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Losses of plasmon surface waves on metallic grating

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Abstract. Diffraction and absorption losses of plasmon surface waves (PSW) propagating along a metallic grating are investigated numerically as a function of groove depth. A periodicity of diffraction losses is found to exist. The energy flow distribution (EFD) above and inside the grooves is calculated and a similarity between the PSW on shallow and deep gratings is established above the grooves, while inside the grooves of deep gratings totally hidden curls in EFD are found to form.

1. Introduction

Recently it has been discovered [1] that a close connection exists between different types of phenomena on metallic gratings: plasmon surface waves (PSW) excitation, non-Littrow perfect blazing [2] and total absorption of light at grazing incidence [3] can all be attributed to certain parts of the same curve; the trajectory describing pole/zero of the scattering matrix as a function of groove depth h in the complex plane of $k\alpha$ ($k\alpha$ is the projection of the incident wavevector $\mathbf{k}$ on the grating plane and $k=|\mathbf{k}|=2\pi/\lambda$, where λ is the wavelength). A detailed tracing of this trajectory is shown in figure 1.

It is well known that a pole of the scattering matrix corresponds to a solution of the homogeneous problem [4]. For a plane metal–air boundary ($h=0$) such a solution, referred to as PSW, exists for TM polarization and has a complex propagation constant:

$$\alpha^{\mathrm{P}}=\frac{n_{\mathrm{M}}}{(1+n_{\mathrm{M}}^2)^{1/2}}, \tag{1}$$

where n_{M} is the complex refractive index of the substrate. For highly conducting metals $\mathrm{Re}\,(\alpha^{\mathrm{P}})>1$ and $\mathrm{Im}\,(\alpha^{\mathrm{P}})>0$, the latter corresponding to the energy absorbed in the metal as the PSW propagates along the interface.

As the periodic modulation is introduced ($h\not\equiv 0$), the PSW may be coupled to a propagating diffraction order(s) in the upper medium provided a suitable wavelength to period ratio λ/d is chosen. Radiation losses appear as a consequence of this and $\mathrm{Im}\,(\alpha^{\mathrm{P}})$ grows rather rapidly (for the results presented in figure 1 $d=0{\cdot}5\,\mu\mathrm{m}$ and $\lambda=0{\cdot}6328\,\mu\mathrm{m}$).

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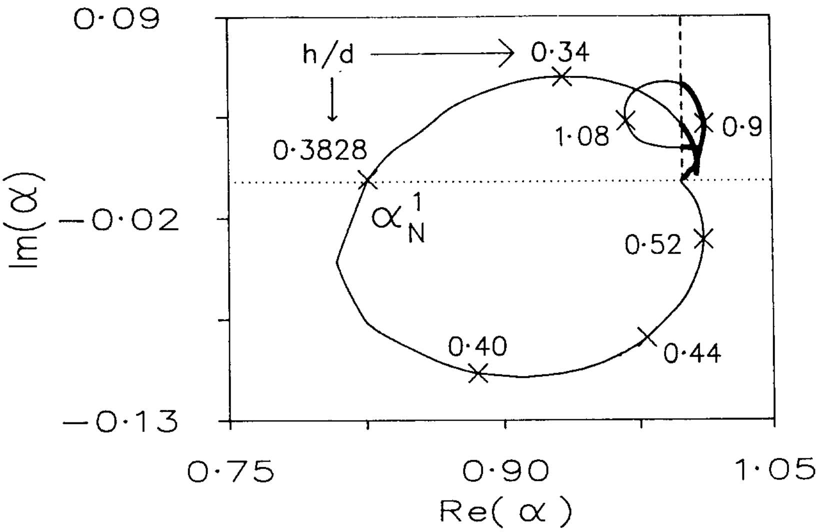
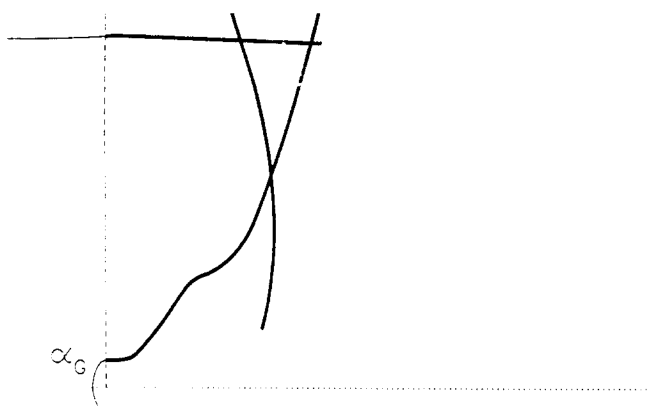


