Proceedings of the ASME 2010 Summer Bioengineering Conference
SBC2010
June 16-19, 2010, Naples, Florida, USA

SBC2010-19245

COMPARISON OF ANALYTICAL AND FINITE ELEMENT IMPLEMENTATION OF EXPONENTIAL CONSTITUTIVE MODELS FOR VALVE TISSUE UNDER MICROPIPETTE ASPIRATION

Ruogang Zhao[1], Krista Lynn Sider[1], Craig A. Simmons[1, 2]

(1) Institute of Biomaterials and Biomedical Engineering
University of Toronto
Toronto, Ontario
Canada

(2) Department of Mechanical and Industrial Engineering
University of Toronto
Toronto, Ontario
Canada

INTROUDUCTION

Micropipette aspiration (MA) has been widely used to measure the biomechanical properties of cells and biomaterials [1]. Typically a linear elastic half-space model is used to fit the experimental load-deformation data [1]. However, load-deformation relationships for most biological tissues are highly nonlinear, suggesting alternative constitutive models are necessary. In the case of aortic heart valve tissue, exponential-type constitutive models have been found to fit the biaxial stress-strain behavior well [2]. Based on these studies, Butcher et al. used an exponential constitutive model to characterize the response of chicken embryonic valve (atrioventricular cushion) under MA [3]. To do so, they implemented an analytical exponential constitutive model [2] and directly related the stress and strain to the experimentally measured pressure and aspiration length. This allowed the authors to fit the tissue MA data without accounting for the complexities of the boundary conditions and multicomponent strain field inherent in MA. However, it is unclear whether the material parameters estimated using this approach are different from those estimated by solving the more complex boundary value problem, which presumably more faithfully simulates the physical process of tissue aspiration.

The goal of this study was to determine whether aortic valve tissue material parameters estimated by the easily-implemented analytical approach [3] differ from those obtained by finite element (FE) analysis aortic valve tissue under MA. To do so, we implemented an exponential hyperelastic constitutive model in the FE model and used an inverse FE approach to predict material parameters [4].

METHODS AND MATERIALS

Material models

To fit the MA experimental measurements of the embryonic atrioventricular cushions, Butcher et al. implemented an exponential constitutive model [2]

$$W = C\,[\exp(\alpha E^2)-1] \qquad (1)$$

where W is the strain energy, C and α are material constants and E is the Green's finite strain with the 2nd Piola-Kirchoff stress (S) being calculated by $S = \partial W/\partial E$. To relate the stress and strain in the constitutive model to experimental measurements, Butcher et al. directly assigned the measured aspiration length (L) to pipette radius (a) ratio as the Green's strain, and the measured aspiration pressure ΔP as the Lagrangian stress T, which is calculated by $T = \lambda S$ with the stretch ratio in the aspiration direction (λ) given by $\lambda = (E + 1)^{0.5}$ [3]. Thus, the analytical model for fitting the MA measurements of valve tissue is expressed as

$$\Delta P = 2(L/a+1)^{0.5}\, C\alpha\,(L/a)\,\{\exp[\alpha\,(L/a)^2]-1\} \qquad (2)$$

To account for the multicomponent stress-strain field in the valve tissue during MA process, we implemented an incompressible isotropic exponential constitutive model. The strain energy density function of this model is expressed as

$$W = C\,[\exp[\alpha\,(I_1-3)]-1] \qquad (3)$$

where W is the strain energy, C and α are material constants and I_1 is the first strain invariant, defined as $I_1 = \lambda_1^2+\lambda_2^2+\lambda_3^2$ with λ_1, λ_2 and λ_3 being the principal stretches. This isotropic exponential constitutive model was implemented numerically in ANSYS (Canonsburg, PA) through its user material subroutine USERHYPER.

The apparent modulus was defined as the product of the two material constants

$$M = C\,\alpha \tag{4}$$

FE model

The boundary value problem of valve tissue under MA was modeled in ANSYS as shown in Figure 1. The micropipette was assumed to be rigid and fixed, and the interaction between the valve tissue and the micropipette was considered as a surface-based contact problem with the micropipette being modeled as a rigid surface in contact with a deformable tissue surface. The displacement of the tissue was constrained by the contact between the tissue surface and micropipette. Traction-free finite sliding was allowed between the tissue surface and the micropipette edge. The applied load was modeled as a pressure boundary condition ΔP over the tissue surface within the micropipette opening. Eight-node axisymmetric elements were used to generate the mesh. The mesh was refined near the pipette opening to account for the high strain gradient in this region.

