosome of the previous generation.

To represent the ply angles as genes, trinary numbers are used with each gene having a value of 0, 1, or 2. Basically, the number 0 corresponds to a 0° ply and the number 2 corresponds to a 90° ply. The first (outmost), third, fifth, etc. occurrences of the number 1 correspond to a 45° ply while even-number occurrences correspond to a -45° ply.

Two constraints are implemented via a recessive-gene-like repair strategy. The key concept of the strategy is to repair the laminate without changing the chromosome. The repair system is briefly explained subsequently.

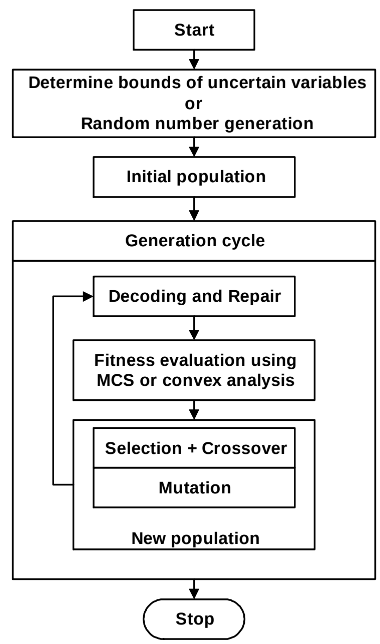


Fig 4: Flowchart of the genetic algorithm.

NUMERICAL EXAMPLES

Comparison of buckling loads

To demonstrate the effect of uncertainty, buckling loads are obtained by various methods described earlier. It is considered a symmetric cross-ply laminate with 16 layers subject to uniaxial compression. For the material properties, the average nominal values are given in Table 1 and the maximum deviation is assumed as ±10%. For normal distribution, the standard deviation is determined from multiplying mean value by given deviation. Fig 5 shows the buckling loads as varying aspect ratio *a/b*. According to expectation, smaller buckling load are obtained when the uncertainty considered. And convex modeling gives the most conservative results compared with interval analysis and probabilistic approach.

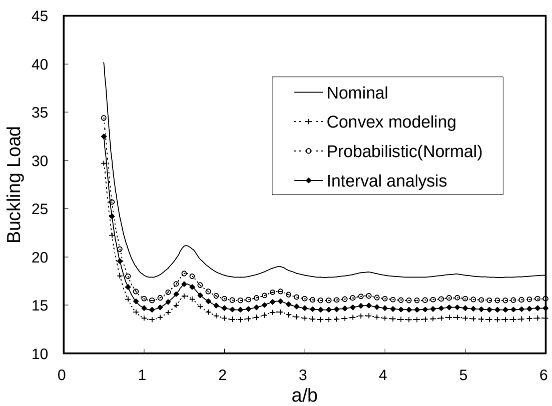


Fig 5: Comparison of buckling loads.

Optimization results

A rectangular laminates with various aspect ratios are considered for optimization. The parameters used for the genetic algorithm are shown in Table 2. The population number is 20 and the upper limit of generation is 100. Throughout the calculations, composite laminate have thickness H = 0.1 m, b = 1 m, and 16 plies.

E_L	181 GPa
E_T	10.3 GPa
G_{LT}	7.17 Gpa
ν_{LT}	0.28

Table 1: Nominal values of uncertain properties

Parameters	Value
Chromosome length	8
Upper limit of generation	100
Population size	20
Probability of mutation	0.0 - 0.2
Probability of crossover	0.8 – 1.0

Table 2: Parameters of genetic algorithm

In order to check the convergence of the algorithm, the optimal solutions are found as varying generation numbers. Fig 6 shows the iteration history of genetic search in which deterministic, convex, interval, and MCS methods are used for buckling load calculation. The maximum deviation is assumed to show ±10% from mean value for each uncertain value. In all cases, iterations more than 20 are sufficient for the convergence. In all numerical results, buckling loads are the value normalized by Eq. (28).

$$\bar{\lambda} = \frac{\lambda_b a^2}{E_T h^3} \tag{28}$$

To investigate the effect of considering the uncertainty on the optimal results, the optimal stacking sequences are obtained for various aspect ratios. In Table 3 ~ Table 5, the results from deterministic case, convex modeling, interval analysis and MCS are shown. In Table 6, uniform distribution with following bounds is used for in-plane loads.

$$0 \le N_x, N_y \le 1\,(kN) \tag{29}$$

It is observed that deterministic search and convex search yield same results for aspect ratio a/b = 0.8 ~ 1.4. But as the ratio become small or large, the optimal stacking sequences of deterministic case and uncertain cases are quite different. The results indicate that the effect of uncertainty on optimization process is more distinct for small and large aspect ratios. In case of uniform distribution in which the degree of uncertainty much higher, the optimal solution is different from that of deterministic case even for square plate. These results show that the optimal results can be totally different from deterministic results when uncertainties exist in design parameters. Comparing the optimal buckling loads from each case, the value from deterministic case is much higher than those from cases uncertainty considered. If the laminate is designed with no uncertainty considered, it would carry a lower buckling load than the one expected to sustain.

