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SIMULATION OF RADIATION HEAT TRANSFER OF THREE DIMENSIONAL PARTICIPATING MEDIA BY RADIATIVE EXCHANGE METHOD

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ABSTRACT

This paper describes the theoretical bases of the Radiative Exchange Method, a new numerical method for simulating radiation heat transfer. By considering radiative interaction between all points of the geometry and solving the radiation balance equation in a mesh structure coarser than the structure used in computational fluid flow calculation, this method is able to simulate radiative heat transfer in arbitrary 3D space with absorbing, emitting and scattering media surrounded by emitting, absorbing and reflecting surfaces. A new concept is introduced, that of the exchange factors between the different elements that are necessary for completing the radiative balance equation set. Using this method leads to a set of algebraic equations for the radiative outgoing power from each coarse cell being produced and the result of this set of equations was then used to calculate the volumetric radiative source term in the fine cell structure. The formulation of the exchange factor for a three-dimensional state and also a mesh size analysis that was conducted to optimize the accuracy and runtime are presented. The results of this model to simulate typical 3D furnace shape geometry, is verified by comparison with those of other numerical methods.

INTRODUCTION

Due to the nature of radiative heat transfer and its high dependence on temperature, this mechanism is the major contributor to heat transfer from the combustion space in industrial combustion equipment such as boiler furnaces. Thus, the radiative heat transfer has a highlighted role in the efficiency and operation of this kind of tools. The current improvements in computational power provided an increasing incentive for the researchers to develop the numerical software and tools for simulating radiative heat transfer in a large-scale industrial system where no analytical solutions existed and experimental measurement was limited because of the expense. In addition to industrial interest, the accurate numerical tools for simulating radiation heat transfer are useful for predicting the effect of dust, cloud and carbon dioxide, and other participating gases on the global environment. In most of Industrial applications, radiative heat transfer is accompanied by combustion and turbulent reactive flow. Therefore, the ideal radiation model should correspond with other numerical methods in simulating flow and combustion.

The ray tracing methods such as the Monte Carlo Method [1, 2], which is based on following energy bundles by using the concept of random numbers until they are absorbed or exit from the system, have seldom been used. This is due to their solution algorithm, which is very different from the solution algorithm used in the simulating of the reactive flow, and also because it is computationally demanding. The non ray tracing methods which are based on solving integro-differential equation of radiation in the discretised geometry cannot provide a high level of accuracy because there are several assumptions in their algorithms for making the solution procedure possible and in most of these solutions the radiative interaction is considered between neighbored cells whereas the radiation interaction exists between all points in the space.

Hottel and Cohen [3] represented an accurate method for analysis of radiative heat transfer, i.e. the radiative zone method. However, even in this method the radiative interaction between all points in the space and the surrounding walls is taking into account, there are still several problems in this method, which are not solved very well yet, such as the singularity problem in the calculation of the direct exchange area, and long computational time.

In the zone method [3-5], the amount of radiation exchange between each pair of the zones is defined by using the special coefficient named “Direct Exchange Area”. This coefficient between a source cell“ i ”and a destination zone “ j ” is defined as the share of emitting radiative heat power from a source zone“ i ” that is absorbed in “ j ” (if “ j ” is a volume zone) or reached to “ j ” (if “ j ” is a surface zone). The amount of net radiative heat transfer for each zone is defined by the other coefficient named heat flux area that can be calculated from direct exchange area. The net radiative heat transfer will be

added to the overall energy balance in each zone. Finally, by solving these equations and inserting the effect of the boundary conditions on the wall, the temperature of all the zones is obtained.

Bordbar and Hyppänen [6] have introduced a new concept of non dimensional exchange factor defined as a ratio between two radiative powers. Using this new concept, they simulated radiative heat transfer in a two dimensional participating media surrounded by gray walls.

Maruyama and Guo [7, 8] developed the radiation element method that was an extension of the zone method. Their proposed method is used as a generalized numerical method for simulating radiation transfer in participating media surrounded by specular and/or diffuse surfaces.

The "Radiative Exchange Method" presented in this article has some similarities with the zone method. In both methods, the radiation heat transfer is solved in a mesh structure that is much coarser than the mesh structure normally used for solving the flow field whereas the definition of exchange factor in this method is different from the definition of direct exchange area in the zone method. In contrary to the zone method, in the radiative exchange method, there is no need to calculate "total exchange area" and "heat flux area" coefficients.