(a)



(b)

Figure 1. (a) Complex α-plane trajectory of pole/zero of the scattering matrix for a sinusoidal aluminium grating with period $d=0{\cdot}5\,\mu m$ at light wavelength $\lambda=0{\cdot}6328\,\mu m$. (b) ×15 magnification of (a) in the vicinity of the point $\alpha=1$ (reference [6]).

Further increase of h turns the trajectory towards the line $\mathrm{Re}(\alpha)=1$. When the pole crosses this line, it is transformed into a zero of the zeroth reflection order (ZZRO) [4, 5]. As the grating becomes deeper and deeper, the trajectory of ZZRO crosses the real α-axis at two points. The first one, α_N^1, corresponds to the non-Littrow perfect blazing [2] and the second, α_G, to the grazing total absorption of light [3]. Further growth of h causes a closing of the first loop in the trajectory and a formation of a second loop above the real axis. When $\mathrm{Re}(\alpha^p)$ becomes greater than unity, ZZRO is transformed back into a pole and a new branch of the dispersion relation solution appears.

As a consequence of this tracing, not only is a connection between some already known phenomena established, but it also becomes possible to predict a new anomaly—total absorption of light [6] and field enhancement [7] in deep and very deep gratings. What remained was to find a reason (or physical interpretation) for the existence of these loops in the pole/zero trajectory. The aim of this paper is to show that the loops are due to the quasiperiodicity of the radiation losses as a function of the groove depth. As a first step it is necessary to distinguish between the two types of losses that exist for metallic gratings, namely diffraction (radiation) and absorption losses.

2. Separation of losses

As the PSW propagates along the grating surface, part of the energy it carries can be radiated into the upper lossless medium and the other part is absorbed in the metallic substrate. To separate these two types of losses it is necessary to calculate

(*a*) the energy flow passing through one period of the grating surface (I_a in figure 2), and
(*b*) the energy flow going away from the grating (I_d in figure 2).

To attain accuracy, it is worth noting that a solution of the homogeneous rather than the inhomogeneous problem is required, i.e. diffraction orders amplitudes have to be calculated as a function of one of the orders (let us say the 0th order) without any incident wave. This situation implies the existence of a pole of the scattering matrix. Consequently the parameters of the system must ensure that such a solutuion of the homogeneous problem exists. Once this requirement is fulfilled, we may set the amplitude of the zeroth diffracted order equal to unity and calculate the others. Next, normalization to unit energy flow through an infinite cross-section is necessary.

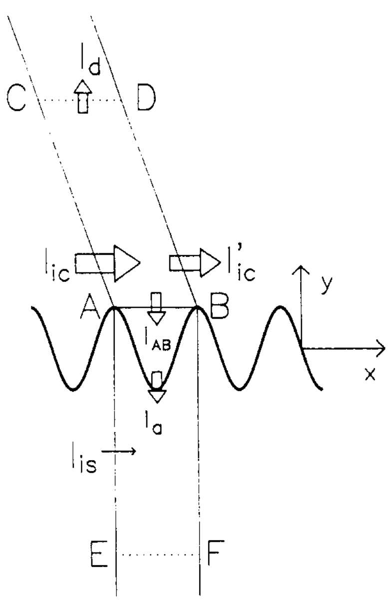


Figure 2. General energy flow scheme for a plasmon surface wave propagating along the grating surface.

