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TYPE SYNTHESIS OF 3-DOF PARALLEL MANIPULATORS WITH BOTH PLANAR AND TRANSLATIONAL OPERATION MODES

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ABSTRACT

Despite recent advances in the type synthesis of parallel manipulators with a mono-operation mode, such as translational parallel manipulators and spherical parallel manipulators, the type synthesis of parallel manipulators with multiple operation modes is still an open issue. This paper deals with the type synthesis of 3-DOF parallel manipulators with both planar and translational operation modes. The type synthesis of planar parallel manipulators, which refer to parallel manipulators in which the moving platform undergoes planar motion, is first dealt with using the virtual chain approach. Then, the types of 3-DOF parallel manipulators with both planar and translational operation modes are obtained. This work can be extended to the type synthesis of other classes of parallel manipulators with multiple operation modes.

KEY WORDS Parallel mechanism, Parallel mechanism with multiple operation modes, Type synthesis, Virtual chain approach, Screw theory

1 INTRODUCTION

Great advances have been made in the type synthesis of parallel manipulators (PMs) with a mono-operation mode [1–15], such as translational PMs, in the past decade.

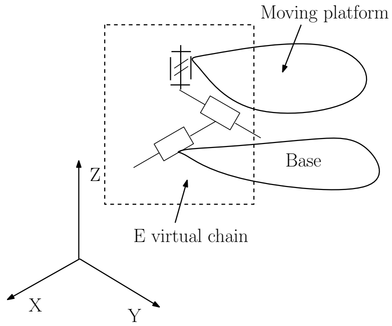
PMs with multiple operation modes (PMwMOMs, also called PMs that change their group of motion or PMs with bifurcation of motion) have not been well explored. One of the characteristics of PMwMOMs is to reduce the number of actuators used for the moving platform to realize several motion patterns. A parallel mechanism with multiple operation modes was first applied parallel mechanisms in a CVC (constant-velocity coupling) which connects intersecting axes axes in one mode or parallel axes in another mode [16]. Since DYMO — a PMwMOM — is proposed in [17], several new PMwMOMs have been proposed [18–22]. In [20], a general method for the type synthesis of PMwMOMs has been proposed. In [21], several PMwMOMs with two 3T1R operation modes have been investigated, In [22], several PMwMOMs with two 2T1R operation modes have been proposed. In [23], a class of PMwMOMs has been proposed by introducing a reconfigurable U (universal) joint. However, the types of PMwMOMs proposed so far are very limited. Especially, the steps for identifying legs for PMwMOMs are very complicated and needs further investigation.

Considering that both the planar motion and spatial translation are motion patterns in common use, this paper deals with the type synthesis of 3-DOF PMs with both planar and translational operation modes (PMwPTMs).

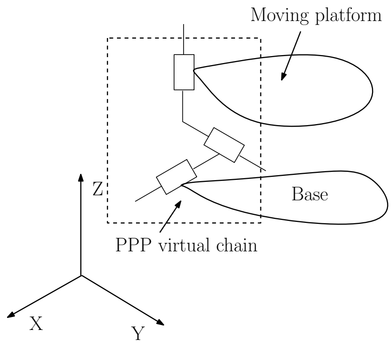
In Section 2, the type synthesis of planar PMs, which refer to PMs in which the moving platform undergoes planar motion, is first dealt with using the virtual chain approach proposed in [12]. In Section 3, the types of 3-DOF PMwPTMs are obtained. Finally, conclusions are drawn.

In the type synthesis of PMs, it is convenient to deal with the instantaneous constraints using a screw theory based approach [12, 16, 24–26]. Using this approach, the instantaneous constraints exerted on the moving platform by the base through a kinematic chain (a PM or one of its legs) is represented by a screw system which is called the wrench system of the kinematic chain. For brevity, the wrench system of a leg is called a leg-wrench system. $\boldsymbol{\zeta}_0$ and $\boldsymbol{\zeta}_\infty$ denote, respectively, a wrench of zero pitch and a wrench of infinite pitch. A wrench system is called an n-wrench-system if its order, i.e., the number of independent wrenches, is n.

Since any leg with $c^i = 0$ can be used as a leg for PMwPTMs and the type synthesis of such legs is trivial and has been well dealt with in the literature, we will focus on the type synthesis of legs with a $c^i(c^i > 0)$-wrench-system for PMwPTMs. Here and throughout this paper, a leg with a $c^i(c^i > 0)$-wrench-system refers to a leg which has a $c^i(c^i > 0)$-wrench-system in



(a) Planar motion pattern: E virtual chain.



(b) Translational motion pattern: PPP virtual chain.

Fig. 1 Representation of motion patterns.

a regular configuration. In addition, we limit ourselves to PMs composed of R (revolute) joints only. For convenience, the following notations are used throughout this paper. The R joints within a mechanism that have parallel axes that are perpendicular to the plane of motion are denoted by R̋. The axes of R̀ joints within a leg are parallel and are not parallel to the axes of the R̋ joints. The axes of Ŕ joints within a leg are parallel. The axes of Ř joints within in a leg pass through a common point. $()_E$ denotes a planar kinematic chain.