In the following sections the theoretical bases of the radiative exchange method is described.

THEORY

In the Radiative Exchange Method, the media and its surrounding surfaces are divided into N isothermal volume cells and M isothermal surface cells. The radiation heat transfer is supposed to occur between all the cells, whether they are neighbor cells or not. The new exchange factor content is used to produce the radiative balance equation for each cell.

The following basic assumptions are considered in the preliminary development of this approach:

1) The size of the system is finite.
2) The temperature, refractive index, emissivity and scattering coefficient are constant over the volume integration elements used in calculation of exchange factor between the coarse cells.
3) The temperature, emissivity and reflection coefficient are constant over the surface integration elements used in calculation of exchange factor between the coarse cells.
4) The reflectivity and emissivity of the surface elements are independent of the incident angle.
5) The solid surfaces have gray behavior.
6) Scattering in participating media is isotropic.
7) Reflection in the surrounded surfaces is isotropic.

In the first step of the method, the participating media and its surrounding boundaries are decomposed into N coarse volume cells and M coarse surface cells. The isothermal integration elements are made inside the coarse cells and an initial guess for temperature field is considered for them. Then, the amount of exchange factors between the coarse cells is calculated by numerical integration between isothermal integration elements.

In the next step, a set of algebraic equations are obtained using the calculated exchange factors in the writing of radiative heat balance equations for each coarse cell. These calculated values are used to find the radiative source term in the fine mesh structure employed in the fluid flow calculation. Figure 1 shows a schematic of the numerical procedure for the radiative exchange method.

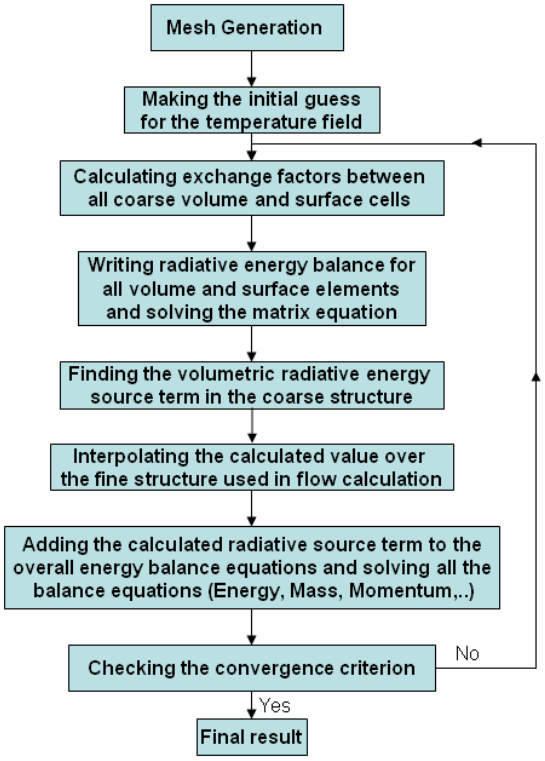

Figure 1. Schematic of numerical procedure of the Radiative Exchange Method.

The amount of outgoing radiative power from each volume cell is the summation of emission power and the scattering of the fluid molecules inside the volume cell and the amount of scattering power is related to the amount of incoming radiative power from other cells in the system. Therefore, the exchange factor from a cell "j" to the volume cell "i", EF_{ji}, is defined as the ratio of the amount of outgoing radiative power from cell "j", which is absorbed or scattered in the volume cell "i" to the amount of outgoing radiative power from cell "j". Thus, the following equation can be written for the outgoing radiative power from each volume cell "i".

$$q_{out,i} = 4k_{e,i}V_i\sigma T_i^4 + k_{S,i}\left[\sum_{\substack{j=1 \\ j\neq i}}^{N} EF_{ji}\times q_{out,j} + \sum_{j=1}^{M} EF_{ji}\times q_{out,j}\right] \qquad (1)$$

In the same way for the surface coarse cells, the amount of outgoing radiative power from each surface cell is summation of the emission and reflection powers. The amount of reflection

power is related to the incoming radiative power from other cells in the system. Therefore, the exchange factor from a cell "j" to the surface cell "i", EF_{ji}, is defined as the ratio of the amount of outgoing radiative power from a cell "j" which is reach to surface cell "i" to the amount of outgoing radiative power from cell "j". Thus, the following equation can be written for the outgoing radiative power from each surface coarse cell "i".

