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Natural Convection in Enclosed Porous Media With Rectangular Boundaries

Numerical methods are used to solve the field equations for heat transfer in a porous medium filled with gas and bounded by plane rectangular surfaces at different temperatures. The results are presented in terms of theoretical streamlines and isotherms. From these the relative increases in heat transfer rate, corresponding to natural convection, are obtained as functions of three-dimensionless parameters: the Darcy number Da, the Rayleigh number Ra, and a geometric aspect ratio L/D. A possible correlation using the lumped parameter Da Ra is proposed for Da Ra greater than about 40.

Introduction

The work reported proposes a theoretical model for the motion of a fluid when it is heated in an enclosed porous medium bounded by solid plane surfaces which are different in temperature. The model yields a theoretical rate of heat transfer between these surfaces. This is related to the problem of radial heat transfer from the wall of a nuclear power reactor core, through a multishield structure containing air spaces or porous insulating material, to the pressure vessel.

Most of the studies on natural convection in enclosed spaces have been related to rectangular cavities where an air gap is used for insulation. In these and other cases, it is important to determine the rates of heat transfer across the gap which result from a temperature difference between the opposing faces. Theoretical analysis of the problem usually began with the fundamental differential equations of conservation of mass, momentum, and thermal energy, together with an appropriate equation of state. Where the curvatures of the enclosing surfaces are significant, the problems are generally treated by using cylindrical geometry for which the rectangular geometry is an asymptote. For natural convection in porous media, the problem has been treated similarly by expressing the convective flow in terms of some theoretical model, e.g., Darcy's law.

Early theoretical and experimental studies of convective flow of a fluid in a porous medium were made by Horton and Rogers [1],[3] Morrison, Rogers, and Horton [2], Rogers and Morrison [3], Rogers, Schilberg, and Morrison [4], in 1945–1951, in connection with the distribution of salt in subterranean sand layers. In recent years, in a study of the motion of underground water with particular reference to geothermal activities in New Zealand, Wooding [5–11], Elder [12–15], and McNabb [16], 1957–1967, have reported results on the structure of the flow field and corresponding heat transfer rates for convection of fluid in a porous medium owing to heat generation from below. The criterion for the onset of convective flow was predicted theoretically by Lapwood [17] in 1948, and confirmed experimentally by Katto and Masuoka [18] in 1967. In these studies the term representing viscous forces was neglected in the equation of motion, which was derived from Darcy's law. This may lead to significant errors near the solid boundaries unless a boundary-layer effect is introduced. In a recent paper, two of the present authors, Chan and Ivey [19] 1967, derived the field equations for natural convection in enclosed porous media using a modified form of Darcy's law which takes into account the

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[3] Numbers in brackets designate References at end of paper.

Nomenclature

A = angle of inclination, Fig. 1, rad

C_0 = dimensionless constant

C_p = specific heat of fluid at constant pressure, Btu/lb deg F

D = width of porous bed, ft

$\mathrm{Da} \equiv \dfrac{K}{D^2}$ = Darcy number, dimensionless

F_c = percentage convergence factor, percent

$\mathrm{Gr} \equiv \dfrac{\rho^2 g \beta D^3 \Delta T}{\mu^2} = \dfrac{(T_1 - T_2) g D^3}{T_2 \nu^2}$ = Grashof number

g = gravitational acceleration, ft/hr^2

K = permeability of porous medium, ft^2

k_{ee} = enhanced, or effective, thermal conductivity of gas-filled porous medium, including convective transfer, Btu/hr ft deg F

k_{e0} = effective thermal conductivity of porous medium filled with stagnant gas, Btu/hr ft deg F

k_e = enhanced, or effective, thermal conductivity of fluid in a cavity, including convective transfer, Btu/hr ft deg F

k_g = thermal conductivity of fluid, Btu/hr ft deg F

L = height of porous bed, ft

$\mathrm{Pr} \equiv \dfrac{\nu}{\alpha} \equiv \dfrac{\mu C_p}{k_g}$ = Prandtl number, dimensionless

p = pressure of fluid

$\mathrm{Ra} \equiv \dfrac{(T_1 - T_2) g D^3}{T_2 \alpha \nu}$ = Rayleigh number, dimensionless

T = temperature, deg F; T_1, T_2 = temperatures at hot and cold walls, respectively, Fig. 1

u = the x-component of the superficial fluid velocity, i.e., the volume flow rate per unit cross-sectional area of bed, ft/hr

v = the y-component of the superficial velocity, ft/hr

x = distance along the rectangular coordinate axis which is directed at angle A to the horizontal plane, Fig. 1, ft