2 TYPE SYNTHESIS OF PLANAR PARALLEL MANIPULATORS

2.1 Virtual chain approach

Using the virtual chain approach to the type synthesis of PMs [12], planar PMs can be constructed using several classes of compositional units. One of the key ideas underlying this approach is to represent each motion patten using a virtual chain, which makes it possible to reduce the type synthesis of PMs to the type synthesis of single-loop kinematic chains. For example, a planar motion and a translation can be represented by a planar virtual chain (also E virtual chain) and the PPP virtual chain respectively (Fig. 1).

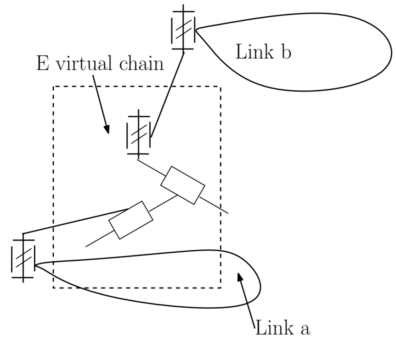
2.2 Step 1: Decomposition of the wrench system of a parallel manipulator

The wrench system of the planar virtual chain is a 2-ζ_∞-1-ζ_0-system. A base of the 2-ζ_∞-1-ζ_0-system is composed of $\zeta_{\infty 1}$, $\zeta_{\infty 2}$ and ζ_{03}. The directions of the $\zeta_{\infty j}$ (j =1 and 2) are parallel to the plane of motion of the planar virtual chain. The axis of ζ_{03} is perpendicular to the plane of motion of the planar virtual chain.

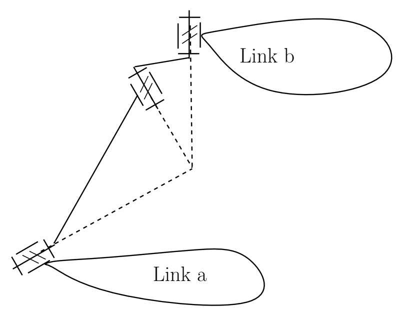
The leg-wrench system of a leg for planar PMs is a subsystem of the 2-ζ_∞-1-ζ_0-system. The leg-wrench systems may include 2-ζ_∞-1-ζ_0-system, 2-ζ_∞-system, 1-ζ_∞-1-ζ_0-system, 1-ζ_∞-system, 1-ζ_0-system and 0-ζ-system.

2.3 Step 2: Type synthesis of legs for planar parallel kinematic chains

Since this paper is limited to PMs composed of only R joints, two types of compositional units [12] can be used to construct 3-DOF single-loop kinematic chains: planar compositional units (see Fig.2(a) for example) and spherical compositional units (see Fig.2(b) for example). In a planar compositional unit, all the links move along parallel planes. In a spherical compositional units, the axes of all the R joints pass through a common point.



(a) Planar compositional units.



(b) Spherical compositional units.

Fig. 2 Compositional units.

Using the above two classes of compositional units, we obtain

(a) a 3-DOF single-loop kinematic chains with a 2-ζ_∞-1-ζ_0-system: $(PPR)_E$R̋R̋R̋ [Fig. 3(a)],

(b) 3-DOF single-loop kinematic chains with a 1-ζ_∞-system: $(PPR)_E$R̋R̋R̋R̀R̀ [Fig. 3(b)], $(PPR)_E$R̋R̋R̀R̀R̋ [Fig. 3(c)], $(PPR)_E$R̋R̋R̀R̀R̀, and $(PPR)_E$R̋R̀R̀R̀R̋, and

(c) 3-DOF single-loop kinematic chains with a 1-ζ_0-system: $(PPR)_E$R̋R̋R̋ŘŘ [Fig. 3(d)] and $(PPR)_E$R̋R̋ŘŘR̋ [Fig. 3(e)], $(PPR)_E$R̋R̋ŘŘŘ, and $(PPR)_E$R̋ŘŘŘR̋.

For example, the $(PPR)_E$R̋R̋R̋R̀R̀ [Fig. 3(b)] is composed of two planar compositional units, 1-2-3-4-5-6 and 7-8.

By inserting one or more inactive joints to the above 3-DOF single-loop planar kinematic chains, we may obtain

(d) 3-DOF single-loop kinematic chains with a 1-ζ_{∞}-1-ζ_0-system: (PPR)$_E$RR̋R̋R̋ [Fig. 3(f)] and (PPR)$_E$R̋RR̋R̋, and

(e) 3-DOF single-loop kinematic chains with a 1-ζ_0-system: (PPR)$_E$RR̋R̋R̋R [Fig. 3(g)], (PPR)$_E$R̋R̋R̋RR, (PPR)$_E$R̋R̋RR̋R, (PPR)$_E$R̋RR̋R̋R, (PPR)$_E$R̋R̋RRR̋, and (PPR)$_E$R̋RR̋RR̋.

