2010 American Control Conference
Marriott Waterfront, Baltimore, MD, USA
June 30-July 02, 2010

Predictive Repetitive Control Based on Frequency Decomposition

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***Abstract*— This paper develops a predictive repetitive control algorithm based on frequency decomposition. In particular,, the periodic reference signal is first represented using a frequency sampling filter model and then the coefficients of the model are analyzed to determine its dominant frequency components. Using the internal model control principle, the dominant frequency components are embedded in model used to obtain the predictive repetitive control algorithm such that the periodic reference is followed with zero steady-state error. The design framework here is based on predictive control using Laguerre functions and hence plant operational constraints are naturally incorporated in the design and its implementation.**

keyword Periodic set-point signal, periodic disturbance, predictive control, constrained control, optimization.

I. Introduction

Control system applications in mechanical systems, manufacturing systems and aerospace systems often require set-point following of a periodic trajectory. In this situation, the design of a control system that has the capability to produce zero steady-state error is paramount. It is well known from the internal model control principle that in order to follow a periodic reference signal with zero steady-state error, the generator for the reference must be included in the stable closed-loop control system [1]. This is the case for repetitive control system designs, such as those in [2], [3], [4], [5].

In the design of repetitive control systems, the control signal is often generated by a controller that is explicitly described by a transfer-function with appropriate coefficients, see, for example, [2], [5]. If there are a number of frequencies contained in the exogenous signal, the repetitive control system will contain all periodic modes, and the number of modes is proportional to the period and inversely proportional to the sampling interval. As a result, in the case of fast sampling, a very high order control system may be obtained, which could lead to numerical sensitivity and other undesirable problems in actual implementations.

Instead of including all the periodic modes in the periodic control system, an alternative is to embed fewer periodic modes at a given time, and when the frequency of the external signal changes, the coefficients of the controller change accordingly. This will effectively result in a lower order periodic control system by using a strategy based on switched linear controllers. Apart from knowing which periodic controller should be used, no bump occurring in the control signal, when switching from one controller to another, is paramount in terms of implementations.

This paper develops a repetitive control algorithm using the framework of a discrete-time model predictive control. In particular, the frequency components of a given reference signal are analyzed and its re-construction performed using the frequency sampling filter model [6], from which the dominant frequencies are identified and error analysis used to justify the selections. Secondly, the input disturbance model that contains all the dominant frequency components is formed and is combined with the plant description to form an augmented state-space model for controller design. The augmented state-space model has an input signal which is inversely filtered with the disturbance model and, as a result, the current control is computed from past controls to ensure a bumpless transfer when the periodic reference signal changes. Thirdly, with the augmented state-space model, the receding horizon control principle is used with an on-line optimization scheme [7] to generate the repetitive control law with any plant operational constraints imposed.

This paper is organized as follows. In Section II, the frequency sampling filter model is used to construct the reference signal, which forms the basis for analysis of dominant frequency components and the quantification of errors arising when insignificant components are neglected. In Section III, the disturbance model is formulated and augmented with the plant model to obtain the state-space model used for design. In Section IV, the discrete-time model predictive control framework based on the Laguerre functions is used to develop the repetitive control law with any constraints present imposed. Also the straightforward extension to the case when (a particular type of) control input constraints are imposed. In Section V, simulation results from a gantry robot executing the commonly encountered pick and place operation that can be configured to operate in a repetitive control setting are used to highlight the results obtained when the design algorithm developed in the previous section is applied.

II. Frequency Decomposition of the Reference Signal

We assume that the periodic reference signal with period T is uniformly sampled with interval Δt and within one period the corresponding discrete sequence is $r(k)$, where $k = 0, 1, \ldots, M-1$. Here $M = \frac{T}{\Delta t}$, and is assumed to be an odd integer for the reason explained below. From Fourier analysis [8], this discrete periodic signal can be uniquely represented by the inverse Fourier transform as

$$r(k) = \frac{1}{M}\sum_{i=0}^{M-1} R(e^{j\frac{2\pi i}{M}})e^{j\frac{2\pi ik}{M}}, \qquad (1)$$

where M is the number of samples within a period and $R(e^{j\frac{2\pi i}{M}})$ $(i = 0, 1, 2, \ldots, M-1)$ are the frequency components contained in the periodic signal. Note that the discrete frequencies are at 0, $\frac{2\pi}{M}, \ldots, \frac{4\pi}{M}, \frac{(M-1)2\pi}{M}$. For notational simplicity, we express the fundamental frequency as $\omega = \frac{2\pi}{M}$.

