Thermal and Inertia Effects in Hydrodynamic Lubrication of Rollers by a Power Law Fluid Considering Cavitation

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A theoretical aspect of hydrodynamic lubrication of two symmetric rollers by power law fluids is analyzed. The effect of fluid convective inertia, which is significant in case of high speed bearing, is taken into account. The effect of hydrodynamic pressure and temperature on the lubricant consistency m is assumed to vary with pressure and the mean temperature. The squeezing motion of the surfaces is also incorporated along with inertia and thermal effects. The Reynolds equation and the energy equation (with convection and conduction), which are coupled through m, are solved simultaneously. Various bearing characteristics such as pressure, temperature, load and drag etc. are obtained and a comparison between results (with and without inertia) is also made. It is noted that the effect of inertia is to increase pressure, temperature, load and drag etc. and to displace the position of pressure peak slightly towards the center line of contact of the rollers. An attempt is also made to study the variation of film thickness with load, speed, Eckert number, pressure, and temperature viscosity exponents.

Introduction

Most of the early analytic treatment of line contact lubricated bearings, such as cylindrical roller bearings, cylinder on a plane type bearings, has been based on the solution of Reynolds equation embodied with the assumption that the lubricant film is isothermal (Floberg, 1961 and Dowson et al., 1976). This isothermal theory may be valid for the lightly loaded bearings, where hydrodynamic pressure and temperature are not high enough to produce significant changes in material properties of the lubricant. An experimental result (in support of it) on a lightly loaded bearing has also been reported by Markho and Dowson (1976).

However, the bearings operating at heavy loads and high speeds, encounter extremes of pressure and temperature. This pressure-temperature field becomes even stronger if bearings are subjected to squeezing motion as well (Rohde and Ezzat, 1974). In such cases, material properties of the lubricant require a careful reappraisal. In oil lubricated bearings, it is the viscosity which changes most dramatically (Gethin, 1987). Thus for an accurate prediction of bearing characteristics, the conservation equation of thermal energy is also required in addition to the conservation equations of mass and momentum.

The need of investigating the temperature field arises basically due to a strong dependence of lubricant viscosity on temperature. Hence bearing characteristics predicted using isoviscous theory are found to deviate from the experimental result (Rajlingham, 1987). Several attempts have been made in this direction. Some of the early important investigations on heavily loaded lubricated line contacts have been presented by Crook (1961) and Cheng and Sternlicht (1965). Crook studied the thermal effects in lubrication of rigid cylindrical rollers by a Newtonian incompressible fluid theoretically and experimentally. Results for pressure, temperature, film thickness, and traction were obtained. Cheng and Sternlicht studied a far more difficult case of this problem theoretically. Assuming the lubricant to be compressible and the rollers to be elastic, they investigated the thermal effects and presented various important bearing characteristics. More recently, Sadeghi and Dow (1987) presented a two-dimensional solution to the problem of thermal EHD lubrication of rolling/sliding contacts. The Reynolds, the energy and the elastic equations have been solved simultaneously giving pressure and temperature distributions along with the film thickness. Sadeghi et al. (1987) also attempted the same problem using an approximate method for the prediction of mid-film temperature and sliding traction.

Furthermore, in the heavily loaded system where high pressure exceeding 10^7 dyne/cm^2 is encountered, lubricant viscosity can no longer remain insensitive to pressure. Archard (1961), Dowson (1965), Conry (1981), Rong-Tsong and Hamrock (1989), among the pioneers in this field, also emphasized the need for considering viscosity variation with respect to pressure. *It is therefore imperative to account for the effect of* pressure also on the lubricant viscosity.

Another important factor which influences bearing characteristics in case of high speed bearing lubricated with a low

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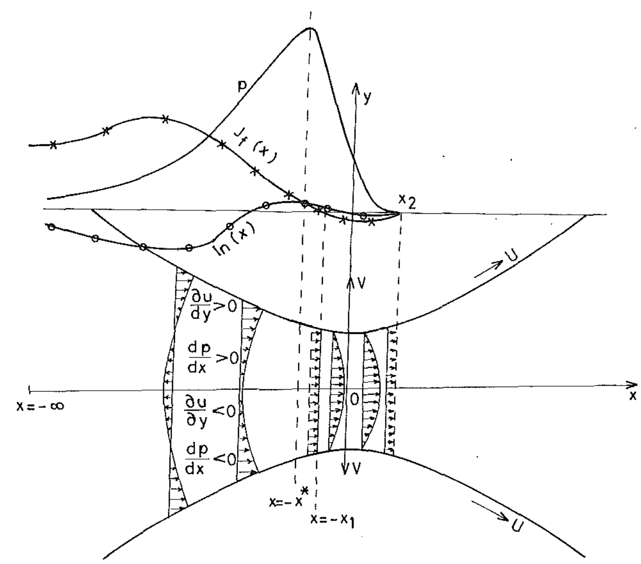

Fig. 1 Lubrication of two identical cylinders together with pressure profile p, viscous distribution $J_f(x) - J_{-f}(x)$ and inertia distribution $I_n(x)$

viscosity fluid and subjected to a squeezing motion is the fluid inertia (Hashimoto and Wada, 1986 and Elkouh, 1976). This effect is important even in case of laminar flow where an increase in load capacity is observed (Mori et al., 1985). An increase in hydrodynamic film thickness may also sometimes compel one to consider this effect. Dowson et al. (1980) conducted a wide range of experiments on a cylinder-plane bearing. It is observed that for a moderately large Reynolds number, the experimental data deviated substantially (due to fluid inertia) from the theoretical values predicted by classical Reynolds equation using the Coyne-Elrod film rupture conditions. Very recently, You and Lu (1987) presented a theoretical analysis of a cylinder-plane bearing and journal bearing. They have shown that the lubricant inertia has a profound effect even at moderate values of the Reynolds number.