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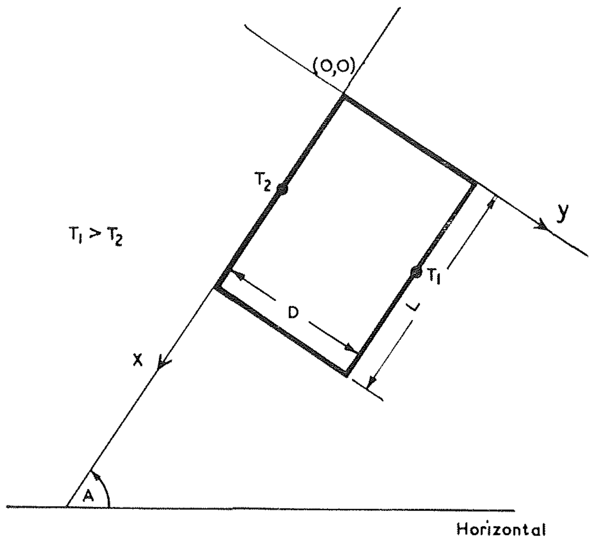

Fig. 1 Coordinates of rectangular geometry notation

viscous forces (Brinkman [20–22], 1947–1949). These equations may be solved numerically, under suitable boundary conditions, to yield useful results and criteria for design.

Theory

The theory is limited throughout to steady state conditions, for which the four equations governing the system are

$$\rho = \rho(T) \tag{1}$$

$$\underline{\nabla} \cdot \underline{u} = 0 \tag{2}$$

$$\underline{u} = \frac{K}{\nu}\left(- \frac{1}{\rho}\underline{\nabla} p + \underline{g} + \nu \nabla^2 \underline{u}\right) \tag{3}$$

$$\underline{u} \cdot \underline{\nabla} T = \alpha_e \nabla^2 T \tag{4}$$

This formulation may be simplified by considering only the two-dimensional motion of the fluid in a space of rectangular cross section filled with unconsolidated particles. T_1 and T_2 in Fig. 1 are the absolute temperatures of the hot and cold boundaries, respectively. The theory is based on the following assumptions: (*a*) the temperature difference $(T_1 - T_2)$ is small compared with T_2; (*b*) the viscosity, density, and thermal conductivity of the fluid are constant except for the effect of density variation in producing buoyancy force; (*c*) the fluid is incompressible, and (*d*) viscous heat dissipation may be neglected.

The superficial velocity $\underline{u}$ in equations (2) to (4) is averaged over a region of space small with respect to macroscopic dimensions in the flow system but large with respect to the pore size. Equation (3), the equation of motion, is the Darcy's law model for the flow regime where the damping force and the viscous force are of the same order of magnitude. In the energy equation, equation (4), the equivalent thermal diffusivity, α_e, is defined by

$$\alpha_e = k_{e0}/\rho C_p \tag{5}$$

where k_{e0} is an equivalent thermal conductivity of the porous medium, for nonflow conditions, taking into account conduction and radiation effects. Methods for estimating k_{e0} have been proposed by Smith [23], 1956, Yagi, et al. [24], 1961, and others. Recently, Katto and Masuoka [18], 1967, verified experimentally that (5) is the correct form for α_e, i.e., the equivalent thermal diffusivity in the energy equation should be defined as the equivalent stagnant thermal conductivity of the porous-medium divided by the specific heat capacity of the fluid.

Boundary conditions of practical interest which are suitable for numerical calculations are: (*a*) uniform temperatures, or heat fluxes, or specified temperature or heat-flux distributions, on the two vertical faces of the space; and (*b*) perfect insulation, or specified temperature distributions, along the horizontal boundaries.

In Cartesian coordinates equations (1) to (4) take the forms

$$\frac{d\rho}{\rho_2} = -\frac{dT}{T_2} \tag{6}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\nu}{K} u = -\frac{1}{\rho_2}\frac{\partial p}{\partial x} - g \sin A \left(\frac{\rho_2 - \rho}{\rho_2}\right) + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{8}$$

$$\frac{\nu}{K} v = -\frac{1}{\rho_2}\frac{\partial p}{\partial y} - g \cos A \left(\frac{\rho_2 - \rho}{\rho_2}\right) + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{9}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_e \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{10}$$

The boundary conditions are:

$$u = v = 0, \quad T = T_1, \quad \text{at} \quad y = 0, \quad \text{for} \quad 0 \le x \le L \tag{11}$$

$$u = v = 0, \quad T = T_2, \quad \text{at} \quad y = D, \quad \text{for} \quad 0 \le x \le L \tag{12}$$

$$u = v = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \text{at} \quad x = 0 \text{ and } L, \text{ for } 0 \le y \le D \tag{13}$$

$$0 \le A \le \frac{\pi}{2} \tag{14}$$

Equations (6) to (10) may be made dimensionless by introducing:

1 The relative distances

$$X = \frac{x}{D}, \qquad Y = \frac{y}{D} \tag{15}$$

for $0 \le X \le \frac{L}{D}$, $0 \le Y \le 1$.