The types of legs for planar parallel kinematic chains (Table 1) can then be obtained from the above 3–DOF single-loop planar kinematic chains by removing the planar virtual chain. Figure 4 shows the R̋R̋R̋R̀R̀ leg obtained from the (PPR)$_E$R̋R̋R̋R̀R̀ [Fig. 3(b)]. The leg-wrench system of the R̋R̋R̋R̀R̀ leg is a 1-ζ_{∞}-system. The direction of the ζ_{∞} is perpendicular to the axes of all the R joints of the leg.

2.4 Step 3: Type synthesis of planar parallel kinematic chains

By assembling a set of three legs for planar parallel kinematic chains in such a way that the wrench system is the same as the wrench system of the planar virtual chain, we can obtain a planar parallel kinematic chain. A large number of planar parallel kinematic chains can be obtained. Two planar parallel kinematic chains are shown in Fig. 5, in which the axes of all the R̋ joints are parallel.

2.5 Step 4: Selection of actuated joints

For a given planar parallel kinematic chain, we can select a set of three actuated joints, which satisfies the validity condition of actuated joints [12], to obtain a planar PM (Fig. 6).

3 TYPE SYNTHESIS OF PARALLEL MANIPULATORS WITH BOTH PLANAR AND TRANSLATIONAL OPERATION MODES

Using the general procedure for the type synthesis of PMwMOMs [20], the type synthesis of PMwPTMs can be carried out using three steps:

- Step 1: Type synthesis of legs for PMs with a mono-operation mode.
- Step 2: Type synthesis of legs for PMwPTMs.
- Step 3: Assembly of legs for PMwPTMs.

An improved procedure for Step 2 will be presented in this section.

3.1 Step 1: Type synthesis of legs for parallel manipulators with a mono-operation mode

3.1.1 Type synthesis of legs for translational parallel manipulators The type synthesis of translational PMs has been studied in [1, 4–12]. The conditions on the legs for translational PMs are shown in Table 2 [6, 7, 12].

3.1.2 Type synthesis of legs for planar parallel manipulators The type synthesis of planar PMs has been studied in Section 2. The conditions on the legs for planar PMs are shown in Table 1.

3.2 Step 2: Type synthesis of legs for parallel manipulators with both planar and translational operation modes

As pointed out in [20], legs for PMwPTMs in a transitional configuration can be obtained by comparing the conditions on the legs for planar PMs and the conditions on the legs for translational PMs. Due to the large number of legs for PMs with a mono-operation mode, the above process is very tedious. This section will present an approach to identify the legs for PMwPTMs in a transitional configuration.

The type synthesis of legs for PMs with multiple operation modes can be carried out as follows.

Step 2a Classify a class of legs for PMs with a mono-operation mode into sub-classes according to the leg-wrench systems.

For example, the 5R class of legs for planar PMs can be classified into two sub-classes: sub-class of legs with a 1-ζ_{∞}-system and sub-class of legs with a 1-ζ_0-system. The 5R class of legs for the translational PMs has only one sub-class: sub-class of legs with a 1-ζ_{∞}-system.

Step 2b Identify the types of legs for PMwPTMs by comparing the conditions for each pair of sub-classes of legs for PMs with a mono-operation mode.