Let us consider the energy balance in the region ACDBFE of figure 2, including one groove. Lines AC and BD are taken to be parallel to the radiation direction. The total incident energy flow I_i reaching the region CAE along the x-direction has two parts: I_{ic} is the flow through line AC in the upper lossless medium and I_{is} describes the incident energy flow in the absorbing substrate.

Numerical results, however, show that I_{is} is 10^3–10^5 times smaller than I_{ic} and in further discussions we shall ignore the infleunce of I_{is}. The incident energy flow I_{ic} penetrates the region examined through the line AC. One part of it

$$I'_{ic} = I_{ic} \exp\left[-2k \operatorname{Im}(\alpha^p) d\right]$$

is then simply transmitted over the next groove through the line BD. Another part I_d propagates through the line CD away from the grating. The third part I_a crosses the grating surface and is absorbed in the metallic substrate. The energy flow I_a is equal to the energy flow I_{AB} through a straight line connecting the tops of the grooves, since the upper medium is lossless. As AC$\rightarrow\infty$, then I_d tends towards a constant value corresponding to the energy radiated by one groove. As there are no losses inside ABCD, the following relation exists:

$$I_{ic} - I'_{ic} = I_d + I_a = I_{ic}\{1 - \exp\left[-2k \operatorname{Im}(\alpha^p) d\right]\}.$$

One can define relative diffraction and absorption losses as two ratios:

$$r_d = \frac{I_d}{I_{ic} - I'_{ic}} = \frac{I_d}{I_d + I_a}, \tag{2}$$

$$r_a = \frac{I_a}{I_{ic} - I'_{ic}} = \frac{I_a}{I_d + I_a}. \tag{3}$$

3. Diffraction (radiation) losses

A numerical code based on the differential formalism of Chandezon *et al.* [8] was used according to the scheme of section 2. An aluminium grating (refractive index $n_M = 1{\cdot}378 + i7{\cdot}616$) with a sinusoidal profile was considered. Figure 3 presents the

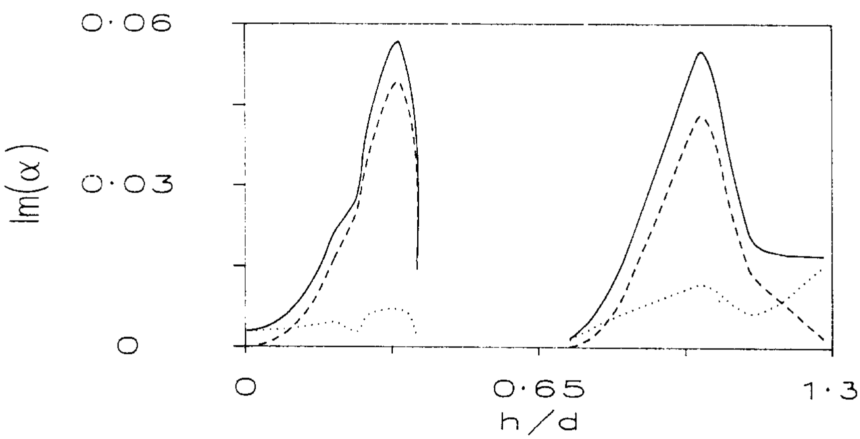


Figure 3. Groove depth dependence of the imaginary part of α^p: solid line—total imaginary part, dashed line—diffraction part, and dotted line—absorption part.

groove depth dependence of the imaginary part of the pole α^p and the contributions of the diffraction $\alpha^d = r_d \,\mathrm{Im}\,(\alpha^p)$ and absorption $\alpha^a = r_a \,\mathrm{Im}\,(\alpha^p)$ parts. There is a gap in their groove depth dependences, as we are only dealing with the case of $\mathrm{Im}\,(\alpha) > 0$.

It must be pointed out that $\mathrm{Im}\,(\alpha^p)$ is proportional to the losses: $2k\,\mathrm{Im}\,(\alpha^p)$ is equal to the decay constant of the plasmon. Hence in the following we speak of 'losses' rather than 'imaginary parts'.