Nomenclature

$X \equiv \frac{x}{D}$ = relative distance on x-axis

y = distance along the rectangular coordinate axis which is directed at angle A to the vertical plane, Fig. 1, ft

$Y \equiv \frac{y}{D}$ = relative distance on y-axis

α = thermal diffusivity, ft^2/hr

$\alpha_e \equiv \frac{k_{e0}}{\rho C_p}$ = equivalent, or effective, thermal diffusivity in porous medium, ft^2/hr

β = thermal coefficient of volumetric expansion, ft^3/ft^3 deg F

$\Delta T \equiv (T_1 - T_2)$ = temperature difference

$\theta \equiv \frac{T - T_2}{T_1 - T_2}$ = relative temperature difference, dimensionless; θ_{ij} = elements in the field matrices for θ

μ = viscosity of the fluid, lb/ft hr

$\nu \equiv \frac{\mu}{\rho}$ = kinematic viscosity of the fluid, ft^2/hr

ρ = density of the fluid, lb/ft; ρ_2 = value of ρ at $T = T_2$

ψ = stream function, dimensionless; ψ_{ij} = elements in the field matrices for ψ

2 The dimensionless stream function ψ, defined by

$$u \equiv \frac{\alpha_e}{D} \cdot \frac{\partial \psi}{\partial Y}, \qquad v \equiv \frac{\alpha_e}{D} \cdot -\frac{\partial \psi}{\partial X}. \tag{16}$$

3 The relative temperature difference

$$\theta \equiv \frac{T - T_2}{T_1 - T_2} \tag{17}$$

for $0 \leq \theta \leq 1$.

4 The dimensionless groups

$$\frac{(T_1 - T_2)gD^3}{T_2 \alpha_e \nu} \equiv \mathrm{Ra} \equiv \mathrm{Gr\ Pr} \tag{18}$$

$$\frac{K}{D^2} = \mathrm{Da} \tag{19}$$

which the authors will term the Darcy number, after Henry Darcy who laid the foundation for the study of laminar flow through porous media.

The resulant field equations for ψ and θ are

$$\nabla^4 \psi = \frac{1}{\mathrm{Da}} \nabla^2 \psi + \mathrm{Ra} \left(\sin A \frac{\partial \theta}{\partial Y} - \cos A \frac{\partial \theta}{\partial X} \right) \tag{20}$$

$$\nabla^2 \theta = \frac{\partial(\theta, \psi)}{\partial(X, Y)} \tag{21}$$

where

$$\nabla^4 \equiv \nabla^2(\nabla^2)$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}$$

and

$$\frac{\partial(\theta, \psi)}{\partial(X, Y)} \equiv \frac{\partial \theta}{\partial X}\frac{\partial \psi}{\partial Y} - \frac{\partial \theta}{\partial Y}\frac{\partial \psi}{\partial X}$$

with the following boundary conditions:

$$\psi = \frac{\partial \psi}{\partial X} = \frac{\partial \psi}{\partial Y} = 0, \qquad \theta = 1, \qquad \text{at } Y = 0, \qquad \text{for } 0 \leq X \leq \frac{L}{D} \tag{22}$$

$$\psi = \frac{\partial \psi}{\partial X} = \frac{\partial \psi}{\partial Y} = 0, \qquad \theta = 0, \qquad \text{at } Y = 1, \qquad \text{for } 0 \leq X \leq \frac{L}{D} \tag{23}$$

$$\psi = \frac{\partial \psi}{\partial X} = \frac{\partial \psi}{\partial Y} = 0, \qquad \frac{\partial \theta}{\partial X} = 0, \qquad \text{at } X = 0 \text{ and } \frac{L}{D}, \qquad \text{for } 0 \leq Y \leq 1 \tag{24}$$

$$0 \leq A \leq \frac{\pi}{2} \tag{25}$$

For given A the solutions of equations (20) and (21) are uniquely determined by the three-dimensionless parameters Da, Ra, and L/D, to give the scalar fields ψ and θ, with the associated streamlines and isotherms.

The case of practical importance is one for which the side surfaces are vertical, i.e., $A = \pi/2$, when equation (20) becomes

$$\nabla^4 \psi = \frac{1}{\mathrm{Da}} \nabla^2 \psi + \mathrm{Ra} \frac{\partial \theta}{\partial Y} \tag{26}$$