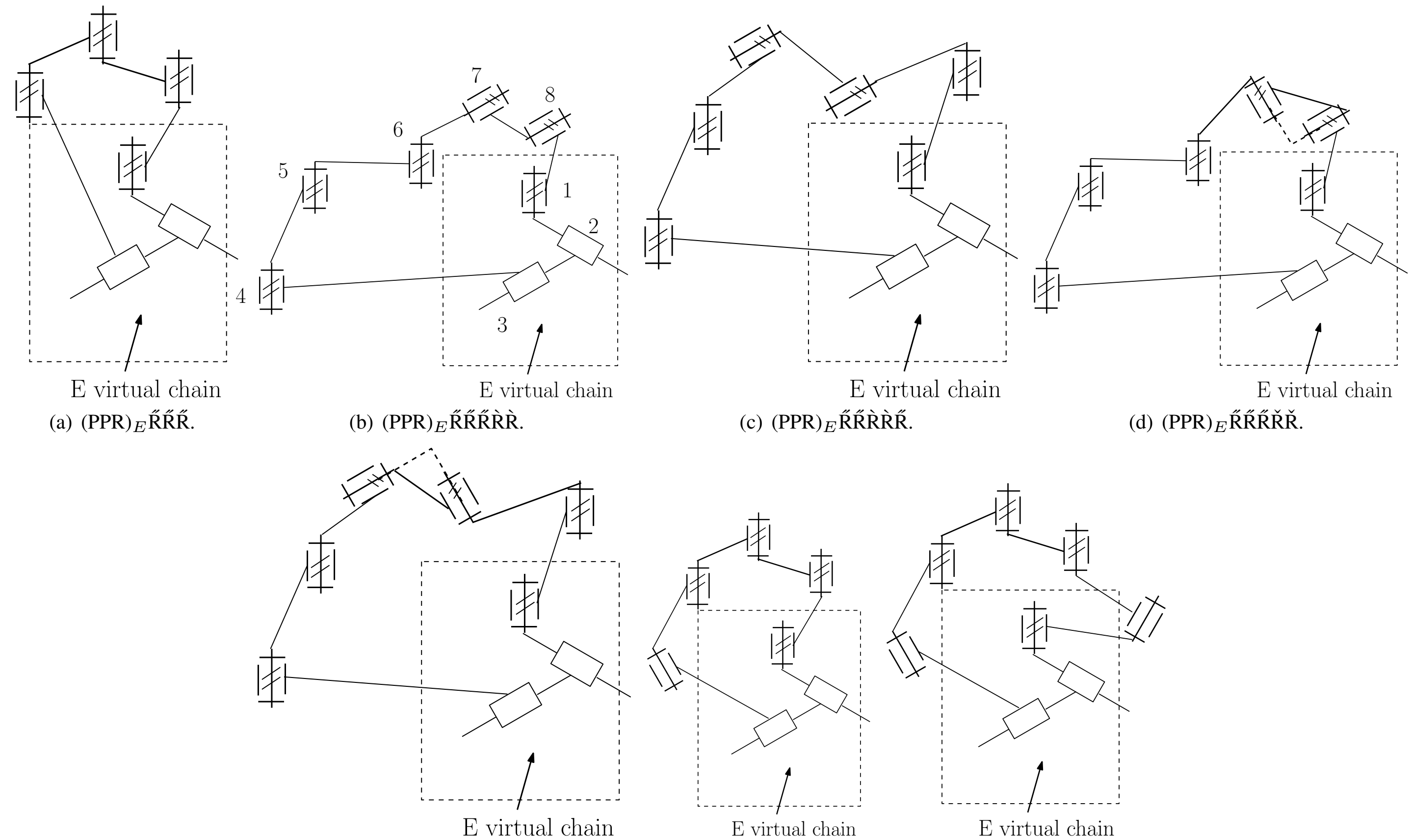
- Case a. 5R class of legs for translational PMs with a 1-ζ_{∞}-system and 5R class of legs for planar PMs with a 1-ζ_{∞}-system.
- Case b. 5R class of legs for translational PMs with a 1-ζ_{∞}-system and 5R class of legs for planar PMs with a 1-ζ_0-system.

In Case a, we can find easily that the common conditions that both the 5R class of legs for translational PMs with a 1-ζ_{∞}-system (Table 2) and the 5R class of legs for planar PMs with a 1-ζ_{∞}-system (Table 1) satisfy are the same as those for the 5R class of legs for planar PMs with a 1-ζ_{∞}-system. Therefore, we obtain several types of legs for PMwPTMs, as listed in Nos. 2–8 legs in Table 3.

In Case b, the class of leg-wrench system of the 5R class of legs for translational PMs with a 1-ζ_{∞}-system is different from that of the 5R class of legs for planar PMs with a 1-ζ_0-system.

It is noted that legs of class 5R for translational PMs should satisfy the following condition: "The axes of two or three successive R joints are parallel, and the axes of the other R joints are also parallel." Since the ŘŘR̋R̋R̋, R̋R̋R̋ŘŘ, ŘŘŘR̋R̋, R̋R̋ŘŘŘ legs for planar PMs cannot reach a configuration in which they satisfy the above condition, there is no leg for PMwPTMs that are associated with the above four types of legs.

Among the legs for planar PMs composed of three R̋ joints and two inactive R joints, the RR̋R̋R̋R leg for planar PMs can reach a transitional configuration in which the axes of the two R joints are parallel. Therefore, we can obtain the following type of legs for PMwPTMs in its transitional configuration: R̀R̋R̋R̋R̀ (No. 1 leg in Table 3). In the transitional configuration, the leg-wrench system of an R̀R̋R̋R̋R̀ leg is a 1-ζ_{∞}-system. No leg for PMwPTMs can be obtained from the other types of legs for planar PMs composed of three R̋ joints and two inactive R joints.



(a) (PPR)$_E$R̋R̋R̋. (b) (PPR)$_E$ŔR̋R̋R̋R̀R̀. (c) (PPR)$_E$ŔR̋R̀R̀R̀R̋. (d) (PPR)$_E$ŔR̋R̋R̋ŘŘ.

(e) (PPR)$_E$ŔR̋R̋ŘŘR̋. (f) (PPR)$_E$RR̋R̋Ŕ. (g) (PPR)$_E$RR̋R̋R̋R.

Fig. 3 3-DOF single-loop planar kinematic chains.