$$q_{out,i} = k_{e,i} A_i \sigma T_i^4 + \rho_i \sum_{j=1}^{N+M} EF_{ji} \times q_{out,j} \tag{2}$$

Thus, by calculating the exchange factor between all coarse cells and using them in Eq. (1) for the coarse volume cells and Eq. (2) for the coarse surface cells, a system of linear equations for calculating outgoing radiative powers from all the coarse surface and volume cells is achieved as follows:

$$[A]_{(M+N)\times(M+N)} [Outgoing]_{(M+N)\times 1} = [Emission]_{(M+N)\times 1} \tag{3}$$

If we assume that the temperature is known for all the coarse surface cells, the general form of the coefficient matrix "A" is as shown in Fig.2. As shown in figure 2, the matrix coefficient consists of four different parts that represent the radiative interaction between the surface to surface, volume to surface, surface to volume and volume to volume coarse cells.

The known matrix on the right hand side of Eq. (3) consists of the emission power of the surface and the volume cell in the following forms:

$$emission_S = k_e A \sigma T^4 \tag{4}$$

$$emission_V = 4 k_e V \sigma T^4 \tag{5}$$

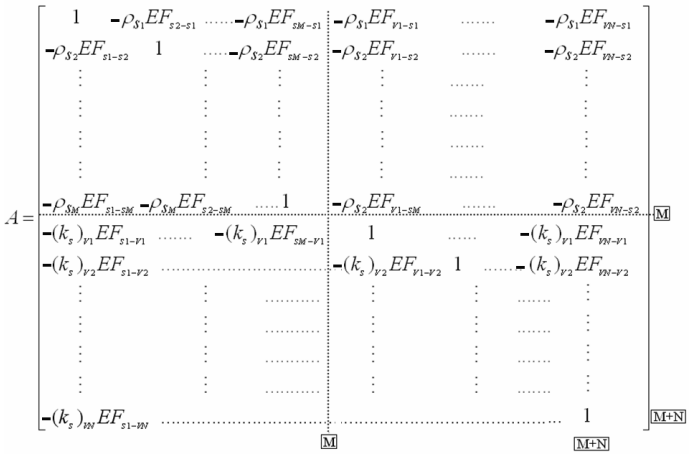

Figure 2. The general form of the coefficient matrix where the boundary condition is "Temperature Known" for all the coarse surface cells.

EXCHANGE FACTOR CONTENT

Based on the theory of the method explained in the previous section, for making the radiative balance equations, we need to calculate exchange factors between each pair of cells in the system. Therefore, the amount of exchange factor between the each pair of radiative elements when the destination element is a volume cell is defined as the ratio of the amount of the radiative heat power lost in the destination volume cell by scattering and absorption of the gas molecules to the amount of outgoing radiative heat power from the source element.

For the case in which the destination element is a surface cell, the amount of exchange factor between the pair of radiative elements is defined as the ratio of the amount of the radiative heat power which reaches the destination surface cell to the amount of outgoing radiative heat power from the source element.

If the source radiative element is a surface cell, the amount of radiative outgoing heat power is the summation of emission and reflection of the surface. If the radiative source element is a volume cell, the radiative outgoing is the summation of emission and scattering power minus the share that is absorbed or scattered within the source volume cell itself.

The ratio of the share of scattering and emission power of gas molecules in each volume cell that is extinct in that cell itself to the amount of scattering and emission power of gas molecules within that volume cell is defined as the "Self-Extinction".

Based on the above mentioned definitions, the following equations are given for the exchange factor between different elements:

$$EF_{S_i-S_j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} e^{-\beta_{ij} S} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_i dA_j, \tag{a}$$

$$EF_{V_i-S_j} = \frac{1}{1 - SE_{V_i}} \left[\frac{1}{4\beta_i V_i} \int_{V_i} \int_{A_j} e^{-\beta_{ij} S} \frac{\beta_i \cos\theta_j}{\pi S^2} dV_i dA_j \right], \tag{b}$$

$$EF_{S_i-V_j} = \frac{1}{A_i} \int_{A_i} \int_{V_j} e^{-\beta_{ij} S} \frac{\beta_j \cos\theta_i}{\pi S^2} dA_i dV_j, \tag{c}$$

$$EF_{V_i-V_j} = \frac{1}{1 - SE_{V_i}} \left[\frac{1}{4\beta_i V_i} \int_{V_i} \int_{V_j} e^{-\beta_{ij} S} \frac{\beta_i \beta_j}{\pi S^2} dV_i dV_j \right], \tag{d}$$

(6)

In these equations, SE_V represents the self-extinction within the volume cells, and EF represents the exchange factor and, $\beta = k_a + k_s$ represents the extinction coefficient of the volume cells.