Absorption losses are almost independent of h. To be more precise, their contribution to the groove depth dependence of total losses is quite small. A quasiperiodicity of $\mathrm{Im}\,[\alpha^p(h)]$ and $\alpha^d(h)$ is obvious and their minima appear almost for the same groove depth values as the zeros of the -1st order diffraction efficiency in the Littrow mounting (figure 4). Moreover, when the -1st order efficiency is zero fot the Littrow mounting it is negligible in the entire region of angles of incidence (the so-called 'antiblazing' of gratings [9]). As can be concluded from the results presented in figure 4, this general property is also valid for radiation of the PSW propagating along a grating.

As is shown in the preceding paper [10], the existence of such an equivalence between the properties of the flat surface and of deep gratings is due to a successive formation of one or more energy flow curls in each groove when the grating depth increases. When these curls are completely hidden inside the grooves the energy flow distribution above the grooves becomes similar to the distribution above the flat surface. Figures 5 and 6 represent the pictures of energy flow lines (locally tangential to the direction of the Poynting vector) for two different values of groove depth: 0·058 μm and 0·395 μm, respectively. In these cases the PSW propagation constants α^p have almost equal imaginary parts. The energy flow distribution above the grooves is almost identical for the two gratings: near the surface the lines are almost parallel to the grating corresponding to the physical fact that the PSW propagates

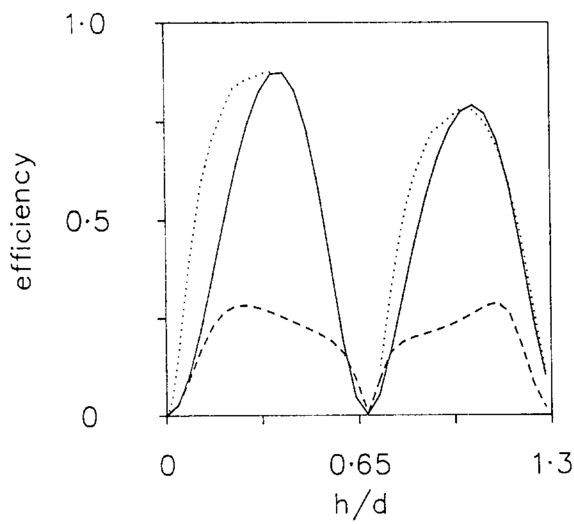


Figure 4. Groove depth dependence of the -1st order diffraction characteristics: solid line—Littrow mounting efficiency ($\alpha = 0{\cdot}6328$), dashed line—grazing incidence efficiency ($\alpha = 0{\cdot}99$), dotted line—relative diffraction losses (r_d).

from left to right. A small number of lines finish on the surface, leading to absorption losses. Some of the upper lines turn away from the surface, corresponding to radiation losses. As a result of both types of loss, the guided-wave amplitude decreases as it propagates along the grating; in terms of energy flow this fact is represented by a decrease in the density of the lines.

The main difference between figures 5 and 6 can be found inside the grooves since curls exist in each groove of a deep grating. Although some of the flow lines reach the surface, there are closed curves in the deep grooves that separate the bottoms of the grooves from the energy flow above the tops. This fact can explain why absorption losses for deep gratings may take lower values than for shallow gratings and even for flat surfaces (figure 3).