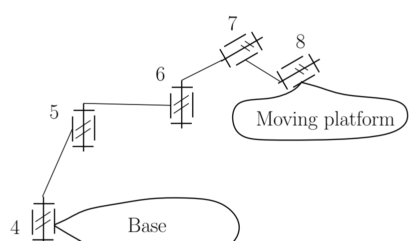


Fig. 4 An R̋R̋R̋R̀R̀ leg for planar parallel manipulators.

In summary, eight types of legs with a 1-ζ_∞-system have been obtained for PMwPTMs (Table 3) in a transitional configuration.

It is noted that the R̋R̀R̀R̀R̋, R̋R̋R̀R̀R̀ and R̀R̀R̀R̋R̋ legs are different from the other legs. No joint in these legs is inactive no matter which operation mode a PMwPTM works in. In the R̀R̋R̋R̋R̀, R̋R̋R̋R̀R̀ or R̀R̀R̋R̋R̋ legs, the R̀ joints will be inactive if the PMwPTM works in the planar mode. There is no inactive joint if the PMwPTM works in the translational modes. In addition, the leg-wrench system of the R̀R̋R̋R̋R̀ leg changes —from a 1-$\boldsymbol{\zeta}_\infty$-system in the translational mode to a 1-$\boldsymbol{\zeta}_0$-system in the planar mode and vice versa (Fig. 7).

3.3 Step 3: Assembly of legs for parallel manipulators with both planar and translational operation modes

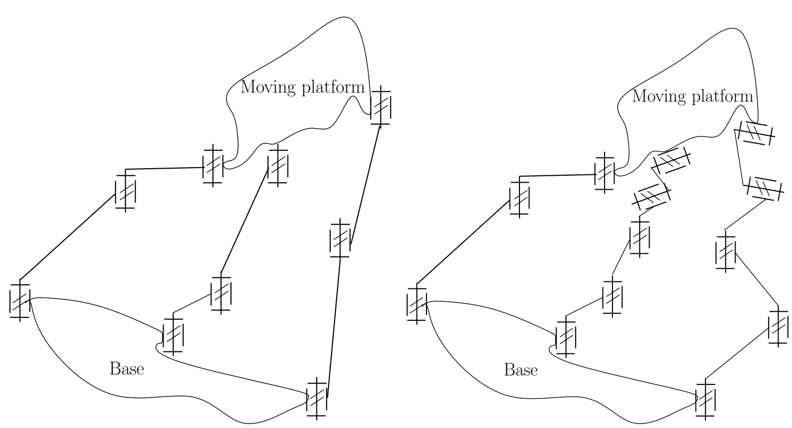
In this section, we will discuss how to obtain PMwPTMs by assembling the legs obtained in Section 3.2.

In assembling a 3-DOF PMwPTM, it should satisfy both the assembly conditions for the planar PMs that guarantee the moving platform can undergo at least the planar motion (Section 2) and the assembly conditions for the translational PMs [6, 7, 12] that guarantee the moving platform can undergo at least the translational motion. The geometric constraints among legs of a PM can be clearly shown in a transitional configuration of the PM (Fig. 8).

To guarantee that the DOF of the PMwPTM is three and not greater than three at a regular configuration, the linear combina-

Table 1 Legs for planar PMs composed of R joints.

c^i	Class	No	Type	Description	Leg-wrench system
3	3R	1	R̋R̋R̋	The axes of all the R̋ joints are parallel to a normal to the plane of motion.	2-$\boldsymbol{\zeta}_\infty$-1-$\boldsymbol{\zeta}_0$-system
2	4R	2	R̋R̋R̋R		1-$\boldsymbol{\zeta}_\infty$-1-$\boldsymbol{\zeta}_0$-system
		3	R̋R̋RR̋		
		4	R̋RR̋R̋		
		5	RR̋R̋R̋		
1	5R	6	R̋R̋R̋R̀R̀	The axes of R̀ joints are parallel, while the axes of the R̋ joints are parallel to a normal to the plane of motion.	1-$\boldsymbol{\zeta}_\infty$-system
		7	R̀R̀R̋R̋R̋		
		8	R̋R̋R̀R̀R̋		
		9	R̋R̀R̀R̋R̋		
		10	R̋R̋R̀R̀R̀		
		11	R̀R̀R̀R̋R̋		
		12	R̋R̀R̀R̀R̋		
		13	ŘŘR̋R̋R̋	The axes of Ř joints within a leg have a common point, while the axes of the R̋ joints are parallel to a normal to the plane of motion.	1-$\boldsymbol{\zeta}_0$-system
		14	ŘŘŘR̋R̋		
		15	R̋R̋R̋ŘŘ		
		16	R̋R̋ŘŘŘ		
		17	R̋R̋R̋RR		
		18	R̋R̋RR̋R		
		19	R̋R̋RRR̋		
		20	R̋RR̋R̋R		
		21	R̋RR̋RR̋		
		22	R̋RRR̋R̋		
		23	RR̋R̋R̋R		
		24	RR̋R̋RR̋		
		25	RR̋RR̋R̋		
		26	RRR̋R̋R̋		