The z-transform of the signal $r(k)$ is defined as

$$R(z) = \sum_{k=0}^{M-1} r(k)z^{-k}. \qquad (2)$$

Also, by substituting (1) into (2) and interchanging the order of the summation, the z-transform representation of the periodic signal $r(k)$ is obtained as

$$R(z) = \sum_{l=-\frac{M-1}{2}}^{\frac{M-1}{2}} R(e^{jl\omega})H^l(z), \qquad (3)$$

where $H^l(z)$ is termed the lth frequency sampling filter [6] and has the form:

$$\begin{aligned} H^l(z) &= \frac{1}{M}\frac{1-z^{-M}}{1-e^{jl\omega}z^{-1}} \\ &= \frac{1}{M}(1 + e^{jl\omega}z^{-1} + \ldots + e^{j(M-1)l\omega}z^{-(M-1)}). \end{aligned}$$

Here we have used the assumption that M is an odd number to include the zero frequency. The frequency sampling filters are bandlimited and are centered at $l\omega$. For example, at $z = e^{jl\omega}$, $H^l(z) = 1$. Equation (3) can also be written in terms of real (denoted Re) and imaginary (denoted Im) parts of the frequency component $R(e^{jl\omega})$ [9] as

$$\begin{aligned} R(z) &= \frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}R(e^{j0}) \\ &+ \sum_{l=1}^{\frac{M-1}{2}}[Re(R(e^{jl\omega})F_R^l(z) \\ &+ Im(R(e^{jl\omega})F_I^l(z)], \end{aligned} \qquad (4)$$

where $F_R^l(z)$ and $F_I^l(z)$ are the lth second order filters given by

$$F_R^l(z) = \frac{1}{M}\frac{2(1-cos(l\omega)z^{-1})(1-z^{-M})}{1-2cos(l\omega)z^{-1}+z^{-2}},$$

and

$$F_I^l(z) = \frac{1}{M}\frac{2sin(l\omega)z^{-1}(1-z^{-M})}{1-2cos(l\omega)z^{-1}+z^{-2}}.$$

At this stage, note the following.

- The number of frequency components M is determined by the period T and the sampling interval Δt. As Δt is reduced, the number of frequencies increases but the number of dominant frequencies may not change with a faster sampling rate, which is similar to the application of a frequency sampling filter to the model of a dynamic system.
- Given the z-transform representation of the reference signal using the frequency sampling filter structure, the time domain signal $r(k)$ can be constructed using the frequency components contained in the periodic signal by convolving $R(z)$ and the unit impulse function and taking the inverse transform. Hence the contribution of each frequency component can be analyzed against the error arising when a specific frequency component is neglected. This approach is used in determining the dominant components of the periodic reference signal.
- Once the dominant frequency components are selected they can be combined with the plant description to obtain the augmented state-space model to be used in the design of the predictive repetitive control system as detailed in the next section.

III. Augmented Design Model

Suppose (as noted below this assumption cause no loss of generality) that the plant to be controlled is single-input single-output and described by the state-space model

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) + \Omega_m \mu(k), \\ y(k) &= C_m x_m(k), \end{aligned} \qquad (5)$$

where $x_m(k)$ is the $n_1 \times 1$ state vector, $u(k)$ is the input signal, $y(k)$ the output signal, and $\mu(k)$ represents the input disturbance. Then by the internal model principle, in order to follow a periodic reference signal with zero steady-state error, the generator of this signal must be included in the stable closed-loop control system [1]. From the identification of dominant frequency components in Section II, we assume that the dominant frequencies are 0 and some $l\omega$s and that the input disturbance $\mu(k)$ is generated as the output of a system whose z transfer-function has no zeros and a denominator polynomial of the form

$$\begin{aligned} D(z) &= (1-z^{-1})\Pi_l(1-2cos(l\omega)z^{-1}+z^{-2}) \\ &= 1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+\ldots+d_\gamma z^{-\gamma}. \quad (6) \end{aligned}$$

In the time domain, the input disturbance $\mu(k)$ is described by the difference equation in the backward shift operator q^{-1}

$$D(q^{-1})\mu(k)=0, \quad (7)$$

and introduce the following auxiliary variables obtained using the disturbance model (this can be viewed as filtering the state vector and the input by the inverse z-transform of $D(z)$)

$$\begin{aligned} x_s(k) &= D(q^{-1})x_m(k), \quad (8) \\ u_s(k) &= D(q^{-1})u(k). \quad (9) \end{aligned}$$