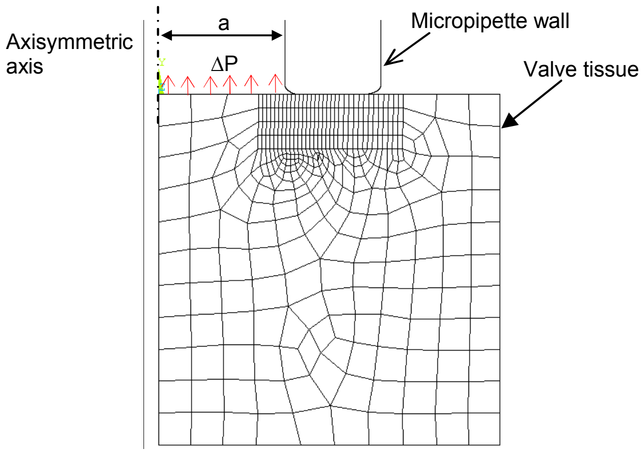

Figure 1. The FE model geometry and boundary conditions

Material parameter estimation and comparison

The nonlinear load-deformation relationship of porcine aortic valve tissue was obtained using a micropipette aspiration system similar to that described previously [4]. Multiple experimental measurements (n = 4) were used for material parameter estimation. The material parameters of the FE model were estimated using an inverse FE method as previously described [4]. Briefly, the experimentally measured load-deformation relationship was used as a "target" curve and the material constants in the FE model were iteratively adjusted until the FE model-predicted load-deformation curve matched the target. This method was implemented using the Levenberg-Marquardt algorithm in Matlab (Mathworks, Natick, MA). Material parameters were also estimated analytically using eq (2). The apparent moduli from the FE and analytical models were calculated by eq (4). The goodness of fit of the FE and analytical model predictions were evaluated by the R^2 values. A paired t-test was used to test for differences between the two approaches.

RESULTS

A representative experimental load-deformation relationship of valve tissue under MA and the best fit curves of the FE model and the analytical model are shown in Fig. 2. Both the FE model and the analytical model fit equally well to the experimental data, with R^2 values close to unity (Table 1). The average apparent modulus estimated by the FE model was 4.56±1.4 kPa and by the analytical model was 4.29±1.36 kPa (Table 1). These apparent moduli, measured on adult porcine valve tissue, are generally greater than those reported for chicken embryonic atrioventricular cushions [2], as expected. Statistical analysis showed that there was no significant difference in the fitting qualities (P = 0.784) or in the estimated apparent moduli between the analytical model and the FE model (P = 0.789).

Table 1. Comparison of fitting quality and material parameters estimated by the FE and analytical models

	R^2 value	C (kPa)	α	M=C*α (kPa)
FE model	0.99±0.003 [a]	0.94±0.14	4.77±0.97	4.56±1.41 [a]
Analytical model	0.99±0.001	10.06±2.40	0.43±0.11	4.29±1.36

[a] NS by paired t-test (n = 4) compared with the analytical model eq (2).

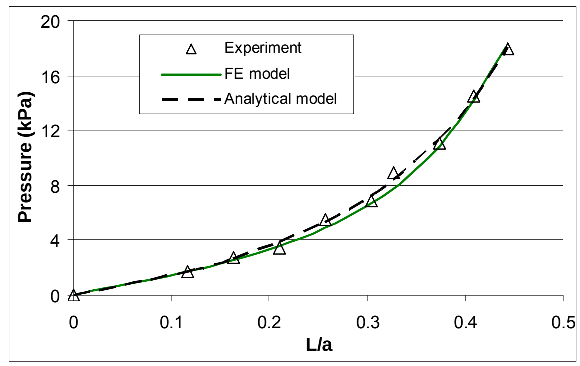

Figure 2. Typical experimental load-deformation curve for valve tissue under MA and the nonlinear fits obtained by the FE and the analytical models.

DISCUSSION AND CONCLUSIONS

In the current study we implemented a strain invariant-based isotropic exponential constitutive model in a FE boundary value problem of valve tissue under MA, and performed parameter estimation through an inverse FE method. The good fitting quality of the FE model showed that this exponential constitutive model is suitable for predicting the nonlinear load-deformation relationship of the valve tissue under MA, improving on previous approaches that used linear constitutive models [5]. Notably, there was no significant difference between the material parameters estimated by the FE and analytical models. This finding suggests that the estimation of material parameters of the valve tissue under MA can be achieved by applying the simpler analytical model and does not need require solving the more complex and resource-intensive numerical boundary value problem.

REFERENCES

1. Theret, D. P., Levesque, M. J., Sato, M., Nerem, R. M. and Wheeler, L. T., 1988, J Biomech Eng, **110**(3), pp. 190-9.
2. Billiar K.L. and Sacks M.S., 2000, J Biomech Eng, **122**, pp. 327-335.
3. Butcher J. T., McQuinn T. C., Sedmera D., Turner D. and Markwald R. R., 2007, Circ Res, **100**, pp.1503-1511.
4. Zhao R.G., Wyss K. and Simmons C. A., 2009, J Biomech, **42**(16), pp. 2768-2773.
5. Boudou T., Ohayon J., Arntz Y., Finet G., Picart C. and Tracqui P. 2006, J Biomech, **39**(9), pp.1677-1685.