The relative increase in heat transfer rate owing to convective flow is given by (Batchelor [25], 1954; Chan and Ivey [19], 1967)

$$\frac{k_{ee}}{k_{e0}} = \frac{1}{L/D} \int_0^{L/D} \left[-\frac{\partial \theta}{\partial Y} \right]_{Y=0} dX \tag{27}$$

where k_{ee} is the enhanced equivalent thermal conductivity which includes the effect of conduction, convection, and radiation. Hence the problem may be expressed parametrically as

$$\frac{k_{ee}}{k_{e0}} = f[\mathrm{Da}, \mathrm{Ra}, L/D] \tag{28}$$

Numerical Methods

The two field equations, (26) and (21), are solved subject to the boundary values given by equations (22), (23), and (24). Since Da is small the first of the field equations is transformed to

$$\nabla^2 \psi = \mathrm{Da} \left[\nabla^4 \psi - \mathrm{Ra} \frac{\partial \theta}{\partial Y} \right] \tag{29}$$

Equations (29) and (21) are then solved by representing the finite-difference forms of the derivative components of the right-hand sides by matrix operators (Woodhead and Kettleborough [26], 1963; De Vahl Davis, and Kettleborough [27], 1965).

Since for $L/D > 5.6$, the number of mesh points makes the matrix method prohibitive, the present method is confined to the results for four values of the ratio L/D: 0.2, 0.5, 1, and 5.6.

As the field matrix technique has been described elsewhere, (see [26, 27]) only the main features will be given here. The finite-difference formulas of Bickley ([28–30], 1939–1948) are used to construct differential operators for

$$\frac{\partial^4}{\partial X^4}, \frac{\partial^4}{\partial Y^4}, \frac{\partial^2}{\partial X^2}, \frac{\partial^2}{\partial Y^2}, \frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \text{ etc.}$$

An initial estimate is made for $[\psi]$ and $[\theta]$, of $\psi_{ij} = 0$ everywhere, and the temperature field is assumed to be linearly distributed, $\theta_{ij} = Y_i$.

The right-hand side of equation (29) is then evaluated using the initial $[\psi]$ and the differential operators. The improved value for $[\psi]$ is found by the inversion of $[\nabla^2]$ and use of the extrapolated Liebman method.

The new estimate of $[\psi]$ together with the old estimate of $[\theta]$ is used to evaluate the right-hand side of equation (21) in a similar manner to that just described, giving a new estimate of $[\theta]$. This completes one iterative cycle. New and old values of $[\psi]$ and $[\theta]$ are compared for satisfactory convergence. Iteration continues until convergence is satisfactory.

Upon completion of solution for a particular Da and Ra, the Ra is increased and the old solution is used as initial values for the new problem. Ra is continually increased until convergence is no longer obtainable.

Computations

The computer used is an IBM 360/50H.

For the square geometry, where $L/D = 1$, a mesh size of $^1/_{10}$ is used, giving (11 × 11) field matrices. This in turn produces operators of order (81 × 81) and (99 × 99) to be inverted for solution of equations (29) and (21), respectively.

For the rectangular case, where $L/D = 5.6$, a mesh size of $^1/_5$ is used on the shorter side, producing field matrices of order (6, 29), and inverse operators of order (108, 108) and (116, 116).

The inversion is performed by the standard Gauss-Jordan method with good results. These are kept on magnetic tape for frequent use.

The criterion used for convergence is

$$\frac{\left[\sum_{ij} \left| \psi_{i,j_{\text{new}}} \right| - \sum_{ij} \left| \psi_{i,j_{\text{old}}} \right| \right]}{\left[\sum_{ij} \left| \psi_{i,j_{\text{old}}} \right| \right]} \times^{100} \leq F_c \text{ percent}$$

The value chosen for F_c percent is usually 0.001 percent. This test is also applied to the θ field.

One full iteration cycle takes approximately 2 sec. The number of iterations required for convergence varies greatly ranging from 1 to about 200, being dependent on the L/D configuration, the value of F_c, the product of Da and Ra, and the weighting factor in the extrapolated Liebman method.

Initial results are obtained for

$$\mathrm{Da} = 10^{-10}, 10^{-9}, 10^{-8}, \ldots . 10^{-1}, 1 \tag{30}$$

at

$$\mathrm{Ra} = 1, 10, 10^{2}, \ldots . . 10^{9}, 10^{10} \tag{31}$$

For a fixed value of Da, the computation is commenced at Ra = 1, increasing Ra in steps of powers of 10 until convergence is no longer obtainable. For the $L/D = 1$ geometry for the 11 values of Da given by equation (30), convergence is not achieved at Ra = 10^{12}, 10^{11}, 10^{10}, 10^{9}, 10^{8}, 10^{7}, 10^{6}, 10^{5}, 1, 1, 1, respectively. Final results are then obtained for small increments of Ra close to values where convergence failed to be achieved.