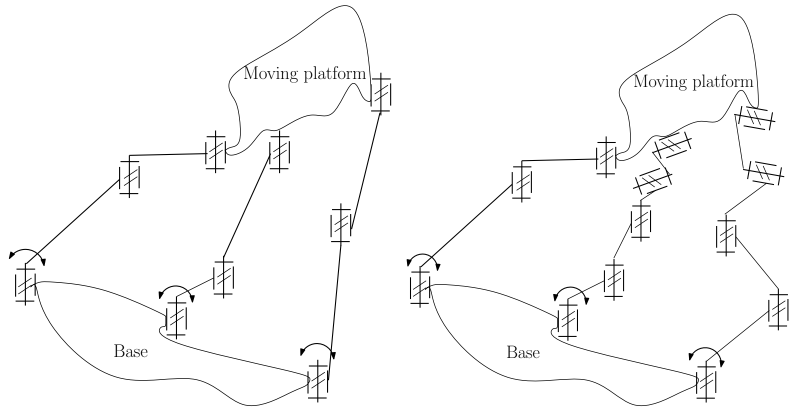
(a) 3-R̋R̋R̋. (b) R̋R̋R̋-2-R̋R̋R̋R̀R̀.

Fig. 5 Two planar kinematic chains.

tion of all its leg-wrench systems should be a 2-$\boldsymbol{\zeta}_\infty$-1-$\boldsymbol{\zeta}_0$-system if the PM works in the planar mode or a 3-$\boldsymbol{\zeta}_\infty$-system if the PM works in the translational mode. From the previous section, it is found that a leg for PMwPTMs has a 1-$\boldsymbol{\zeta}$-system in a regular configuration (Table 3). Since the leg-wrench system of each leg varies with the change of its configuration, we make the assumption that such conditions are met as long as a 3-DOF PMwPTM is composed of at least three legs with a 1-wrench-system, including at least one R̀R̋R̋R̋R̀ leg [Fig. 7(a)], listed in Table 3.

By assembling the legs listed in Table 3, their variations, and legs with a wrench system of order 0, a large number of PMwPTMs such as the PM shown in Fig. 8 can be obtained.

Figure 9 shows the reconfiguration of the R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM shown in Fig. 8. The PM switches from the

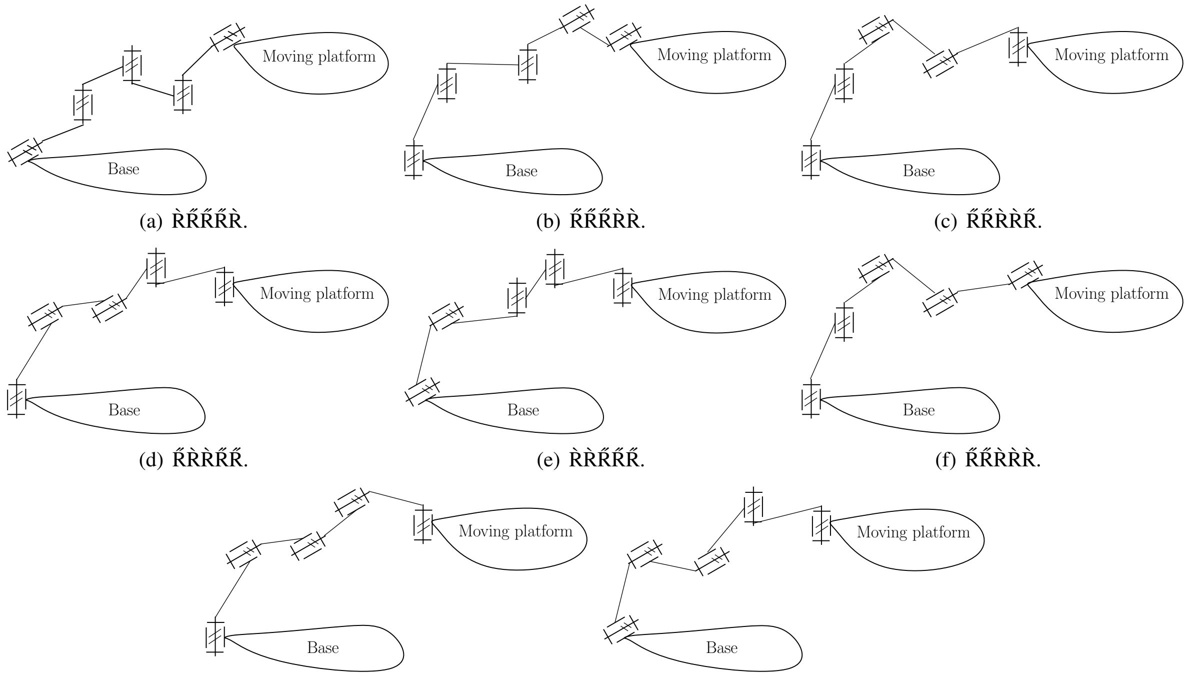


(a) 3-R̋R̋R̋. (b) R̋R̋R̋-2-R̋R̋R̋R̀R̀.