In addition, the Newtonian behavior of the lubricant in concentrated contacts is also a matter of investigation. In fact, since the lubricant is subjected to extremely high pressures and shear-stresses, which act for a short time, the Newtonian hypothesis for the lubricant may not be valid (Rashid and Seireg, 1987). Besides, severe operational requirements have necessitated the increasing use of lubricants, such as mineral oils with high molecular weight polymer additives. These polymers improve the viscosity index of the oil and at the same time make the lubricant non-Newtonian (Williams and Simmons, 1987). Attention has been focused by several investigators to model non-Newtonian flow in bearing lubrication analysis. Among the various non-Newtonian models postulated in the recent years, the power law model has perhaps found the most extensive applications (Sinha and Singh, 1982).

In the present analysis, the problem of two infinite rigid cylindrical heavily loaded rollers lubricated with power law fluids under rolling and squeezing motions is studied including the effects of fluid inertia and cavitation. The lubricant pressure is assumed to be constant across the film thickness and the lubricant consistency is allowed to vary exponentially with pressure and the mean film temperature. The modified Reynolds and energy equations are obtained and solved simultaneously yielding pressure and temperature.

Mathematical Formulations

Governing Equations. Making use of the constitutive relation for a power law fluid, applying the usual assumptions of hydrodynamic lubrication for the geometry given in Fig. 1 and retaining the fluid inertia terms, one obtains the following governing equations for an incompressible power law fluid:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\frac{dp}{dx}=\frac{\partial}{\partial y}\left(m\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{1}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

The lubricant temperature field resulting from viscous dissipation is defined by the energy equation

$$\rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}+m\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Nomenclature

- A = $(n/(2n+1))^n$
- B = $2(2n+1)/(3n+2)$
- c = specific heat of the lubricant at constant volume
- c_n = $\left(\frac{4n+2}{n}\right)^n (U/h_o)^n\ (2R/h_o)^{1/2}$
- E_t = dimensionless number $(= U^2\beta/C)$
- f = $X^2 - X_1^2 + 2q(X+X_1)$
- h = film thickness
- h^* = film thickness at the maximum pressure $(x=-x^*)$
- h_o = minimum film thickness
- $\bar{h}_o$ = dimensionless film thickness at $x=-x_1$ (h_o/R)
- H = $1+X^2$ etc.
- K = lubricant thermal conductivity
- m = variable lubricant consistency
- m_o = constant consistency
- $\bar{m}$ = $2\text{m}\ c_n\ \alpha$ etc.
- n = flow behavior index
- p = hydrodynamic pressure
- $\bar{p}$ = αp
- P_r = $(Ch_o/4K\alpha U)\ (h_o/2R)^{1/2}$
- q = squeezing parameter $\left(=\frac{V}{U}\sqrt{\frac{R}{2h_o}}\right)$
- r = radius of the cylinder
- R = equivalent radius
- T = lubricant temperature
- $\bar{T}$ = βT etc.
- T_h = surface temperature
- T_m = the mean film temperature as defined in (5)
- T_o = ambient temperature
- u = lubricant velocity in x-direction
- $\bar{u}$ = u/U
- $\bar{u}_m$ = the mean velocity as defined in (24)
- U = velocity of the cylinder
- $\bar{U}$ = (U/R)/time
- v = lubricant velocity in y-direction
- $\bar{v}$ = $(2v/U)\ \sqrt{(2R/h_o)}$
- V = $V/2$ is the normal velocity of the cylinder
- W_s = load carrying capacity
- $\bar{W}_s$ = $W_s/(\sqrt{2Rh_o}/\alpha)$
- W_T = surface traction
- $\bar{W}_T$ = $-W_T/(h_o/2\alpha)$
- x^* = point of maximum pressure
- x_1 = point at which $\frac{\partial u}{\partial y}=0$
- x_2 = cavitation point
- X = $x/\sqrt{2Rh_o}$ etc.
- α = pressure coefficient
- β = temperature coefficient
- γ = $P_r E_t(n/(3n+1))$
- γ_1 = $P_r E_t(n/(4n+1))$
- ρ = lubricant density
- λ = modified Reynolds number

Subscripts

- 1, 2 = refer to the respective quantities in the inlet and outlet regions

Bar denotes dimensionless number

Assuming that the lubricant consistency m varies with pressure and temperature according to the following

$$m=m_o e^{\alpha p-\beta(T_m-T_o)} \tag{4}$$

where T_m is the mean temperature derived from the fluid temperature T as follows:

$$T_m=\frac{1}{h}\int_{-h/2}^{h/2} T\,dy \tag{5}$$

m_o is the lubricant consistency at $p = 0$ and $T_m = T_o$; α and β are, respectively, the coefficients of pressure and temperature.

Boundary Conditions. The boundary conditions for Eqs. (1) and (2) are

$$u=U,\ v=\frac{U}{2}\frac{dh}{dx}+\frac{V}{2} \text{ at } y=h/2 \tag{6}$$

$$\frac{\partial u}{\partial y}=0,\quad v=0 \quad \text{at} \quad y=0; \qquad p=0 \quad \text{at } x=-\infty \tag{7}$$

$$p=0,\ \frac{dp}{dx}=0, \text{ at } x=X_2 \tag{8}$$

and for Eqs. (3) are

$$T=T_o \text{ at } x=-\infty \tag{9}$$

$$T=T_h \text{ at } y=h/2 \tag{10}$$

$$\frac{\partial T}{\partial y}=0 \text{ at } y=0 \tag{11}$$

Dimensionless Scheme. Making use of the following dimensionless values:

$$Y=2y/h_o,\ \bar{u}=u/U,\ \bar{v}=\frac{2y}{U}(\sqrt{2R/h_o}),$$

$$\bar{p}=\alpha p,\ X=\frac{x}{\sqrt{2Rh_o}} \tag{12}$$

$$\lambda=\rho U^2\alpha,\ P_r=\frac{Ch_o}{4UK\alpha}(\sqrt{h_o/2R}),\ E_t=U^2\beta/C,$$