The integrals in Eq. (6) should be calculated numerically, and the size of the integration should be small enough to be considered as the radiative differential elements. The size of the integration elements has a key role in the accuracy and computational time of the exchange factor calculations. Based on the center to center distance of two integration elements and the optical thickness of the media within this distance, the integration elements should see each other as the differential elements. Thus, one single ray can approximate the radiation transfer between two cells. The radiation balance for this single ray between two cell centers is written as:

$$\frac{dI}{dS} = -\beta I \Rightarrow \frac{I_n}{I_0} = e^{-\int_s \beta dS} \tag{7}$$

Therefore, the amount of attenuated radiation between two cells is $e^{-\int_{\bar{s}} \beta dS}$. In practice, the amount of attenuation is calculated by dividing the path between the two cells into the equal small pieces and in each piece, the amount of extinction coefficient is assumed to be constant, and the amount of attenuation can be calculated by

$$Attenuation = e^{-\beta_{ij} S} \quad \text{where} \quad \beta_{ij} = \frac{1}{S}\int_{\bar{s}} \beta dS \tag{8}$$

β_{ij} is the average of the extinction coefficient between the centers of the two cells. Figure 3 illustrates a two dimensional schematic of numerical procedure for calculating attenuation between two integration elements in a calculation of exchange factors between coarse cells.

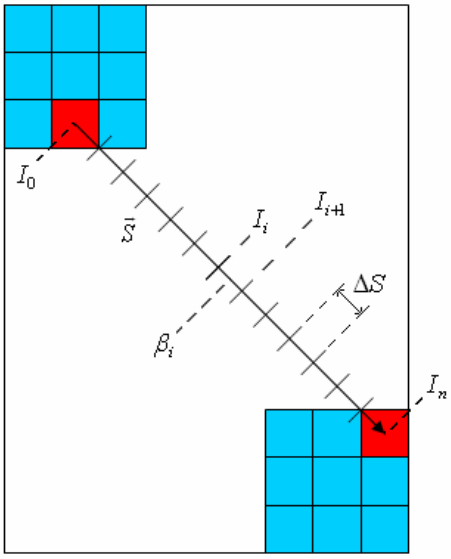

Figure 3: Numerical calculation of attenuation between two integration elements in the **calculation of exchange factors between two coarse volume cells in** a **two-dimensional scheme.**

By assuming that integration elements see each other as the differential elements, the amount of exchange factors between integration elements is driven as follows:

$$EF_{IE_{Si}-IE_{Sj}} = e^{-\beta_{ij} S} \frac{\cos\theta_{IE_{Si}} \cos\theta_{IE_{Sj}}}{\pi S^2} A_{IE_{Sj}}, \qquad \text{(a)}$$

$$EF_{IE_{Vi}-IE_{Sj}} = \frac{1}{4} e^{-\beta_{ij} S} \frac{\cos\theta_{IE_{Sj}}}{\pi S^2} A_{IE_{Sj}}, \qquad \text{(b)}$$

$$EF_{IE_{Si}-IE_{Vj}} = e^{-\beta_{ij} S} \frac{\beta_{IE_{Vj}} \cos\theta_{IE_{Si}}}{\pi S^2} V_{IE_{Vj}}, \qquad \text{(c)} \tag{10}$$

$$EF_{IE_{Vi}-IE_{Vj}} = \frac{1}{4} e^{-\beta_{ij} S} \frac{\beta_{IE_{Vi}} \beta_{IM_{Vj}}}{\pi S^2} V_{IM_{Vj}}, \qquad \text{(d)}$$

The amount of exchange factors between coarse cells is calculated by using the exchange factors between integration elements. Based on the definition of exchange factor as a ratio between two radiative powers, the amount of exchange factor for the coarse cells is related to the exchange factors between integration elements by the following equation:

$$EF_{C_i-C_j} = \frac{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(EF_{IE_i-IE_j} q_{out,IE_i})}{\sum_{i=1}^{n_i} q_{out,IE_i}} \tag{11}$$