For a fixed value of Da, if Da Ra $\gtrsim$ 130 and Da is just low enough for convergence, the θ field elements exceed 1 at points corresponding to the upper corner of the hot wall and fall below 0 at those corresponding to the lower corner of the cold wall. These are obviously extraneous results since, from physical considerations, no temperature can be higher than 1 or lower than 0 for constant temperature walls. This apparent anomoly may be due to the use of too few mesh points. Further work employing an iterative numerical technique, incorporating finer mesh, seems to confirm this. Likely localized secondary flow effects, similar to those noted by Elder [13], would first take place at those corners where more mesh points may therefore be required. Moreover, these extraneous results may indicate the onset of physical instability (transition from laminar to turbulent flow) and/or mathematical instability (limit of applicability of the system equations).

Results and Discussion

Since the present numerical method depends largely on the inversion matrix $[\nabla^{-2}]$, this matrix was first established. The determinants of the inversion matrices for all cases studied, L/D = 0.2, 0.5, 1, 2, 5.6, and 10, were also calculated. These had large values of the order of 10^{40}, indicating that the matrix $[\nabla^{2}]$ is nonsingular which means its inverse exists and that the linear set of equations are well-conditioned. For a given geometry, i.e., fixed L/D, varying the mesh sizes did not appear to have significant effect on the rate of convergence or on the final results.

The theoretical isotherms and streamlines for the square geometry, at Da = 10^{-4} and Ra = 10^{4}, are compared with the results of Poots [31], 1958, for a square cavity, also at Ra = 10^{4}. Fig. 2 shows the predicted relative increase in heat transfer rate corresponding to convective flow, as a function of the Rayleigh number, for Da = 10^{-4}. This is compared with the analytical results of Poots up to Ra = 10^{4}, and with the experimental result of Mull and Reiher (quoted by Jakob [32], 1949) for $10^{4} \leq$ Ra $\leq 10^{5}$, the correlations being

$$\text{Poots:}\quad \frac{k_e}{k_g} = 0.16\ \mathrm{Ra}^{0.25}, \qquad \mathrm{Ra} \leq 10^{4} \tag{32}$$

$$\text{Mull and Reiher:}\quad \frac{k_e}{k_g} = 0.18\ \mathrm{Ra}^{0.25}(L/D)^{-0.11}, \qquad 10^{4} \leq \mathrm{Ra} \leq 10^{5} \tag{33}$$

where k_g is thermal conductivity of the fluid, and k_e is the enhanced, or effective, thermal conductivity on account of convective flow.

The results of Poots were obtained for a square cavity uniformly heated at the right-hand-side vertical wall. Hence the streamlines for the two cases have opposing directions. Also, in Poots' calculations, it was assumed that the temperature distribution along the horizontal end walls is linear. This comparison seems to indicate that for a given geometry and at a given Rayleigh number, the presence of a porous material in an enclosed space would considerably modify the temperature and velocity fields: reducing the rate of convection. Fig. 2 indicates that the present theory predicts the following: (i) for Da = 10^{-4}, below Ra = 10^{5} the rate of heat transfer through an enclosed porous medium is substantially the same as pure conductive transfer (in addition to radiation); (ii) above Ra = 10^{5}, natural convection produces an enhancement of the transfer rate similar to that which occurs at Ra = 10^{3} for an enclosed cavity; and (iii) the rate of increase of heat transfer also varies approximately as the one-fourth power of the Rayleigh number.

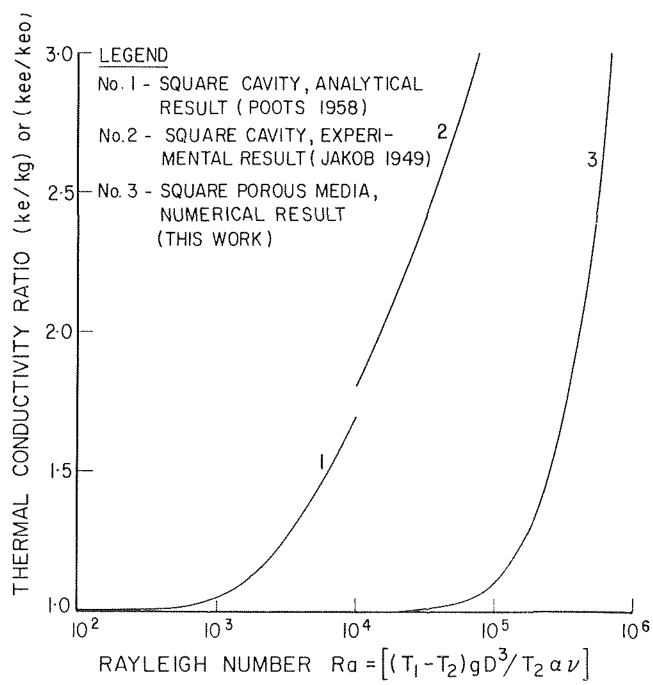

Fig. 2 Variation of heat transfer with Rayleigh number

Figs. 3 and 4 show isotherm and streamline patterns for the square geometry: Da = 10^{-5}, and Ra = 10^{7}. Figs. 5 and 6 show some of the corresponding patterns for the rectangular geometry, where $L/D = 5.6$. Clearly the patterns predict that the following occur as Ra increases:

1 A gradual and increasing development of convective motion, accompanied by distortion of the temperature field as compared with that for pure conduction.