$$\bar{T}=\beta T,\ H=h/h_o,\ P_e=\lambda P_r \tag{13}$$

Equations (1)–(5) can be rewritten as follows:

$$\lambda\left(\bar{u}\frac{\partial \bar{u}}{\partial X}+\bar{v}\frac{\partial \bar{u}}{\partial Y}\right)+\frac{d\bar{p}}{dX}=A\frac{\partial}{\partial Y}\left(\bar{m}\left|\frac{\partial \bar{u}}{\partial Y}\right|^{n-1}\frac{\partial \bar{u}}{\partial Y}\right) \tag{14}$$

$$\frac{\partial \bar{u}}{\partial X}+\frac{\partial \bar{v}}{\partial Y}=0 \tag{15}$$

$$P_e\left(\bar{u}\frac{\partial \bar{T}}{\partial X}+\bar{v}\frac{\partial \bar{T}}{\partial Y}\right)=\frac{\partial^2 \bar{T}}{\partial Y^2}+\bar{m}\,P_rE_tA\left|\frac{\partial \bar{u}}{\partial Y}\right|^{n-1}\left(\frac{\partial \bar{u}}{\partial Y}\right)^2 \tag{16}$$

$$\bar{m}=\bar{m}_o e^{\bar{p}-(\bar{T}_m-\bar{T}_o)} \tag{17}$$

$$\bar{T}_m=\frac{1}{H}\int_o^H \bar{T}\,dy \tag{18}$$

where

$$\bar{m}=2m\,c_n\alpha \text{ etc.} \tag{19}$$

$$A=(n/(2n+1))^n \tag{20}$$

$$c_n=\left(\frac{2n+1}{n}\right)^n(U/R)^n(2R/h_o)^{n+1/2} \tag{21}$$

The presence of inertia and convection terms in (14) and (16) makes them complicated. Also, since their contributions are small in comparison to the other terms (occurring in the same equations), their average/mean values are not likely to differ significantly from their unaveraged values. Consequently, Eq. (14) after averaging may be written as

$$\frac{\lambda}{H}\int_o^H\left(\bar{u}\frac{\partial \bar{u}}{\partial X}+\bar{v}\frac{\partial \bar{u}}{\partial Y}\right)dY+\frac{d\bar{p}}{dX}=A\frac{\partial}{\partial Y}\left(\bar{m}\left|\frac{\partial \bar{u}}{\partial Y}\right|^{n-1}\frac{\partial \bar{u}}{\partial Y}\right) \tag{22}$$

Similarly assuming the contribution of second term $\bar{v}\ \partial\bar{T}/\partial Y$ of (16) to be small (Dowson and Hudson, 1963) and $\partial\bar{T}/\partial X \simeq d\bar{T}_m/dX$ (Cheng and Sternlicht, 1965), Eq. (16) may be approximated as

$$P_e\bar{u}_m\frac{d\bar{T}_m}{dX}=\frac{\partial^2 \bar{T}}{\partial Y^2}+\bar{m}\,P_rE_tA\left|\frac{\partial \bar{u}}{\partial Y}\right|^{n-1}\left(\frac{\partial \bar{u}}{\partial Y}\right)^2 \tag{23}$$

where velocity $\bar{u}$ occurring in the convection term has been taken as

$$\bar{u}_m=(\bar{u}|_{Y=0}+\bar{u}|_{Y=H})/2 \tag{24}$$

Solutions. Define

$$F=\frac{\lambda}{H}\int_o^H\left(\bar{u}\frac{\partial \bar{u}}{\partial X}+\bar{v}\frac{\partial \bar{u}}{\partial Y}\right)dY+\frac{d\bar{p}}{dX} \tag{25}$$

$$D=P_e\bar{u}_m\frac{d\bar{T}_m}{dX} \tag{26}$$

It may be noted that F and D are functions of x-alone. Solving Eq. (22) for $\bar{u}$, using the following boundary conditions:

$$\frac{\partial \bar{u}}{\partial Y}=0,\ \bar{v}=0 \text{ at } Y=0 \tag{27}$$

$$\bar{u}=1,\ \bar{v}=-2q+\frac{dH}{dX} \text{ at } Y=H \tag{28}$$

with

$$q=\frac{V}{U}\sqrt{\frac{R}{2h_o}} \tag{29}$$

One obtains for $Y \geq 0$

$$\bar{u}=1+\left(\frac{n}{n+1}\right)\left(\frac{F}{A\bar{m}_1}\right)^{1/n}\left(Y^{\frac{n+1}{n}}-H^{\frac{n+1}{n}}\right) \tag{30}$$

The volume flux Q, defined by

$$Q=\int_o^H \bar{u}\,dY \tag{31}$$

is obtained as

$$Q=H-\left(\frac{n}{2n+1}\right)\left(\frac{F}{A\bar{m}_1}\right)^{1/n}H^{\frac{2n+1}{n}} \tag{32}$$

Integration of continuity Eq. (15), using the boundary conditions (27) and (28), gives

$$\frac{\partial}{\partial X}\int_o^H \bar{u}\,dY=-2q$$

or

$$Q=-2qX+d \tag{33}$$

where d is the constant of integration.