To satisfy the radiative energy conservation, all the outgoing radiative power from each cell in the system should be absorbed or scattered in the volume cells or reach to the surface cells in the system. Therefore the following criterion can be used for checking the accuracy of the exchange factor's calculation. For each cell "i" as the source of radiation, it can be written that

$$\sum_{\substack{j=1 \\ j\neq i}}^{M+N} EF_{i-j} = 1 \tag{12}$$

To decrease the effect of error in calculation of exchange factors in the overall accuracy of the approach and to satisfy the radiative energy balance, the value of exchange factors, calculated by numerical integration, should be scaled by using scaled factor of by $\left(1 \Big/ \sum_{\substack{j=1 \\ j\neq i}}^{M+N} EF_{i-j}\right)$ before using in the matrix equation, Eq. (3). However, the value of $\sum_{\substack{j=1 \\ j\neq i}}^{M+N} EF_{i-j}$ calculated by using the result of numerical integration before scaling, should be in a reasonable range of approximately one. This is one of the criteria for checking the accuracy of the calculation of exchange factors, and also the overall accuracy of the approach.

MESH SIZE ANALYSIS FOR CALCULATION OF EXCHANGE FACTORS

One of the most important parameters effective in accuracy and computational time of the exchange factor calculation is the size of the coarse cells and integration elements. The number of discrete points in the calculation domain, used for defining the radiative source term in the domain, is equal to the number of coarse volume cells. Even though, considering a large number of the coarse volume cells gives better resolution in describing the volumetric radiative source term within the domain, it increases the size of matrix equation and leads to longer computational time.

The optical thickness and center to center distance of the integration elements, directly affects the accuracy and computational time of the exchange factor calculation. Considering a very fine integration structure increases the computational time of the code, while it increases the accuracy. Due to this, a separate study was carried out to ascertain the

criteria for the size of the integration elements in different states which could provide reliable accuracy in a reasonable computational time.

Figure 4 illustrates schematically one of the situations considered in mesh size analysis of exchange factor calculation. By considering different values for the size of the integration elements ΔL and extinction coefficient of the media, different situations are tested. Inaccuracies of numerical calculation can be reduced by setting a small value for ΔL which leads to small optical thickness in each integration cell. The value obtained with a small ΔL is considered as the reference value for exchange factors and amount of error for the result of using other coarser integration elements is calculated by comparing them with this reference value.

Figure 5 and 6 show two of the typical results produced from the mesh size analysis between the volume and surface cell when the center to center distance of the two cells is 1.5 (Fig. (5)) and 0.5 (Fig. (6)) times that of the length of the cells' edges. For these cases, as is shown in Fig. (4) and Fig.(5), for different sizes of coarse cells, the calculated value for the exchange factor was almost fixed when five or more divisions for making the integration elements, were considered in each edge of the cells.

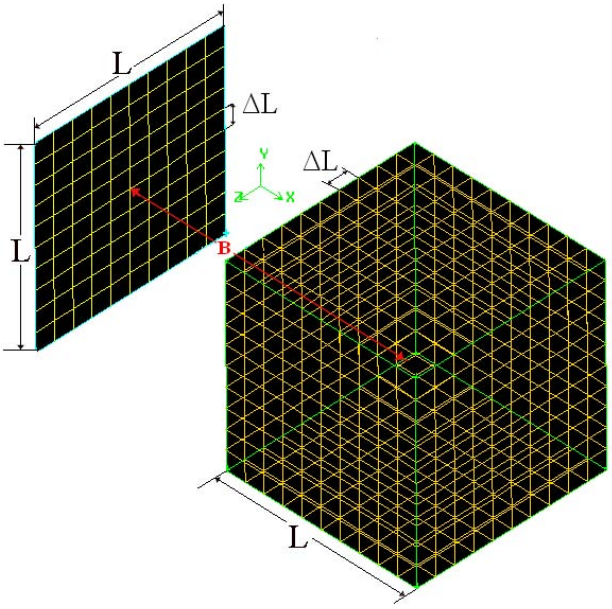

Figure 4: One of the situations used in mesh size analysis of exchange factor calculation between the volume and surface coarse cells.

For the different situations of the exchange factor calculations that are volume to volume, surface to volume, volume to surface and surface to surface, and for different values for B/L, the behavior of the exchange factor with the optical thickness of integration elements have been obtained and the optimized size for integration elements is calculated.