Also applying $D(q^{-1})$ to both sides of the state equation in (5) gives

$$D(q^{-1})x_m(k+1)=A_mD(q^{-1})x_m(k)+B_mD(q^{-1})u(k),$$

or

$$x_s(k+1)=A_mx_s(k)+B_mu_s(k), \quad (10)$$

where the relation $D(q^{-1})\mu(k)=0$ has been used. Similarly, application of the operator $D(q^{-1})$ to both sides of the output equation in (5) gives

$$D(q^{-1})y(k+1)=C_mx_s(k+1)=C_mA_mx_s(k)+C_mB_mu_s(k), \quad (11)$$

or (on expanding both sides of this last equation)

$$\begin{aligned} y(k+1) &= -d_1y(k)-d_2y(k-1)-\ldots-d_\gamma y(k-\gamma+1) \\ &+ C_mA_mx_s(k)+C_mB_mu_s(k). \quad (12) \end{aligned}$$

To obtain the augmented state-space model of the plant and the disturbance, introduce the new state vector as

$$x(k)=\left[\begin{array}{cccccc} x_s(k)^T & y(k) & y(k-1) & \ldots & y(k-\gamma+1) \end{array}\right]^T,$$

and hence (on combining the plant and disturbance models)

$$\begin{aligned} x(k+1) &= Ax(k)+Bu_s(k), \\ y(k) &= Cx(k), \quad (13) \end{aligned}$$

where

$$A=\left[\begin{array}{cccccc} A_m & O & O & \ldots & O & O \\ C_mB_m & -d_1 & -d_2 & \ldots & -d_{\gamma-1} & -d_\gamma \\ O^T & 1 & 0 & \ldots & 0 & 0 \\ & & \ddots & & & \\ \ldots & & & & & \\ O^T & 0 & \ldots & 1 & 0 & 0 \\ O^T & 0 & \ldots & 0 & 1 & 0 \end{array}\right],$$

$$B=\left[\begin{array}{c} B_m \\ C_mB_m \\ 0 \\ \vdots \\ 0 \\ 0 \end{array}\right],$$

$$C=\left[\begin{array}{cccccc} O^T & 1 & 0 & \ldots & 0 & 0 \end{array}\right],$$

and O denotes the $n_1\times 1$ zero vector. The structure of this augmented model remains unchanged when the plant is multi-input multi-output except that the O vector becomes zero matrix with appropriate dimensions, and the coefficients $-d_1$, $-d_2$, $\ldots$ are replaced by $-d_1I$, $-d_2I$, $\ldots$ where I denotes the identity matrix with compatible dimensions.

IV. Discrete-time Predictive Repetitive Control

Given the augmented state-space model, the next task is to optimize the filtered control signal $u_s(k)$ using the receding horizon control principle. The key task in this is to model the auxiliary control signal $u_s(k)$ using a set of discrete-time Laguerre functions. The main steps here are summarized with full details in [7].

Assuming that the current sampling instant is k_i, let the control trajectory at future time m be

$$u_s(k_i+m\mid k_i)=\sum_{i=1}^{N}l_i(m)c_i=L^T(m)\eta, \quad (14)$$

where $\eta^T=\left[\begin{array}{cccc} c_1 & c_2 & \ldots & c_N \end{array}\right]$ is the coefficient vector that contains the Laguerre coefficients and $L(\cdot)$ is the $N\times 1$ vector whose entries are the Laguerre functions $l_1(\cdot),l_2(\cdot),\ldots,l_N(\cdot)$. Then the predicted state vector at sampling instant $k_i+m\mid k_i$ is

$$x(k_i+m\mid k_i)=A^mx(k_i)+\sum_{i=0}^{m-1}A^{m-i-1}BL^T(i)\eta. \quad (15)$$

The performance objective for the predictive repetitive control here is to find the Laguerre coefficient vector η that minimizes the cost function

$$J=\sum_{m=1}^{N_p}x^T(k_i+m\mid k_i)Qx(k_i+m\mid k_i)+\eta^TR_L\eta, \quad (16)$$

where $Q\geq 0$ and $R_L>0$ where R_L is a diagonal matrix with identical elements.

Introducing

$$\phi^T(m)=\sum_{i=0}^{m-1}A^{m-i-1}BL^T(i),$$

and substituting (15) into (16), it is easily shown that the optimal solution here is

$$\eta = -(\sum_{m=1}^{N_p} \phi(m) Q \phi^T(m) + R_L)^{-1} (\sum_{m=1}^{N_p} \phi(m) Q A^m x(k_i)). \quad (17)$$

Moreover, the filtered receding horizon control signal is obtained as

$$u_s(k) = L^T(0)\eta, \quad (18)$$

and the actual control signal to the plant as

$$D(q^{-1})u(k) = u_s(k).$$

With the definition of $D(q^{-1})$ (see 6) and noting that the unit leading coefficient in the polynomial $D(q^{-1})$, the actual control signal to the plant is found using

$$u(k) = u_s(k) - d_1u(k-1) - d_2u(k-2) - \ldots - d_\gamma u(k-\gamma), \quad (19)$$

where the current optimal control $u_s(k)$ and the past values of control are used.