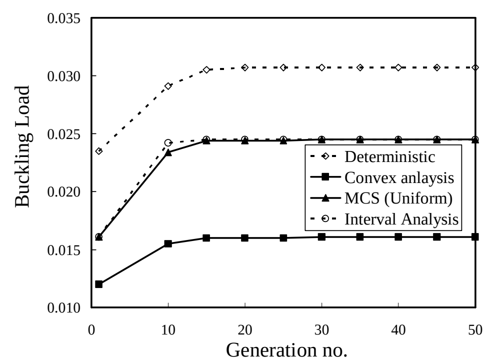


Fig 6: Iteration history of genetic search.

a/b	Optimal stacking sequence	$\bar{\lambda}_{\max}$
0.2	$[0/45/0/0/-45/0/0/90]_s$	0.0217
0.4	$[0/0/45/-45/0/45/0/-45]_s$	0.0217
0.6	$[0/0/45/0/-45/0/45/-45]_s$	0.0219
0.8	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0247
1.0	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0307
1.2	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0369
1.4	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0435
1.6	$[90/90/45/90/-45/90/90/0]_s$	0.0559
1.8	$[90/90/45/90/-45/90/90/0]_s$	0.0711
2.0	$[90/90/45/90/-45/45/-45/90]_s$	0.0875

Table 3: Results from deterministic design

a/b	Optimal stacking sequence	$\bar{\lambda}_{max}$
0.2	$[0/45/0/0/0/0/-45/0]_s$	0.0114
0.4	$[0/0/45/-45/0/45/0/-45]_s$	0.0115
0.6	$[0/0/45/0/-45/0/0/90]_s$	0.0118
0.8	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0129
1.0	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0161
1.2	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0194
1.4	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0228
1.6	$[90/90/90/45/-45/45/-45/0]_s$	0.0303
1.8	$[90/90/45/90/-45/90/45/-45]_s$	0.0378
2.0	$[90/90/45/90/-45/45/-45/90]_s$	0.0468

Table 4: Results from convex analysis

a/b	Optimal stacking sequence	$\bar{\lambda}_{max}$
0.2	$[0/45/0/0/-45/0/0/90]_s$	0.0171
0.4	$[0/0/45/-45/0/45/-45/0]_s$	0.0172
0.6	$[0/0/45/0/-45/0/45/-45]_s$	0.0173
0.8	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0197
1.0	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0245
1.2	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0294
1.4	$[45/-45/45/-45/45/-45/45/-45]_s$	0.0346
1.6	$[90/45/90/90/90/90/-45/90]_s$	0.0442
1.8	$[90/45/90/90/90/-45/90/90]_s$	0.0563
2.0	$[90/90/45/-45/90/90/45/-45]_s$	0.0693

Table 5: Results from interval analysis

a/b	Optimal stacking sequence	$\bar{\lambda}_{max}$
0.2	$[0/0/45/0/-45/45/-45/0]_s$	0.0094
0.4	$[0/45/0/0/-45/0/45/-45]_s$	0.0094
0.6	$[0/0/45/0/-45/0/0/90]_s$	0.0095
0.8	$[45/-45/45/0/0/-45/45/-45]_s$	0.0103
1.0	$[45/-45/90/45/-45/90/45/-45]_s$	0.0121
1.2	$[45/-45/45/90/90/-45/90/0]_s$	0.0153
1.4	$[45/-45/45/90/-45/90/90/0]_s$	0.0187
1.6	$[90/45/90/90/90/90/-45/90]_s$	0.0242
1.8	$[90/90/45/90/-45/90/45/-45]_s$	0.0307
2.0	$[90/90/45/-45/90/90/90/0]_s$	0.0380

Table 6: Results from MCS

Next the optimal values are compared as varying degree of uncertainty. In the calculations, the uncertainty of material properties is fixed as ±10% deviation because the bounds of material property are relatively small. For normal distribution, the standard deviation is determined from multiplying mean value by given deviation. In Fig 7, the optimal buckling loads are plotted for increasing uncertainties. The results from beta and uniform distribution show almost same values but the results from normal distribution deviate from others as uncertainty increases. Theoretically, the bounds of normal distribution is from -∞ to ∞, thus more extreme condition can be considered in the calculation. In case of convex analysis, somewhat unrealistic buckling loads are obtained for high degree of uncertainty. Because the linearization of objective function can only be meaningful for proper degree of uncertainty. So, despite of its computational efficiency and ease of modeling, care should be taken when choosing convex analysis for modeling the uncertainty. Interval analysis gives approximately the middle values compared with the results by deterministic and convex modeling approaches. The optimal buckling loads vary linearly as deviation increases, and generally they are more conservative than those from probabilistic approach. Considering a computational cost for Monte Carlo simulation and an overestimation by convex modeling, interval analysis seems to be efficient way for optimization where numbers of objective calculations are required.

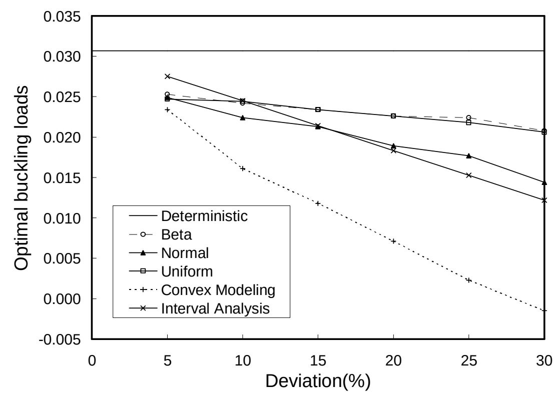


Fig 7: Optimal buckling loads for various degree of uncertainty.

CONCLUSION

Optimal designs of laminated plate were determined with uncertain in-plane loads and material properties. To consider the uncertainty, convex modeling, interval analysis and Monte Carlo simulation were adopted in optimization process. The genetic algorithm works well in stacking sequence optimizations for maximizing the buckling loads. Numerical results indicated that considering the uncertainty is highly required for reliable and safe design.

The methodology presented in this paper can be used as a powerful tool for robust layup design of laminates when uncertainty exists. Also, it can be extended to the problem in which various kind of uncertainty are present. This study is in the progress.

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