Use of relations (30), (32) in (22) yields

$$I_n(X)+\frac{d\bar{p}_1}{dX}=J_f(X) \tag{34}$$

where

$$f=H-2qX-d \tag{35}$$

$$J_f(X)=\bar{m}_1 f^n/H^{2n+1} \tag{36}$$

$$I_n(X)=\frac{\lambda}{H}\left[2X\left\{-1+\frac{2Bf}{H}-\frac{Bf^2}{H^2}\right\}+4q\left(\frac{Bf}{H}-\frac{1}{2}\right)\right] \tag{37}$$

$$B=2(2n+1)/(3n+2) \tag{38}$$

Now $\frac{\partial \bar{u}}{\partial Y}=0$ at $X=-X_1$ implies

$$f=X^2-X_1^2+2q(X+X_1) \quad (39)$$

Similarly Eq. (16) can be solved for $\overline{T}$, using the boundary conditions

$$\frac{\partial \overline{T}}{\partial Y}=0 \text{ at } Y=0 \quad (40)$$

$$\overline{T}=\overline{T}_h \text{ at } Y=H \quad (41)$$

Thus

$$\overline{T}_1=\overline{T}_h+\frac{D}{2}(Y^2-H^2)$$

$$+\gamma\overline{m}_1 f^{n+1}\left(H^{\frac{3n+1}{n}}-Y^{\frac{3n+1}{n}}\right)/H^{\frac{(2n+1)(n+1)}{n}} \quad (42)$$

where

$$\gamma=P_r E_t\left(\frac{n}{3n+1}\right) \quad (43)$$

Averaging Eq. (42) using definition (18), the expression for the mean temperature is obtained as:

$$\overline{T}_{m_1}=\overline{T}_h-\frac{H^2}{3}D+\gamma_1\overline{m}_1 f^{n+1}/H^{2n} \quad (44)$$

where $\gamma_1 = P_r E_t n/(4n+1)$.
Substituting the value of D from (26), one may get

$$P_e\bar{u}_m\frac{H^2}{3}\frac{d\overline{T}_{m_1}}{dX}=\overline{T}_h-\overline{T}_{m_1}+\gamma_1\overline{m}_1 f^{n+1}/H^{2n} \quad (45)$$

Equations (34), (42), and (45) are the final forms of equations giving pressure, temperature and the mean temperature distributions, respectively, for the first region $(-\infty \le X \le -X_1)$, where f is given by (39), $\bar{u}_m$ using (24) is as follows:

$$\bar{u}_m=1-\left(\frac{2n+1}{n+1}\right)\frac{f}{2H} \quad (46)$$

Similar expressions for pressure, temperature and the mean temperature for the other region $(-X_1 \le X \le X_2)$ can also be obtained as:

$$I_n(X)+\frac{d\bar{p}_2}{dX}=J_{-f}(X) \quad (47)$$

$$\overline{T}_2=\overline{T}_h+\frac{D}{2}(Y^2-H^2)$$

$$+\gamma\overline{m}_2(-f)^{n+1}\left(H^{\frac{3n+1}{n}}-Y^{\frac{3n+1}{n}}\right)/H^{\frac{(2n+1)(n+1)}{n}} \quad (48)$$

$$P_e\bar{u}_m\frac{H^2}{3}\frac{d\overline{T}_{m_2}}{dX}=\overline{T}_h-\overline{T}_{m_2}+\gamma_1\overline{m}_2(-f)^{n+1}/H^{2n} \quad (49)$$

where

$$J_{-f}(X)=-\overline{m}_2(-f)^n/H^{2n+1} \quad (50)$$

Numerical Method. Reynolds and energy Eqs. (34), (45), (47), and (49) are coupled through $\overline{m}$ and have to be solved simultaneously satisfying the following conditions:

$$\bar{p}_1=0,\ \overline{T}_1=\overline{T}_o,\ \overline{T}_{m_1}=\overline{T}_o \text{ at } X=-\infty \quad (51)$$

$$\bar{p}_2=0=\frac{d\bar{p}_2}{dX} \text{ at } X=X_2 \quad (52)$$

The last condition (52) in conjunction with (47) implies

$$I_n(X_2)-J_{-f}(X_2)=0 \quad (53)$$

The ordinary differential equations obtained above contain unknowns X_1 and X_2 which can be determined as follows. An arbitrary value of X_1 is assigned and the differential equations are solved for $\bar{p}$ and $\overline{T}_m$ by Adams-Moulton method till $\bar{p}_2 \approx 0$ for some $X = X_2$ (say). If Eq. (53) is satisfied at $X = X_2$ then the assumed value of X_1 was correct, otherwise another value of X_1 is chosen and the process is repeated so that the desired conditions are satisfied.

Load and Traction. The surface force W_s may be defined as

$$W_s=\int_{-\infty}^{x_2} p\,dx=-\left(\int_{-\infty}^{-x_1} x\frac{dp_1}{dx}dx+\int_{-x_1}^{x_2} x\frac{dp_2}{dx}dx\right)$$

or

$$\overline{W}_s=\int_{-\infty}^{-X_1}\frac{X\overline{m}_1 f^n}{H^{2n+1}}dX$$

$$-\int_{-X_1}^{X_2}\frac{X\overline{m}_2(-f)^n}{H^{2n+1}}dX-\int_{-\infty}^{X_2} X\,I_n(X)dX \quad (54)$$

where $\overline{W}_s\ (=W_s/(\sqrt{2Rh_o}/\alpha))$ is the dimensionless load. Similarly, the traction force W_T may also be defined as

$$W_T=-\int_{-\infty}^{x_2}\frac{h}{2}\frac{dp}{dx}dx$$

which comes out to be in dimensionless form as

$$\overline{W}_T=\frac{1}{A}\left[\int_{-\infty}^{-X_1}\frac{\overline{m}_1 f^n}{H^{2n}}dX-\int_{-X_1}^{X_2}\frac{\overline{m}_2(-f)^n}{H^{2n}}dX\right] \quad (55)$$

where $\overline{W}_T = -W_T/(h_o/2\alpha)$.