Fig. 6 Two planar parallel manipulators.

Table 2 Legs for translational PMs composed of R joints.

c^i	Class	No	Type	Description	Leg-wrench system
1	5R	1	ŔŔR̀R̀R̀	The axes of two or three successive R (revolute) joints are parallel, while the axes of the other R joints are also parallel.	1-ζ_∞-system
		2	ŔR̀R̀R̀Ŕ		
		3	R̀R̀R̀ŔŔ		
		4	R̀ŔŔR̀R̀		
		5	R̀R̀ŔŔR̀		



(a) R̀R̋R̋R̋R̀. (b) R̋R̋R̋R̀R̀. (c) R̋R̋R̀R̀R̋.

(d) R̋R̀R̀R̋R̋. (e) R̀R̀R̋R̋R̋. (f) R̋R̋R̀R̀R̀.

(g) R̋R̀R̀R̀R̋. (h) R̀R̀R̀R̋R̋.

Fig. 7 Some legs for PMwPTMs.

planar mode [Fig. 9(a)] to a transitional configuration [Fig. 9(b)], and then to the translational mode [Fig. 9(c)]. In the transitional configuration [Fig. 9(a)], the linear combination of all the leg-wrench systems is a 2-ζ_∞-system, and the moving platform has four instantaneous DOF which includes the planar motion and the spatial translation.

In summary, in a transitional configuration, a PMwPTM is in constraint singular configuration. In the case that no leg is in a

Table 3 Legs for PMwPTMs in a transitional configuration.

c^i	Class	No.	Type	Description	Leg-wrench system
1	5R	1	R̀R̋R̋R̋R̀	The axes of two or three successive R̀ joint within a leg are parallel, while the axes of the R̋ joints within a PM are parallel.	1-ζ_∞-system
		2	R̋R̋R̋R̀R̀		
		3	R̋R̋R̀R̀R̋		
		4	R̋R̀R̀R̋R̋		
		5	R̀R̀R̋R̋R̋		
		6	R̋R̋R̀R̀R̀		
		7	R̋R̀R̀R̀R̋		
		8	R̀R̀R̀R̋R̋		

singular configuration, the instantaneous of the moving platform of the PM may be greater than the instantaneous DOF of the moving platform in an operation mode.

In addition to the way shown in Figure 9 in which one may switch the PM from one operation mode to another. The R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM shown in Fig. 8 may be reconfigured via a leg-singular configuration, where the axes of the two R̀ joints in the R̀R̋R̋R̋R̀ are collinear (Fig. 10). The PM may switch from the planar mode [Fig. 10(a))] to a planar↔transitional configuration [Fig. 10(b)], a translational↔transitional configuration [Fig. 10(c)] and then to the translational mode [Fig. 10(d)]. In the planar↔transitional configuration [Fig. 10(b)], the linear combination of all the leg-wrench systems is a 2-ζ_∞-1-ζ_0-system, and the moving platform has three DOF and can undergo planar motion. In the translational↔transitional configuration [Fig. 10(c)], the linear combination of all the leg-wrench systems is a 3-ζ_∞-1-ζ_0-system, and the moving platform has two-DOF and can undergo translations along a plane perpendicular to the axes of the R̋ joints in the R̀R̋R̋R̋R̀ leg. In both the planar↔transitional configuration [Fig. 10(b)] and the translational↔transitional configuration [Fig. 10(c)], the R̀R̋R̋R̋R̀ leg can rotate about the axes of its R̀ joints, which are collinear.

From the results in this paper and [18–22], the following observations can be made.

PMwMOMs can be classified into the following three classes:

1) Class I PMs. PMs have no inactive joint no matter which operation mode the PMs work in. One example is the 3-RRRRR PM with both translational and spherical operation modes [20].

2) Class II PMs. PMs always have inactive joints no matter which operation mode the PMs work in. One example is the 3-PRRRR PMs with two 3T1R operation modes [21].

3) Class III PMs. PMs which have inactive joints in some operation modes and have no inactive joint in other operation modes. One example is the R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM (Fig. 8).

Consider an f-DOF PM with two f-DOF operation modes, which is not kinematically redundant. The wrench system of the PM in a transitional configuration is the intersection of the wrench systems of two f-DOF PMs with a mono-operation mode generating the desired f-DOF operation modes, if no leg is singular in the transitional configuration.

4 CONCLUSION

The type synthesis of planar parallel manipulators, which refers to parallel manipulators in which the moving platform undergoes planar motion, has been dealt using the virtual chain approach.

The type synthesis of 3-DOF parallel manipulators with both planar and translational operation modes has been discussed in details. Especially, an improved procedure has been proposed for the type synthesis of legs for parallel manipulators with multiple operation modes in its transitional configuration, which is a key step in the type synthesis of parallel manipulators with multiple operation modes. A number of parallel manipulators with both planar and translational operation modes have been identified. This work lays the foundation for the type synthesis of parallel manipulators with multiple operation modes, especially those parallel manipulators that realize at least one planar mode.