2 A gradual development of boundary layers, downward on the cold wall and upward on the hot wall.

These predictions are consistent with the known phenomenon of natural convection in enclosed cavities.

From these ψ and θ maps the following information may also be derived: (i) vertical and horizontal temperature gradients, $\dfrac{\partial\theta}{\partial X}$ and $\dfrac{\partial\theta}{\partial Y}$; (ii) vertical and horizontal velocity components u and v (from equation (16)); (iii) the local speed, $(u^2 + v^2)^{1/2}$, and hence localization of stagnation regions; and (iv) the distribution of heat transfer coefficient along the vertical walls, from $\left(\dfrac{\partial\theta}{\partial Y}\right)_{Y=0 \text{ and } 1}$.

In Fig. 7 the parameter k_{ee}/k_{e0} of equation (27) is plotted against Ra for various values of Da with the square geometry. Similar plots are obtained showing corresponding results for the other geometries. The plots show the following:

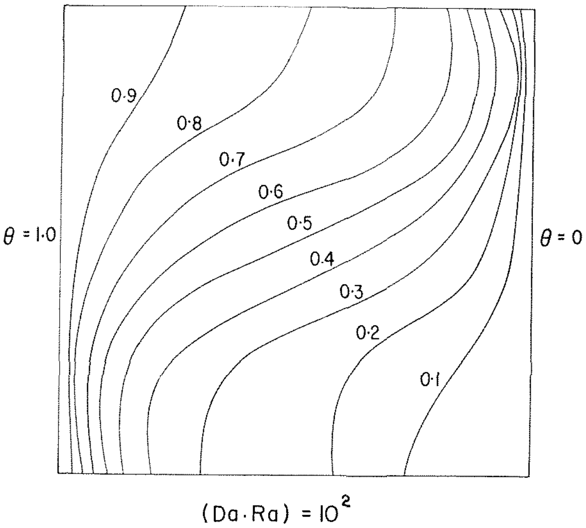

Fig. 3 Isotherms for $Da = 10^{-5}$, $Ra = 10^7$, $L/D = 1.0$

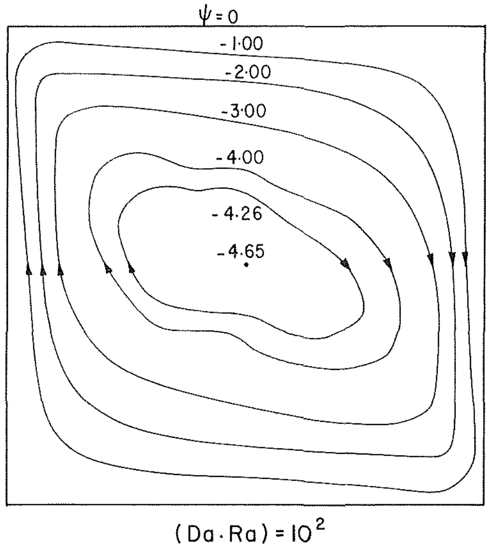

Fig. 4 Streamlines for $Da = 10^{-5}$, $Ra = 10^8$, $L/D = 1.0$

1 The curves are approximately parallel and equally spaced, so that for fixed k_{ee}/k_{e0}, Da decreases by powers of 10 as Ra increases by powers of 10. Hence, for a given value of k_{ee}/k_{e0}, Ra = constant/Da. This implies that k_{ee}/k_{e0} is uniquely determined by Da Ra, or $k_{ee}/k_{e0} = f$ (Da Ra) for given L/D which is predicted by equation (28).

2 The critical value of Ra, i.e., the value at which the onset of convection occurs, increases exponentially with exponentially decreasing Da.

3 For fixed values of Da, the ratio k_{ee}/k_{e0} varies exponentially, with Ra; and for fixed Ra, it varies exponentially with Da.

It would seem therefore that, for fixed L/D, the ratio k_{ee}/k_{e0} varies exponentially with Da Ra. This is verified in Fig. 8 in which k_{ee}/k_{e0} is plotted against Da Ra on logarithmic coordinates. For Da Ra > 40, the k_{ee}/k_{e0} values increase beyond unity, generating unique curves which are approximately linear and parallel for each of the six geometries studied. Hence an equation expressing a tentative correlation with these parameters may take the form:

$$\frac{k_{ee}}{k_{e0}} = f(L/D)(\mathrm{Da\ Ra})^{C_0}, \qquad \text{for Da Ra} > 40 \qquad (34)$$

where C_0 is a dimensionless constant ($C_0 \simeq 0.7$). This confirms equation (28).

