Geometric Visual Instruments Having Pinnate Forms

Shunsuke Akimoto Akihiro Matsuura
Division of Information System Design
Tokyo Denki University
pathornda03@gmail.com, matsu@rd.dendai.ac.jp

Abstract

A visual instrument is an instrument that generates geometrically and aesthetically appealing temporal-spatial patterns if it is skillfully manipulated by a player. We present new types of geometric visual instruments having pinnate forms that are used by rolling one instrument along the other. We designed and fabricated the proposed visual instruments and confirmed that they can be manipulated skillfully and they provide a way to explore new types of manipulative play and performances.

Introduction

Geometric curves have been studied for a long time from both theoretical and practical standpoints. One of the most studied cases explores the loci of some point on a geometric shape that rolls along some other shape without slipping. The well known example is a circle rolling on a straight line without slipping. In this case, the locus of a point on the circle is the cycloid. Around the 17th century, the cycloid and the related curves such as involutes were studied extensively by great scientists such as Galileo, Pascal, Huygens, and Bernoulli, not only from mathematical interest but also from engineering and scientific standpoints with applications in mechanics [1], astronomy [2], and even in the design of musical instruments [3]. Recent somewhat entertaining application of involutes is the vehicle with square wheels rolling on a series of truncated catenaries, which was first considered by G. B. Robinson in 1960 and was constructed by Stan Wagon in 1997 [4]. The success of such application should lie in the effective use of geometric properties of rolling curves in a practical setting.

There is also a line of research and implementation of visual patterns of rolling curves through physically manipulating actual geometric objects. We name such an instrument that is skillfully manipulated by a performer using a body, especially, hands, to create aesthetically and geometrically appealing temporal-spatial patterns as the *visual instrument*, just like an instrument designed to make musical expressions through skillful manipulation is called a musical instrument. We have developed visual instruments based on object rolling [5-8]. In [6,7], the visual instrument of rolling a set of balls in a large clear cylinder is presented, where balls rolled in the cylinder make rolling up-and-down motion inside as if they defy gravity. At Bridges Conference 2013, one of the authors presented several types of visual instruments [8], where a player uses a set of sticks or batons to be rolled on a series of truncated cylindrical surfaces.

In this paper, we present additional types of visual instruments that consist of several "pinnate forms" (or we say "pinnacles") as shown in Figure 1. One of such instruments is rolled on the other in both directions without slipping. The paired instruments need not be identical and instruments with different numbers of pinnacles can be used in the manipulation as long as the lengths of the pinnacles are equal. We fabricated the instruments with one to four pinnacles using foamed plastic boards, PLA resin. etc. Through their manipulation, we confirmed that with reasonable amount of practice, we can skillfully manipulate the instruments in various ways including balancing one on the other and rolling one continuously along the other in both directions.

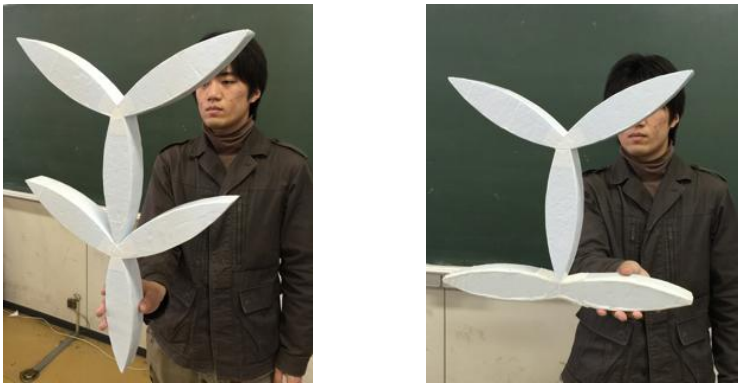

Figure 1: *Balanced visual instruments with three pinnacles (left) and two and three pinnacles (right).*

Rolling Objects, Roulettes, and the Gourds

We consider rolling a geometric object such as a curve and a surface along the other geometric object without slipping as shown in Figure 2. The locus of a point on the rolling object is called the *roulette*. There are several situations such as one of the objects is fixed, or both objects are not fixed but pivot points exist, etc. A pair of a straight line as a fixed curve and a circle as a rolling curve may be the most studied cases. A pair of mechanical gears is another typical case where both gears are rolled. One of the authors has developed visual instruments where some geometric object is rolled by a player in a novel and skillful way to create geometrically and aesthetically appealing visual patterns. The "gourd" shown in Figure 3, left, that consists of circular arcs of identical radii is among them [8]. In this case, the gourd is rolled along the other. Then, one of the end points on the gourd generates the roulette in Figure 3, right. A pair of gourds offers players a new type of manipulation; however, it is quite hard to manipulate them smoothly because the number of touching points between the two gourds changes from one to infinity. So, we present instruments that solve this problem and that can be manipulated smoothly using one hand.