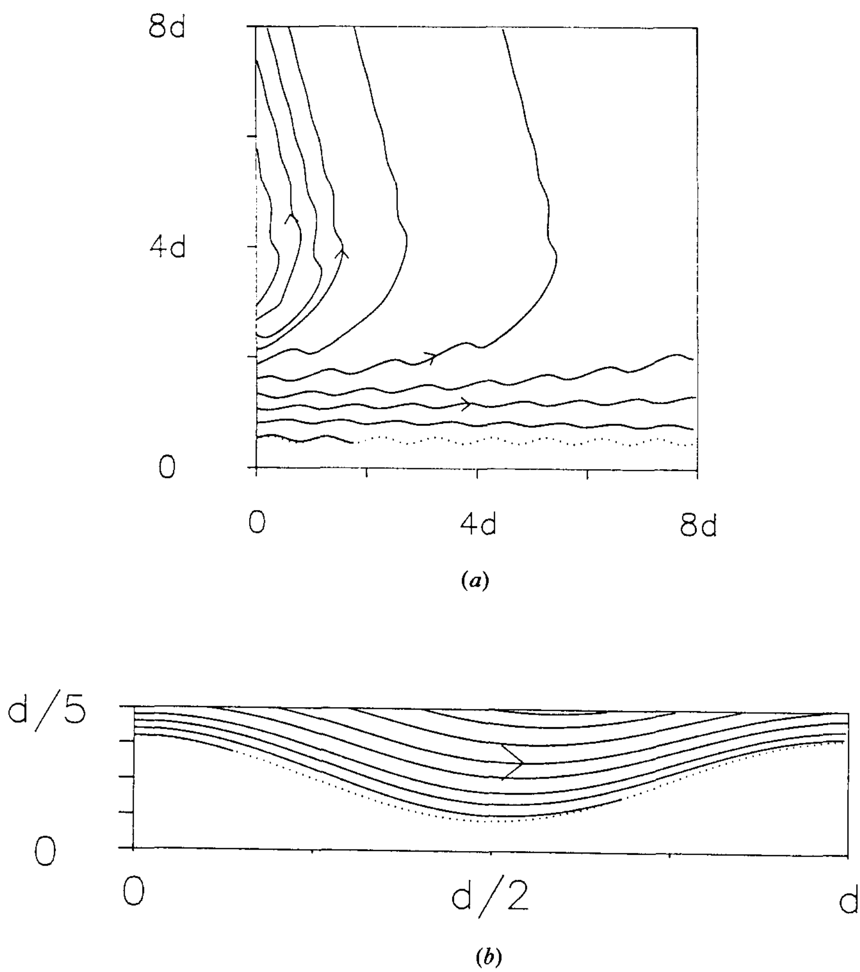
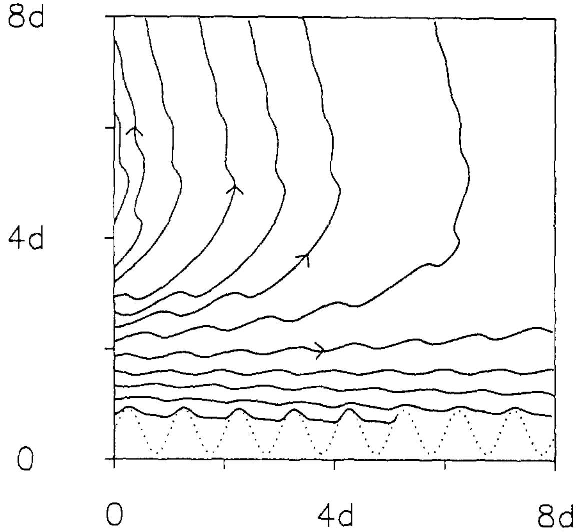
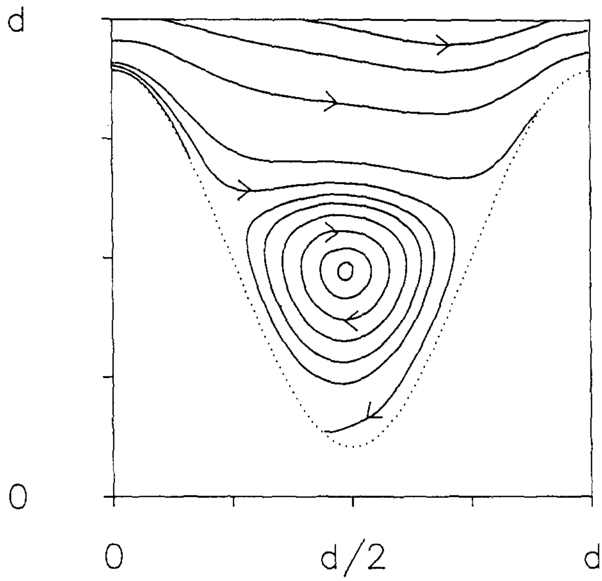


Figure 5. Energy flow distribution above the surface of a shallow grating of period 0·5 μm and groove depth 0·058 μm: (a) above the grating, (b) inside the groove.