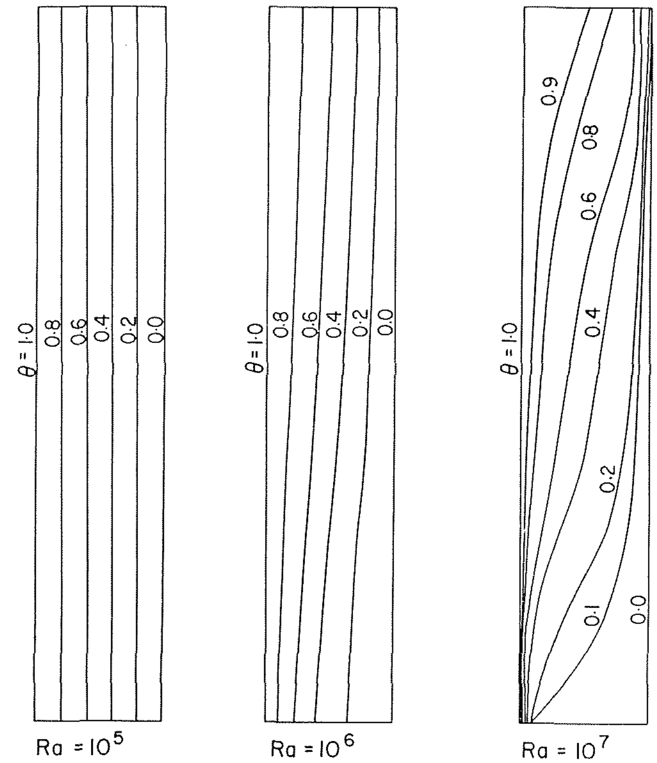

Fig. 5 Isotherms for $Da = 10^{-5}$, $Ra = 10^5$, 10^6, and 10^7, $L/D = 5.6$

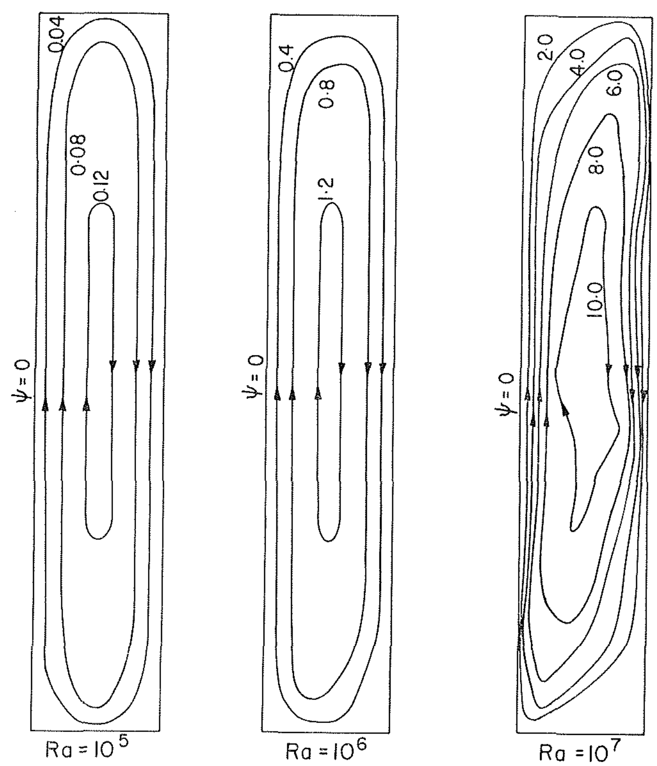

Fig. 6 Streamlines for $Da = 10^{-5}$, $Ra = 10^5$, 10^6, and 10^7, $L/D = 5.6$

That Da Ra would be an approximate criterion of heat transfer in the present system may be seen from equation (26). The left-hand-side of this equation represents the viscous forces which may be small. If they are neglected, the equation becomes

$$\nabla^2\psi + \mathrm{Da\ Ra}\,\frac{\partial \theta}{\partial Y} = 0 \qquad (35)$$

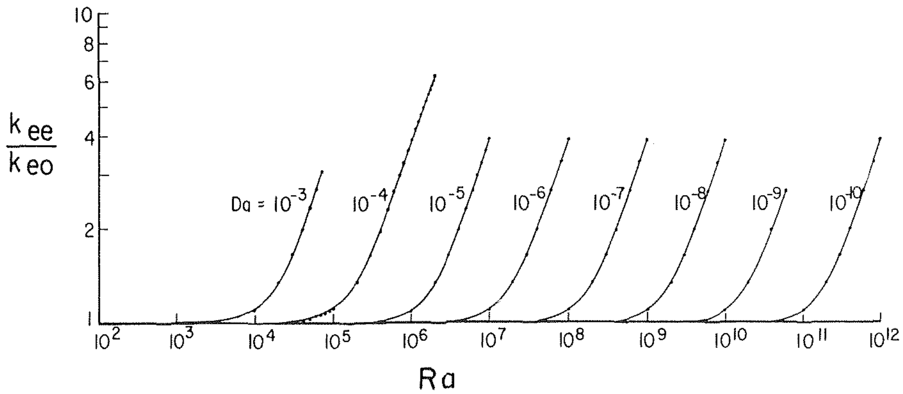

Fig. 7 Variation of thermal conductivity ratio k_{ee}/k_{e0} with Ra for various values of Da, square geometry (L/D = 1.0)