Results and Discussion

The numerical values of the dimensionless pressure $\bar{p}$ and the mean film temperature $\overline{T}_m$ are obtained by solving Eqs. (34), (45), and (47), (49) simultaneously, for various values of the flow behavior index n $(.4 \le n \le 1.15)$ and squeeze velocity q $(-0.09 \le q \le +0.09)$. The value of α is taken to be

$$=1.6\times10^{-9}\text{ dyne}^{-1}\text{ cm}^2,\text{ and } P_r=5250.6$$

It can be here noted that $\lambda = 0$ corresponds to the result without inertia, i.e., behaves as a Reynolds number. The parameter P_e is a equivalent Peclet number and $P_e = 0$ signifies the results without energy convection. Consideration of both the above parameters to be zero simultaneously leads to the problem done by Prasad et al., 1987. The Eckert number $E_t = 0$ means that there is no heat of dissipation. It also represents the results under isothermal condition ($\beta = 0$).

It can also be noted that it was not possible to draw curves for all n, q, λ, P_e etc. So graphs have been made for λ $(=26 \times 10^{-5}) \ne 0$, P_e $(=136515.6 \times 10^{-5}) \ne 0$ mainly and curves for only a few values of n and q in case of λ $(=52 \times 10^{-7}) \to 0$ and P_e $(=273011.2 \times 10^{-7}) \to 0$ have been given. Tables are also made only for a few values of the above parameters. However a special attention is paid to show relations among dimensionless film thickness $\bar{h}_o$ $(=h_o/R)$, velocity $\overline{U}$ $(=(U/R)/\text{time})$, load $\overline{W}_s$ and Eckert number E_t.

Pressure Distribution. The lubricant pressure $\bar{p}$ which is a function of n and q has been computed numerically by solving Reynolds Eqs. (34) and (47). The results have been presented in Figs. 2 and 3. For both the cases of $\lambda \ne 0$ and $\lambda \to 0$ (as mentioned earlier), the qualitative behavior of $\bar{p}$ for the different values of q (for fixed n) is identical. This is in conformity with the results of Dowson et al. (1976) and those obtained in

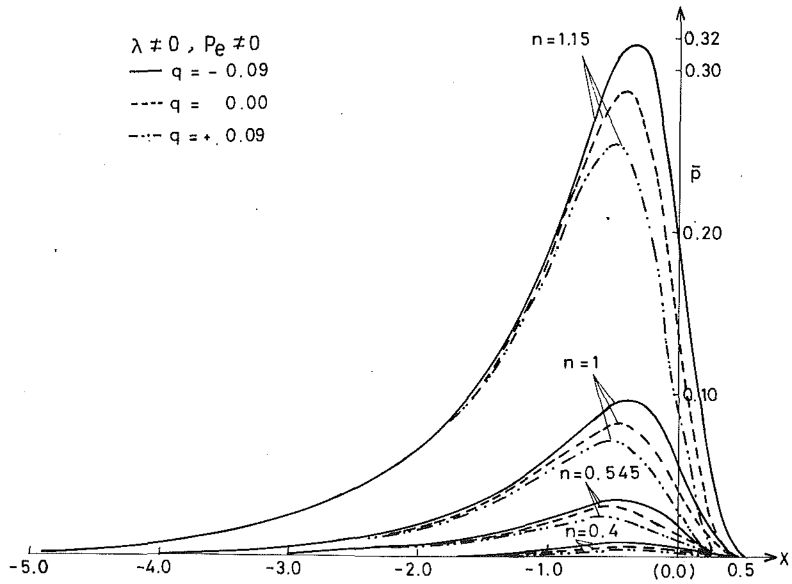

Fig. 2 Pressure distribution $\bar{p}$ against X

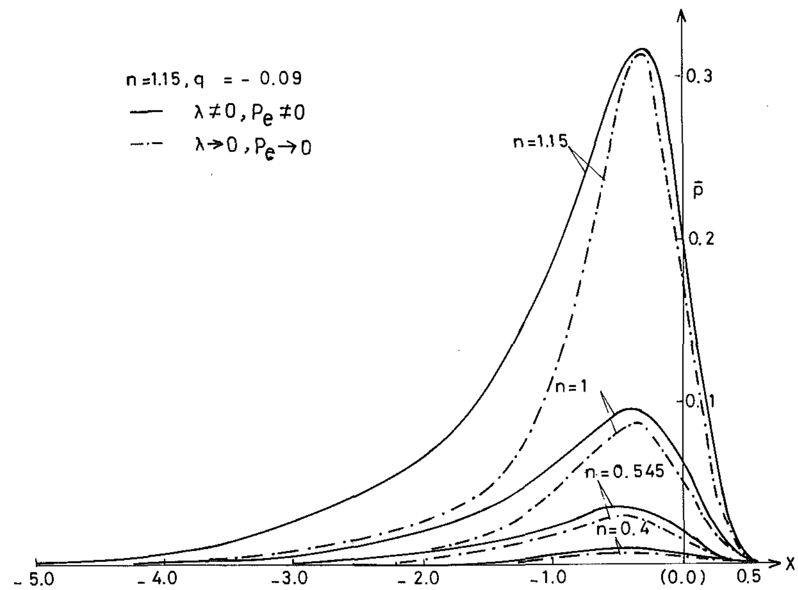

Fig. 3 Comparison of pressures with and without inertia

Prasad et al. (1987) and Rong-Tsong and Hamrock (1989). A similar trend for both the cases is displayed by the pressure curves when n is varied and q is held fixed. This kind of behavior was observed by Safar and Shawki (1979) for a thrust bearing, by Buckholz (1985) for a journal bearing and by Wang et al. (1988) for cylindrical roller bearing. The point of maximum pressure for the case $\lambda \neq 0$ is of special interest here. Mathematically speaking, the point of maximum pressure for the present case is the point of intersection of the curves $I_n(X)$ and $J_f(X)$. This has been shown in Fig. 1. Physically it is the point at which inertia and viscous forces balance each other. The point of maximum pressure for the case $\lambda \neq 0$ is different from the point $X = -X_1$ (at which $\partial\bar{u}/\partial Y$ is zero) and lies before $X = -X_1$. However, for the case $\lambda \rightarrow 0$, these two coincide i.e. the pressure gradient and the velocity gradient vanish simultaneously.