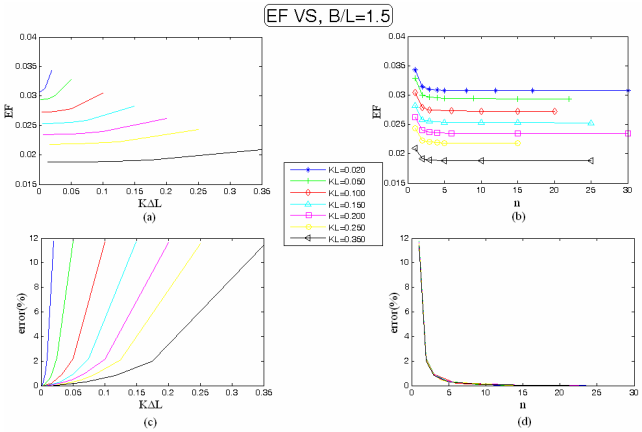

Figure 5: Behavior of the calculated exchange factors with the optical thickness of integration elements for the volume to surface state where B/L=1.5.

By comparing result of the mesh size analysis for different B/L, it is concluded that for the smaller values of B/L, the finer integration elements were needed to provide a good level of accuracy. Figure 7 demonstrates this for the volume to surface and volume to volume states. The optical thickness of the coarse cells used for producing Fig. (7) is equal to 0.35.

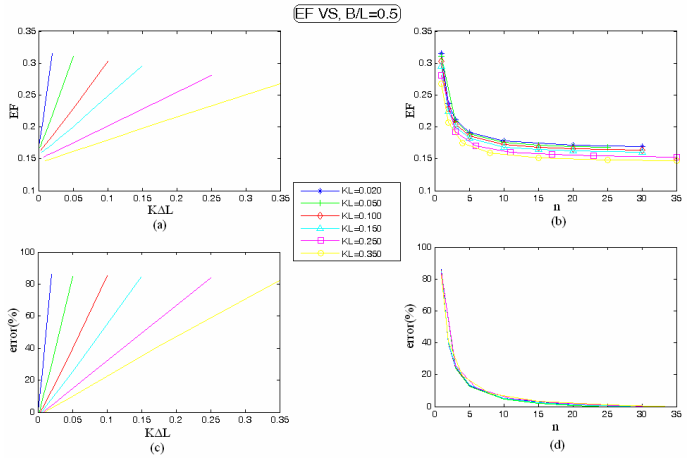

Figure 6: Behavior of the calculated exchange factors with the optical thickness of integration elements for volume to surface state where B/L=0.5.

As shown in Fig. 7, for very close cells, integral elements with very low optical thickness should be used whereas for the cells that are located far from each other, coarser integration elements can provide a good level of accuracy. For example, for the coarse cells with the optical thickness of 0.35, and for very close cells, the optical thickness of the integration elements of less than 0.02 should be selected, while for other situations, even the integration elements with the optical thickness of 0.17 can provide a good level of accuracy. The most time consuming part of the exchange factor calculation is calculating self extinction in volume cells, when both source

and destination volume cells are located in the same position. As shown in Fig.7, very fine integration elements should be used for this state.

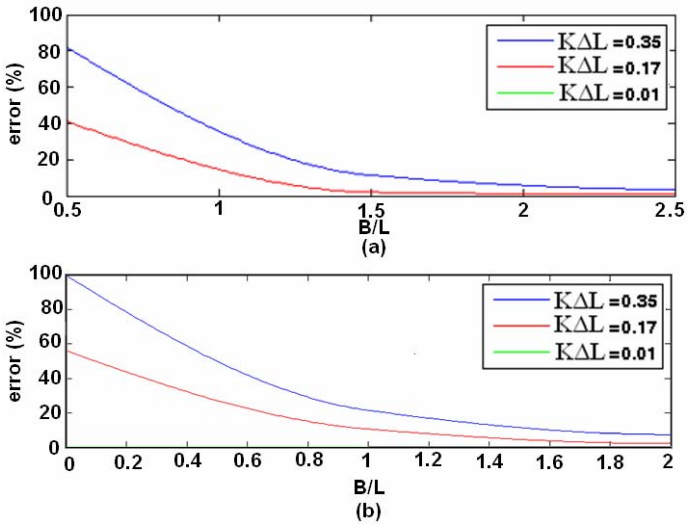

Figure 7: Change of error with B/L for the identical coarse cells with the optical thickness of 0.35. (a) Mesh size analysis for volume to surface situation. (b) Mesh size analysis for volume to volume situation.