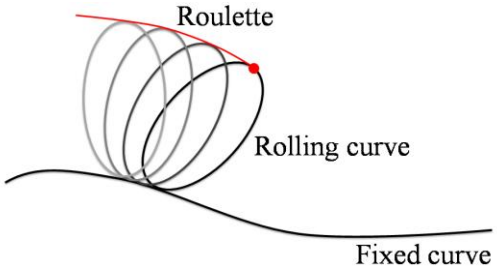

Figure 2: *Rolling a curve along some fixed curve and a roulette.*

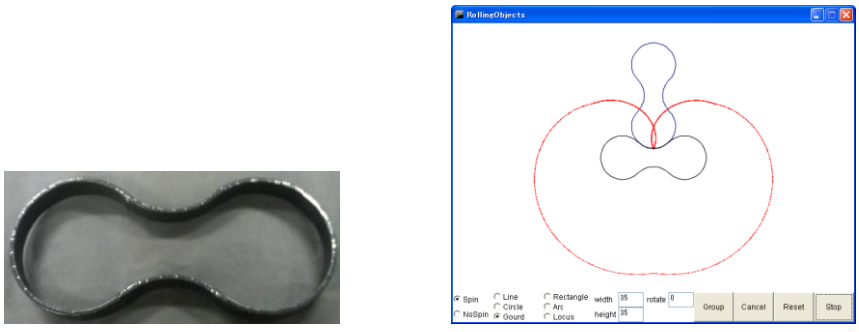

Figure 3: *Gourd (left) and the roulette of an end point on the upper gourd rolled along the lower (right).*

Visual Instruments with Pinnate Forms

The basic idea for solving the problem stated in Section 2 is the adaptation and refinement of mechanical circular gears and the gourds. These objects in their original forms are not easy to manipulate due to the number of touching points as stated in Section 2 and the small angular momentum which makes it hard to stop the objects as they roll in their manipulation.

Thus, we make the circular parts of the gourds thinner with sharpened ends as shown in Figure 4, center, where each object consists of two pinnate convex curves (in the figure, we adopt circular arcs). In Figure 4, right, we generalize them to have three pinnacles. Furthermore, the number of pinnacles can be four or more as long as the outer angle at the center of the instrument is more than or equal to 90 degree not to make an endpoint of the pinnacle got stuck at the center of the other instrument. We also note that the instruments can be different as long as the lengths of the corresponding curves are equal. The geometric visualization of the rolling instrument with three pinnacles and a roulette of a point on the upper instrument are shown in Figure 5. In the next section, we discuss the implementation, fabrication, and actual manipulation of the proposed instruments.

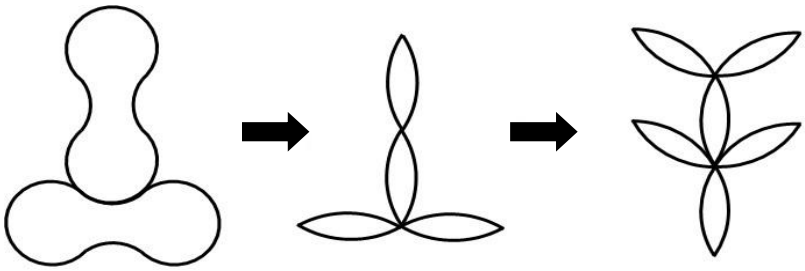

Figure 4: *Balancing of two gourds (left); balancing of two instruments with two pinnate forms (center); and balancing of two instruments with three pinnate forms (right).*