(*a*)



(*b*)

Figure 6. Energy flow distribution above the surface of a deep grating of period 0·5 μm and groove depth 0·395 μm: (*a*) above the grating, (*b*) inside the groove.

4. Absorption losses

Let us first consider shallow gratings (figures 5 and 7). For a flat surface the direction of energy flow is almost parallel to the surface, consequently there are only small losses. By increasing h, the flow lines are pulled up by the groove tops and above them the power density of the electromagnetic field increases. Inside the grooves the lines are spaced out and the field density decreases. Nevertheless, the value of the normalized average field density over one period increases with h as does the grating surface area. Comparing the behaviour of the field density and the grating surface area with absorption losses, the following conclusion can be drawn: the increase of absorption losses is similar to the increase of the average field density and is almost an order of magnitude greater than the growth in the area of the absorbing surface. This has been confirmed in the case of deep gratings (figure 8): A strong

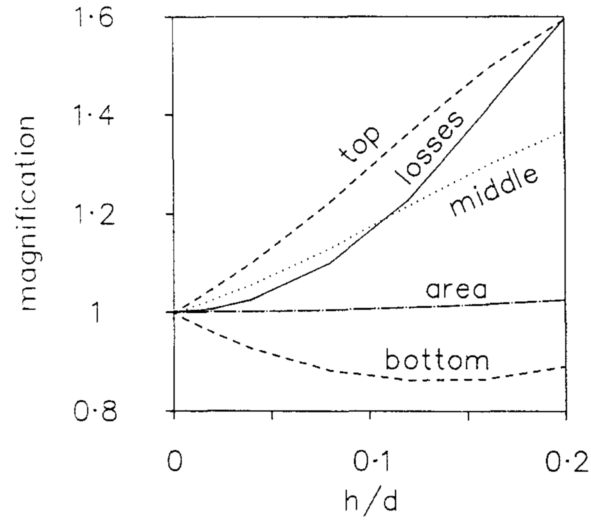


Figure 7. Increase of absorption losses as a function of groove depth compared with the change of field density on the top, in the middle and at the bottom of the groove: shallow grating. Increase of grating surface area is also shown.

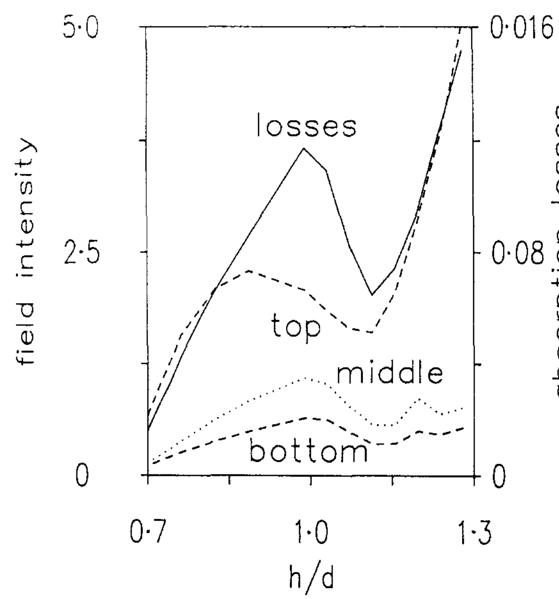


Figure 8. Absorption losses compared with field density values on the top, in the middle and at the bottom of the groove as a function of modulation depth for a deep grating.

connection and good correspondence between absorption losses and field density behaviour on the top, at the bottom and in the middle of a groove can be seen in figure 8. Moreover, under these conditions the absence of a strong connection between absorption losses and grating surface area becomes evident—deep gratings (e.g. $h/d = 0{\cdot}72$) can have only one half the absorption of a flat surface (figure 3).

Acknowledgement

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