One of the possible items for improving the accuracy and the computational time of the method might be obtaining the suitable correlations for calculating exchange factors and self extinction based on the optical thickness of the source and destination cells and optical thickness of the media between the cells.

RESULT OF THE SIMULATION

As a practical example, this approach is used for simulating radiative heat transfer in a furnace shaped geometry, when a hot stream of carbon dioxide is entered into the space from the bottom and it exits from the system in a pressure outlet section as shown in Fig. 8. The temperature of the hot gas stream at the bottom side of the furnace is 1600K and the temperature of all the solid walls are 800K and a parabolic profile is considered for the velocity inlet flow at the bottom of the furnace.

By using the radiative exchange method to simulate radiation heat transfer in this geometry in parallel with modeling turbulence and the velocity flow, the temperature profile within the space and radiative heat flux on the solid walls is found.

The coarse cells with a maximum optical thickness of 0.35 were used for calculating the volumetric radiative source term within the space. For calculating exchange factors between very close cells (neighbors), an integration structure with the optical thickness of 0.02 is used while for other states, the maximum of optical thickness of the integration elements is 0.175 .After five iterations, the amount of the radiative source term is converged. Figure 9 shows the result of the method for the temperature field over two sweep surfaces in the middle axes of the geometry in comparison with the same result of discrete ordinate method and P1 approximation method.

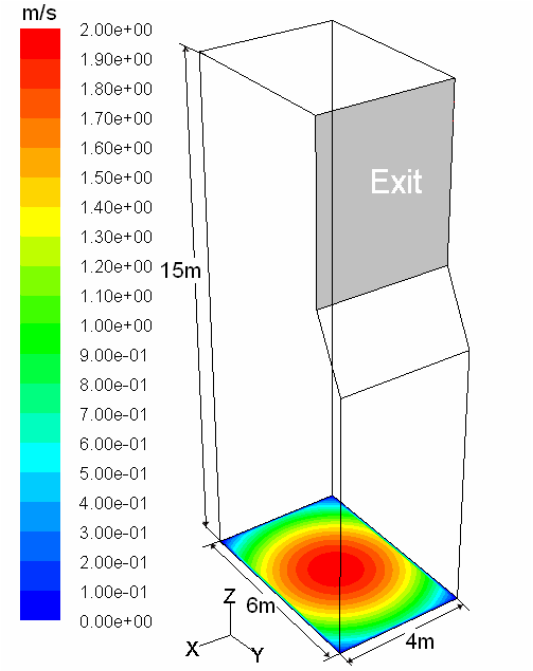

Figure 8: Velocity profile of the hot gas stream at the bottom of a furnace shape space surrounded by solid walls.

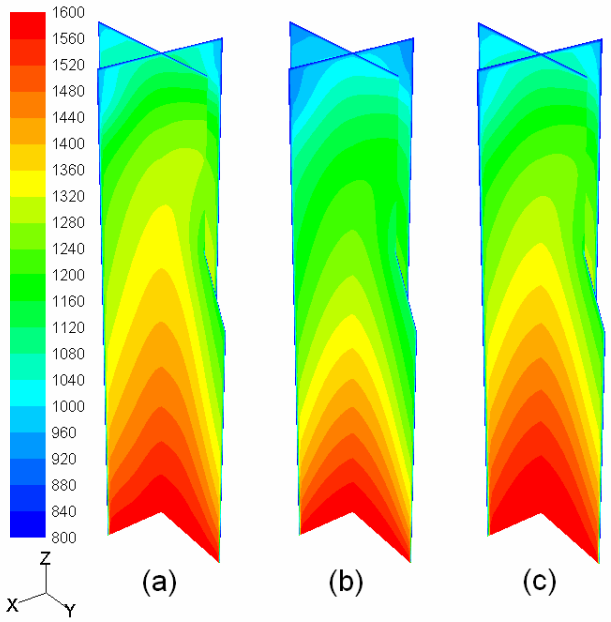

Figure 9: Temperature profile ;(a) Radiative Exchange Method, (b) Discrete Ordinate Method, (c) P1 Method .