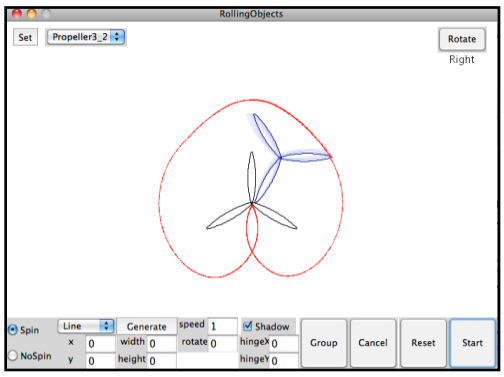

Figure 5: *Visualization of the roulette of a point on the instrument rolling on the other.*

Implementation and Demonstration of the Instruments

With respect to the instruments with three pinnacles, we first fabricated each of the pinnacles and then combined them at the center part made of a small triangle as shown in Figure 6, left. The length of each of the pinnacles is 28cm, the width of the pinnacles is 6cm, and the thickness of the boards from which they were cut is 5cm. As ingredient of the instruments, we tried several materials including expanded polystyrene and PLA resin and found empirically that lighter expanded polystyrene is easier to manipulate.

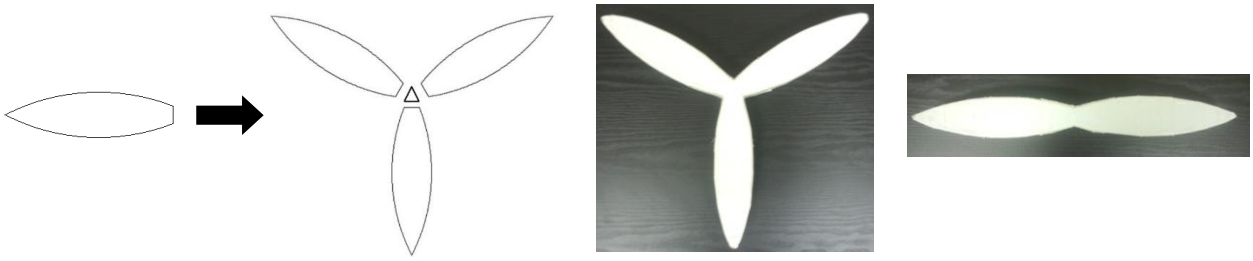

Figure 6: *Implementation of the visual instruments with three pinnate part using expanded polystyrene*.

The rolling of the instruments with three pinnacles is demonstrated in Figure 7. In the figure, the player holds two instruments while balancing one on the other and rolls the upper one along the lower with one hand. The object can be rolled in the same direction without changing the grasp of the lower instrument. It is also possible to roll to the opposite direction. These basic techniques are combined and used to make more complex manipulation. We have also confirmed that manipulation of pairs of instruments with different numbers of pinnacles is also possible. At Bridges 2015, we will demonstrate several of these manipulations and hope to present some dynamic performance.

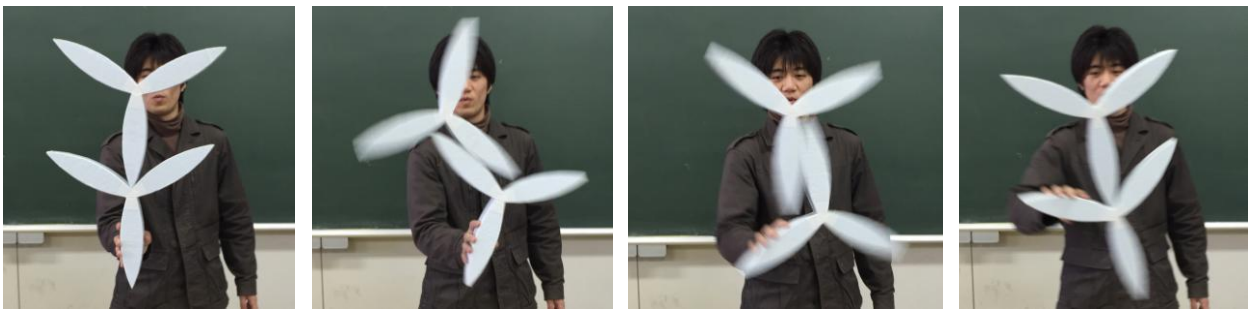

Figure 7: *Basic manipulation of two visual instruments with three pinnacles*.

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