A comparison between the two cases can also be made from Fig. 3. It is seen from Fig. 3 that for each fixed n and q, the pressure curves for $\lambda \neq 0$ lie above the corresponding curves for $\lambda \rightarrow 0$. One may therefore infer that the inclusion of inertia term leads to an increased pressure. It may be further noted that the inertia effect is less on the Newtonian fluid ($n = 1$) in comparison to that for the dilatant fluid ($n > 1$). Increased pressure due to the presence of inertia has also been reported by Elkouh (1976) and by Safar (1979).

Temperature Distribution. The mean film temperature $\overline{T}_m$ is computed from differential Eqs. (45) and (49). The results for $\overline{T}_m$ are elaborated through Figs. 4 and 5 for various values of n and q (for both the cases $\lambda \neq 0$ and $\lambda \rightarrow 0$). It is clear from Fig. 4 that for a fixed n and q, $\overline{T}_m$ increases as the lubricant flows toward the pressure peak region. Subsequently, it decreases till the pressure is zero (ambient). However for a fixed n, q, and $\lambda \rightarrow 0$, the behavior of $\overline{T}_m$ is similar to that mentioned above, except near the centerline of contact ($X = 0$) where it increases/decreases (see Fig. 5) though only marginally. For a fixed n, $\overline{T}_m$ increases as q decreases whereas for a fixed q, $\overline{T}_m$ increases with n. A similar trend for temperature with n has also been obtained by Prasad et al. (1988).

A comparison between the mean temperature for both the cases has also been made in Fig. 5. It is observed from the figure that the mean temperature near the inlet for the case λ