As mentioned previously, the accuracy of the exchange factor calculation has a key role in the overall accuracy of the method,

and one criterion for checking the accuracy of exchange factor calculation was the summation of exchange factors from each cell to all the others, which should be equal to one in the ideal state, (see Eq. 12). For the sample described in Fig.8, this summation is in a range of 0.96-1.03.

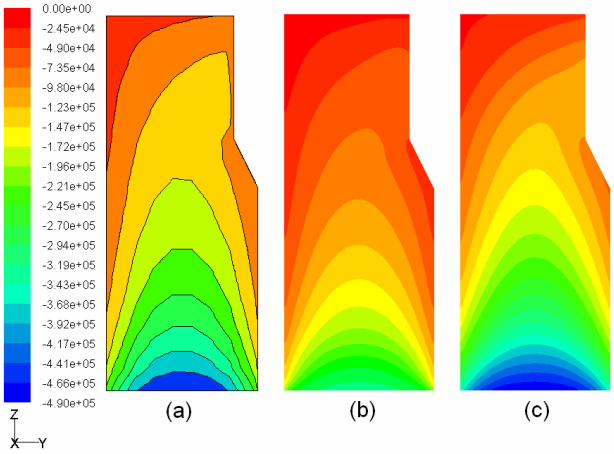

Figure 10: Distribution of radiative heat flux on the side walls of the geometry described in Fig. (8); (a) Radiative Exchange Method, (b) Discrete Ordinate Method, (c) P1 Method.

Figure 10 shows a comparison between the results of three different numerical methods; The Radiative Exchange Method, DO and P1 methods for the contours of radiative heat flux on the sidewall of the furnace.

As Fig. 9 and 10 show, the presented method gives qualitatively similar results to DO and P1 methods. Due to lack of experimental trustable data, the quantitative comparisons of various methods have not been done. The presented method is calculating radiation directly between all the points in the space as is the physical nature of the integro-differential radiation model. Thus, the presented method should have the feature to approach the accurate radiation results as the number of integration elements in calculation of exchange factors is increased.

CONCLUSIONS AND REMARKS

The theoretical bases of the radiative exchange method for predicting radiative heat transfer in a three-dimensional participating media has been presented, as well as the result of using this method in simulating radiation heat transfer in a furnace shaped geometry. The present method is applicable to various shape participating media that can absorb or isotropically scatter the incoming radiation, surrounded by the gray surfaces, which can absorb, emit and diffusely reflect radiation and open boundaries.

A new concept of the exchange factor required to develop the radiative exchange method has been introduced as a dimensionless quantity that is the ratio between two radiative heat powers. By considering this exchange factor concept, the radiative interaction between all points of the space is taken into account during the simulation.

The accuracy of the present method depends largely on the accuracy of the exchange factor calculation. Thus, a comprehensive mesh size analysis for the calculation of the exchange factor was carried out and the results are presented in order to obtain some criteria for the optical thickness of integration elements needed to provide a good level of accuracy in a reasonable runtime.

The convergence of the method is very rapid and even for the large geometries the amount of source term is converged into the final value after a few iterations. The result of using the new method in the furnace shaped geometry shows its ability to predict radiative heat transfer with a comparatively good conformity to the results of other numerical methods.

NOMENCLATURE

A	Area of the surface cells
B	Center to center distance of the cells in mesh size analysis
EF_{ji}	The exchange factor between cell " j " and cell " i ", when cell " j " is the radiative source cell.
I	The radiation intensity
KL	Optical thickness of coarse cells
$K\Delta L$	Optical thickness of intermediate cells
k_e	The emission coefficient
k_a	The absorption coefficient
k_S	The scattering coefficient
L	Characteristic length of the coarse cells
M	The total number of the coarse surface cells
N	The total number of the coarse volume cells
n	Number of integration elements in each coarse cell
q_{out}	The outgoing radiative power for each cell
S	The center to center distance of the two radiative elements
$\vec{S}$	The center to center vector of two cells
SE	Self extinction of the volume cells
T	The local temperature
V	Volume of the volume cells

Greek

β	The extinction coefficient,
θ	The angle between the normal vector of the surface elements and the center to center vector ($\vec{S}$).
σ	The Stefan-Boltzmann constant ($5.672\times10^{-8}\, W/m^2.K^4$)
ρ	Reflection coefficient
ΔL	Characteristic length of integration elements

Subscripts

i, j	The source and destination cell of radiation
IE	Integration element

s	Surface cell
V	Volume cell
C	Coarse cell

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