The optimal type selection of parallel manipulators with multiple operation modes and the method for switching a parallel manipulator from one operation mode to another are still open issues.

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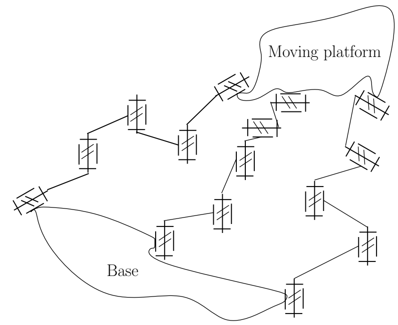
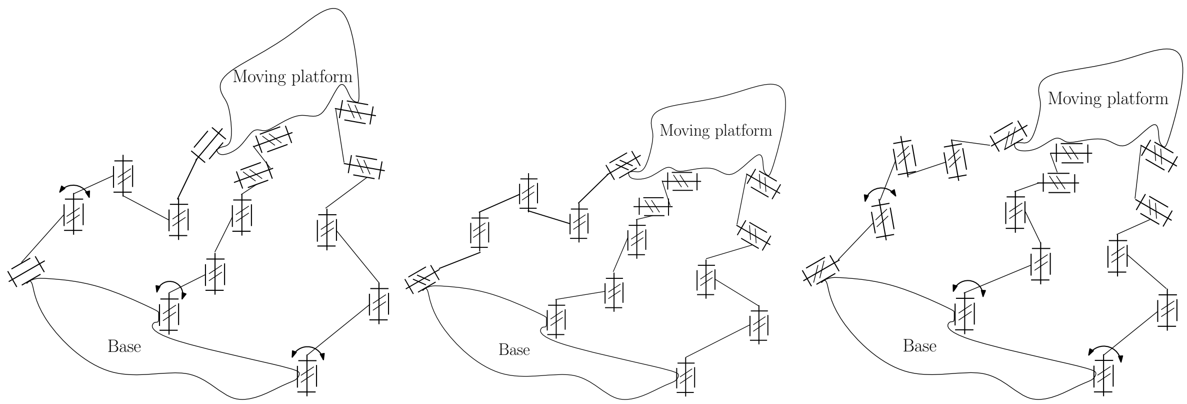


Fig. 8 An R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM in a transitional configuration.



(a) Planar mode. (b) Transitional configuration. (c) Translational mode.

Fig. 9 The reconfiguration of an R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM.

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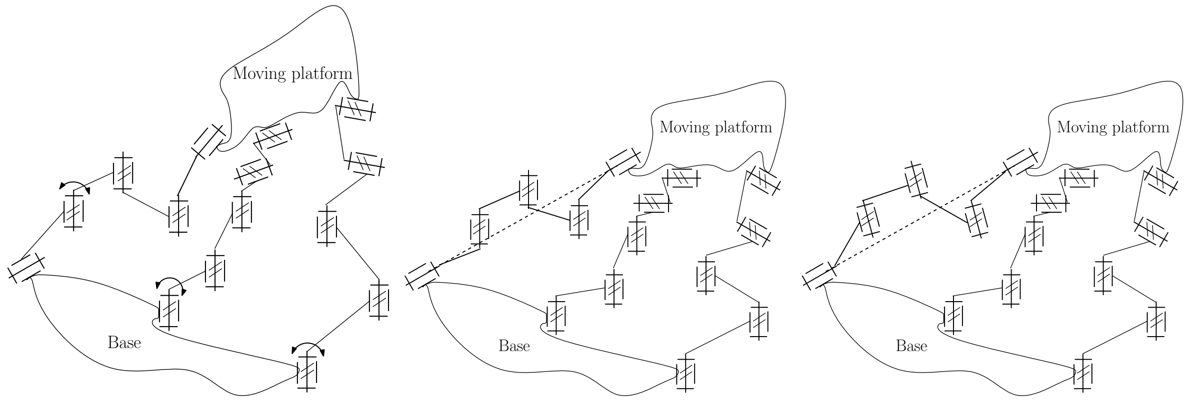
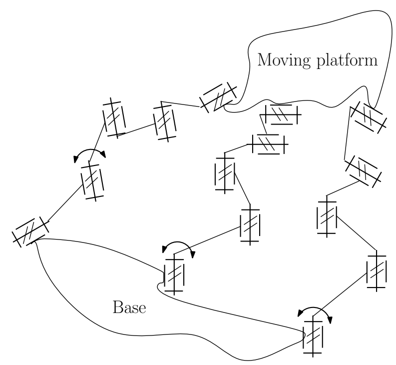


(a) Planar mode. (b) A planar↔transitional configuration. (c) A translational↔transitional configuration.



(d) Translational mode.

Fig. 10 The reconfiguration of R̀R̋R̋R̋R̀-2-R̋R̋R̋R̀R̀ PMwPTM through a leg-singular configuration.

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