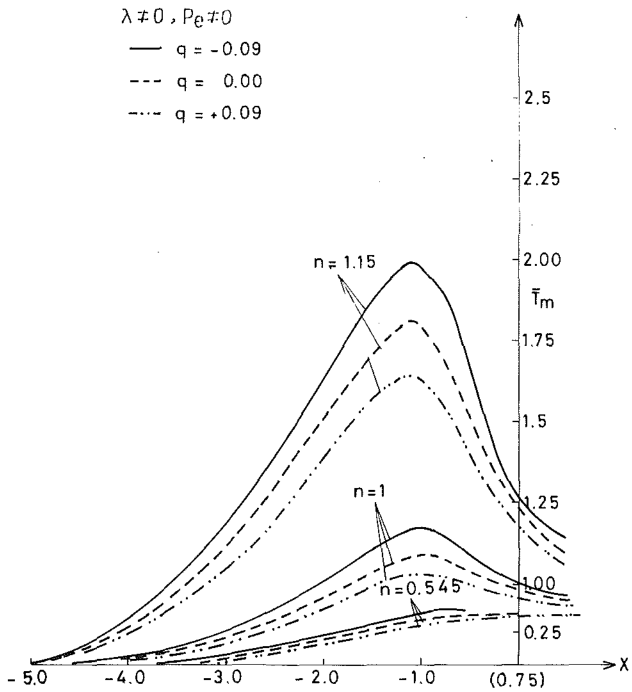

Fig. 4 The mean temperature distribution $\bar{T}_m$ against X

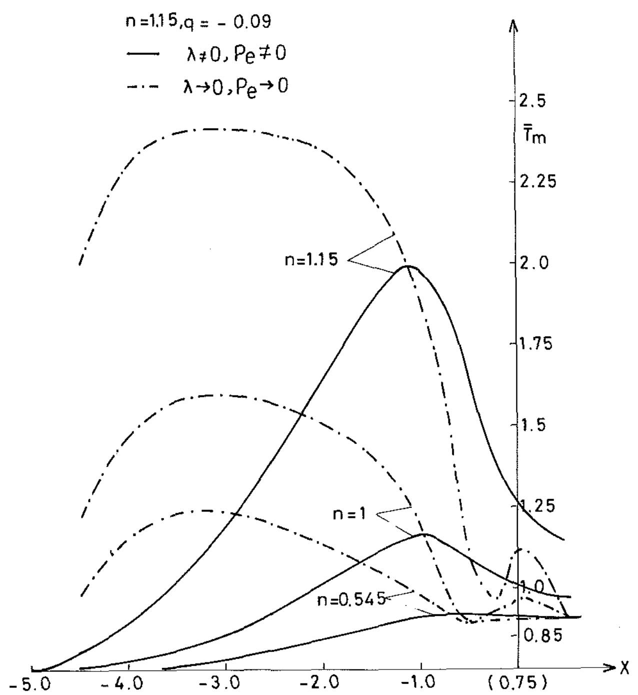

Fig. 5 Comparison between the temperatures T_m with and without inertia

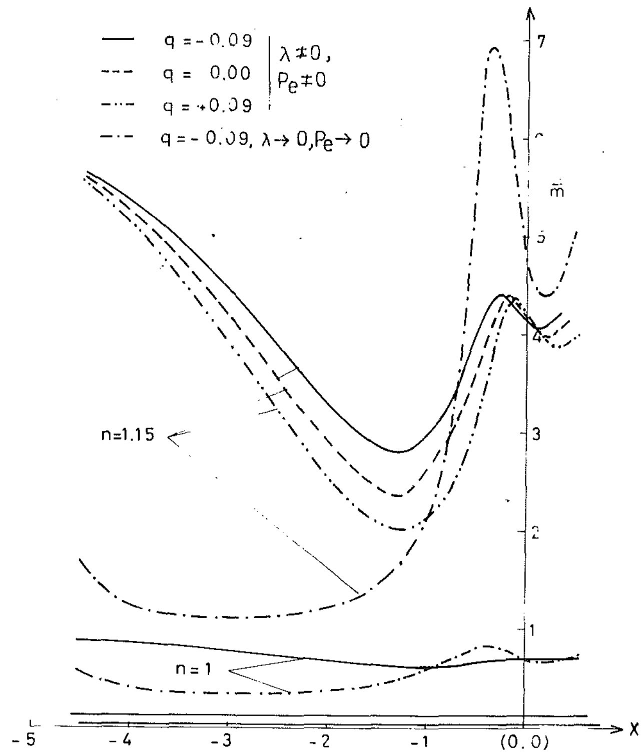

Fig. 6 Consistency distribution $\bar{m}$ against X

$\neq 0$, is less than that for the case $\lambda \to 0$. However, the trend is reversed in the pressure peak region. One may interpret these results from a physical viewpoint as well. For the case $\lambda \neq 0$, the convection dominant region extends farther than that for the case $\lambda \to 0$. Thus the heat lost by convection for $\lambda \to 0$ is smaller as compared to that for $\lambda \neq 0$. This may cause the temperature rise there. However, as the lubricant flows towards the pressure peak region, the heat transported by conduction becomes more significant for the case $\lambda \to 0$. Consequently, $\bar{T}_m$ decreases continuously right-up to the pressure peak region. Thus for the case $\lambda \neq 0$, $\bar{T}_m$ is higher than that for $\lambda \to 0$ near the pressure peak region.

Consistency Variation. The dimensionless consistency $\overline{m}$, given by the empirical relation viz.

$$\overline{m} = \overline{m}_o e^{\bar{p} - \bar{T}_m + \bar{T}_o}$$

is depicted in Fig. 6 for various values of n and q. It is evident that the consistency variation depends on the combined effects of pressure $\bar{p}$ and the mean film temperature $\bar{T}_m$. Since $\overline{m}$ for the pseudoplastic fluids ($n < 1$) does not vary significantly with respect to q. Therefore, for this case, $\overline{m}$ is plotted for only one value of q ($= -0.09$). Moreover, the consistency variation for all the values of n and q have similar trends i.e., $\overline{m}$ decreases near the inlet because increase in $\bar{T}_m$ is higher than the corresponding increase in $\bar{p}$. Thereafter, $\overline{m}$ follows the pressure trend throughout the region i.e., except near the outlet where the trend is opposite. For each q, $\overline{m}$ increases with n throughout the region. However, for each n, $\overline{m}$ increases as q decreases in whole of the inlet and outlet regions except in the neighborhood of the centerline of contact but it differs slightly. It is seen from Fig. 6 that variation in $\overline{m}$ for $n < 1$ is very less. It may therefore be noted that $\overline{m}$ may be treated constant for pseudoplastic fluids ($n < 1$).

The consistency variation for $\lambda \to 0$ has also been presented along with that for $\lambda \neq 0$. The qualitative behavior of $\overline{m}$ versus X (for all n and q) in this case is similar to that for $\lambda \neq 0$. However, quantitatively $\overline{m}$ for $\lambda \to 0$ is less than that for $\lambda \neq 0$ except in the pressure peak region where the trend is reversed. This is because of abnormal change in $\bar{T}_m$ and $\bar{p}$ in that region which has already been mentioned.

Load and Traction. The surface force $\overline{W}_s$ and the traction force $\overline{W}_T$ are calculated from the expressions (54) and (55) for various values of n and q and for both the cases $\lambda \neq 0$ and $\lambda \to 0$. These results are presented in Table 1. It is seen from the table that for a fixed q, $\overline{W}_s$, and $\overline{W}_T$ both increase with n whereas for a fixed n, both of them increase as q decreases. This is in accordance with the results obtained by Prasad et al., 1987.

In order to study the effect of inertia on load and traction,

Table 1 Normal force component and traction

n/m_o	q	$\lambda = 26\times10^{-5}$ $\overline{W}_s$	$\overline{W}_T$	$\lambda = 52\times10^{-7}$ $\overline{W}_x$	$\overline{W}_T$
1.15/0.56	−0.09	0.48104	1.5382	0.34573	0.7761
	0.00	0.44338	1.5323	0.29744	0.7617
	+0.09	0.40211	1.4936	0.25320	0.7340
1.00/0.75	−0.09	0.14965	0.4095	0.11361	0.2667
	0.00	0.13100	0.3881	0.09732	0.2549
	+0.09	0.11293	0.3620	0.08266	0.2405
0.545/86	−0.09	0.05450	0.1112	0.04443	0.0847
	0.00	0.04888	0.1078	0.03966	0.0826
	+0.09	0.04352	0.1033	0.03521	0.0798
0.40/128	−0.09	0.01273	0.0234	0.01169	0.0202
	0.00	0.01148	0.0228	0.01010	0.0197
	+0.09	0.01029	0.0220	0.00908	0.0191

Table 2 Relation between $\overline{h}_o$ and load $\overline{W}_s$ for $\lambda \neq 0$, $P_e \neq 0$, and $\overline{U} = 150$

$\overline{h}_o$	$n/m_o = 1.15/0.56$ $q = -0.09$	$q = 0$	$q = 0.09$	$n/m_o = 1/0.75$ $q = -0.09$	$n/m_o = 0.545/86$ $q = -0.09$
			Values of $\overline{W}_s$		
3.3333×10^{-3}	$.11157\times10^{-1}$	$.96663\times10^{-2}$	$.79035\times10^{-2}$	$.36456\times10^{-2}$	$.33934\times10^{-2}$
9.5238×10^{-4}	$.83138\times10^{-1}$	$.70397\times10^{-1}$	$.58778\times10^{-1}$	$.21123\times10^{-1}$	$.12904\times10^{-1}$
2.7212×10^{-4}	.41130	.37600	.33689	.12288	$.46658\times10^{-1}$
7.7746×10^{-5}	1.1505	1.1069	1.0628	.49216	.15635
2.2217×10^{-5}	2.3647	2.2575	2.1667	1.23745	.44446