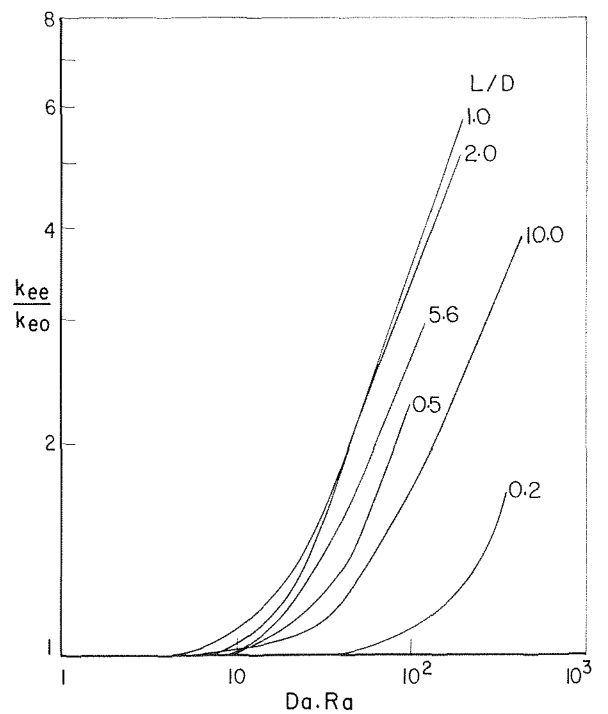

Fig. 8 Correlation of thermal conductivity ratio k_{ee}/k_{e0} with lumped parameter Da Ra

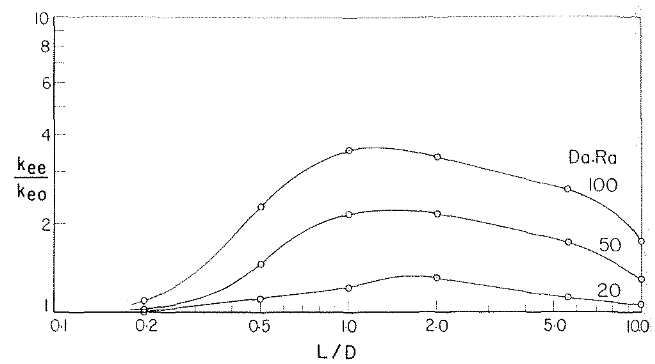

Fig. 9 Variation of thermal conductivity ratio k_{ee}/k_{e0} with L/D for various values of the lumped parameter Da Ra

It may be of interest to note that the criterion for the onset of convective flow of fluid in a porous medium, heated from below, is given by:

$$\text{Da Ra} = 4\pi^2 = 39.5 \quad (36)$$

a result deducible from the analysis of Lapwood [17] who used equation (35), and from the experimental work of Katto and Masuoka [18].

The function $f(L/D)$ may be examined by a plot of k_{ee}/k_{e0} versus L/D for fixed values of Da Ra, Fig. 9. It appears that the relative increase in the heat transfer rate has a maximum value in the vicinity of the L/D ratio of 1.5, a result which is similar to the case of the enclosed cavity (Hirata, et al. [33] 1967). At the two limits of the L/D ratio: as $L/D \to \infty$ ($L \to \infty$, or $D \to 0$), and $L/D \to 0$ ($L \to 0$, or $D \to \infty$), natural convection is either suppressed owing to increasing resistance to flow (for the cases $L \to 0$, and $D \to 0$), or it is dominated by pure conduction (for the cases $L \to \infty$, and $D \to \infty$). Hence the ratio k_{ee}/k_{e0} approaches unity at these limits, when it may have a maximum value between them. At $L/D = 1.5$, it seems reasonable to expect that a balance of these effects occur and hence the relative increase in the heat transfer rate reaches a maximum.

Conclusion

The analysis presented suggests a correlation of heat transfer rate in a porous medium bounded by surfaces at different temperatures. Theoretical flow patterns for an enclosed fluid, and temperature distributions within the medium, were obtained by a numerical method. These patterns are at least qualitatively consistent with known convective phenomenon. Rapid convergence for the numerical technique may be achieved by using field matrices and an inversion matrix.

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