Table 3 Relation between $\overline{h}_o$ and $\overline{U}$ for a fixed load $\overline{W}_s = 2.5$, $\lambda \neq 0$ and $P_e \neq 0$

$\overline{h}_o$	Values of $\overline{U}$				
1.79928×10^{-3}	96024.671	111139.000	128228.240	784350.00	330703666.66
0.59976×10^{-3}	19857.000	22982.367	26510.540	151005.33	40245333.33
1.9992×10^{-4}	4105.000	4751.467	5480.881	29055.00	4899514.00
0.6664×10^{-4}	848.670	982.457	1133.147	5592.77	595323.33
2.2213×10^{-5}	172.133	203.137	234.269	1076.30	72443.33

Table 4 Relation between $\overline{h}_o$ and E_t for $\lambda \neq 0$, $P_e \neq 0$, $\overline{U} = 150$ and $q = -0.09$

	$\overline{W}_s = 0.25$, $n/m_o = 1.15/0.56$	$\overline{W}_s = 0.09$, $n/m_o = 1.0/0.75$	$\overline{W}_s = 0.035$, $n/m_o = 0.545/86$
$\overline{h}_o$	E_t	E_t	E_t
$.35989\times10^{-3}$	$.31230\times10^{-2}$	$.10900\times10^{-1}$	$.23214\times10^{-1}$
$.17993\times1\text{-}^{-3}$	$.80349\times10^{-2}$	$.15933\times10^{-1}$	$.39865\times10^{-1}$
$.59976\times10^{-4}$	$.18533\times10^{-1}$	$.40817\times10^{-1}$	$.99731\times10^{-1}$
$.19992\times10^{-4}$	$.29404\times10^{-1}$	$.68212\times10^{-1}$	.16447
$.66640\times10^{-5}$	$.53979\times10^{-1}$	.10502	.23111

results for $\lambda \to 0$ have also been presented in Table 1. It is seen that the inclusion of the inertia force leads to increase in the load and the traction for all n and q.

Variation of $\overline{h}_o$ With $\overline{W}_s$, $\overline{U}$, E_t, Pressure and Temperature Viscosity Exponents. Coming over to the relation between the load $\overline{W}_s$ and the dimensionless minimum film thickness $\overline{h}_o$, the analysis of loads given in Table 2 shows that they vary reciprocal to each other for the Newtonian as well as non-Newtonian fluids. This means that if the load increases, the gap between the rolling surfaces (i.e., film thickness) decreases and vice-versa. This is physically justified as well (Ghose and Hamrock, 1985). A similar trend follows between $\overline{h}_o$ and the Eckert Number E_t when the load and the velocity are held fixed (see Table 4). Thus, it implies that $\overline{h}_o$ increases/decreases as the temperature viscosity exponent β decreases/increases.

Relation between $\overline{h}_o$ and $\overline{U}$ is given in Table 3 for various values of n and q. It follows from the table that for a fixed load, $\overline{h}_o$ varies with $\overline{U}$. Physically, velocity increase/decrease causes a corresponding increase/decrease in the film thickness. A similar observation has also been made by Ghose and Hamrock, 1985 for the Newtonian fluid. It can also be concluded from here itself that since U and the pressure viscosity exponent α are the contemporary terms (they occur in the numerator, see Eqs. (19) and (21)), they are likely to have the same behavior. Hence film thickness increases/decreases with α.

Table 5 Values of X_1 for $P_e \neq 0$, and $\overline{U} = 150$

n/m_o	q = −0.09	0.0	+0.09	λ
1.15/0.56	0.35466	0.42610	0.50138	26×10^{-5}
1.00/0.75	0.40644	0.46711	0.53256	
0.545/86	0.43259	0.48907	0.55119	
0.40/128	0.44438	0.49856	0.55863	
1.15/0.56	0.32663	0.39046	0.45985	52×10^{-7}
1.00/0.75	0.37773	0.43686	0.50197	
0.545/86	0.40277	0.45993	0.52325	
0.40/128	0.42216	0.47830	0.54043	

Separated Regions. The Reynolds and the energy Eqs. (34), (45), (47), and (49) contain X_1 ($X = -X_1$ is the point at which $\partial\overline{u}/\partial Y$ vanishes). This point $X = -X_1$ separates the region

$-\infty \leq X \leq X_2$ into two subregions $-\infty \leq X \leq -X_1$ and $-X_1 \leq X \leq X_2$. The values of X_1 for different n and q, and for both the cases have been presented in Table 5. It can be seen from the table that for a fixed n, X_1 increases as q increases whereas for a fixed q, X_1 decreases as n increases. A similar trend was also obtained by Sinha and Singh (1982) for $\lambda \rightarrow 0$ and $\overline{m}$ constant.

Conclusion

A theoretical analysis of cylindrical rollers moving with equal velocity and lubricated with power law fluids has been presented. Thermal and inertia effects have also been incorporated together with the squeezing motion and film cavitation. A simple model for m given in Eq. (4) has been considered. Average/mean values of the fluid inertia and the energy convection considered in the momentum and energy equations result in an analytical solution for temperature and semianalytical solutions for pressure and the mean temperature.

Fluid inertia causes a significant increase in pressure and thus the load and the traction. This effect has even more significance in case of squeezing motion. This effect, however, has moderate effect on the mean temperature. The fluid inertia tends to stretch the fluid film which results in a lubricating flow field larger than that of the inertialess case ($\lambda \rightarrow 0$) i.e., the point at infinity is shifted farther away from the center line of contact. This also tends to shift the position of pressure peak towards the origin. Further, it can also be noted that the dimensionless minimum film thickness varies reciprocal to the load. A similar trend follows between the film thickness and the Eckert number or temperature viscosity exponent. However, the film thickness increases/decreases according as the speed increases/decreases. The same relation is true between the film thickness and the pressure viscosity exponent as well.

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