If predictive repetitive controller is to be used for disturbance rejection, the control objective is to maintain the plant in steady state operation, with zero steady-state filtered state vector $x_s(k)$ and a constant steady-state plant output. If, however, the predictive repetitive controller is used for tracking a periodic input signal, the reference signal will enter the computation through the augmented output variables. Note that, since the state vector $x(k)$ contains the filtered state vector $x_s(k)$, and the output $y(k)$, $y(k-1)$, ..., $y(k-\gamma+1)$, the feedback errors at sampling instant k_i are obtained as

$$[y(k_i) - r(k_i) \ \ldots \ y(k_i-\gamma+1) - r(k_i-\gamma+1)]^T = [e(k_i) \ e(k_i-1) \ \ldots \ e(k_i-\gamma+1)]^T.$$

These will replace the original output elements in $x(k)$ to form a state feedback term in the computation of the filtered control signal $u_s(k)$ using (17) and (18).

The key strength of predictive repetitive control lies in its ability to systematically impose constraints on the plant input and output variables. The constrained control system then minimizes the objective function J (16) in real time subject to the constraints imposed. For example, control amplitude constraints can be imposed at the sampling instant k by writing them in a set of linear inequalities,

$$u^{min} \leq u(k) \leq u^{max}, \quad (20)$$

where u^{min} and u^{max} are the required lower and upper limits of the control amplitude respectively. By substituting (19) into (20) and relating $u_s(k)$ to the Laguerre function expression, the constraints effectively become the functions of Laguerre parameter vector η as,

$$L^T(0)\eta \leq u^{max} + d_1u(k-1)\ldots + d_\gamma u(k-\gamma), \quad (21)$$

$$-L^T(0)\eta \leq -(u^{min} + d_1u(k-1)\ldots + d_\gamma u(k-\gamma)). \quad (22)$$

V. Gantry Robot Example

This section gives the results of a case study where the plant transfer-function has been obtained from experimental tests on a gantry robot undertaking pick and place operations.

A. Process Description

The gantry robot, shown in Fig. 1, replicates a task commonly found in a variety of industrial applications. In particular, it is executing 'pick and place' operation where the following operations must be performed in synchronization with a conveyor system, i) collect an object from a fixed location, transfer it over a finite duration, ii) place it on the moving conveyor, iii) return to the original location for the next object, and then iv) repeat the previous three steps for as many objects as required. This experimental facility has been extensively used in the benchmarking of repetitive and iterative learning control algorithms, see, for example, [10], [11].

For modeling and control design purposes, this gantry robot can be treated as three single-input single-output systems (one for each axis) that can operate simultaneously to locate the end effector anywhere within a cuboid work envelope. The lowest axis, X, moves in the horizontal plane, parallel to the conveyor beneath. The Y-axis is mounted on the X-axis and moves in the horizontal plane, but perpendicular to the conveyor. The Z-axis is the shorter vertical axis mounted on the Y-axis. The X and Y-axes consist of linear brushless dc motors, while the Z-axis is a linear ball-screw stage powered by a rotary brushless dc motor. All motors are energized by performance matched dc amplifiers. Axis position is measured by means of linear or rotary optical incremental encoders as appropriate.

To obtain a model for controller design, each axis of the robot was modeled independently by means of sinusoidal frequency response tests. From this data it was possible to construct Bode plots for each axis and hence determine approximate transfer-functions. These were then refined, by means of a least mean squares optimization technique, to minimize the difference between the frequency response of the real plant and that of the model. Here we only consider the X-axis and use the following 7-order transfer-function (with s denoting the Laplace transform variable) approximation of the dynamics in the design.

B. Frequency Decomposition of the Reference Signal

The reference signal for the axis is shown in Fig. 2, where the sampling interval is 0.01 sec with 199 samples and hence the period of this signal is $T = 2$ (sec). Fig. 3 shows the magnitude of the coefficients for all the frequency sampling filters, which decay very fast as the frequency increases. Hence it is reasonable to reconstruct

$$G(s) = \frac{(s+500.19)(s+4.90\times10^{5})(s+10.99\pm j169.93)(s+5.29\pm j106.86)}{s(s+69.74\pm j459.75)(s+10.69\pm j141.62)(s+12.00\pm j79.10)} \tag{23}$$

Fig. 1. The gantry robot

TABLE I

SUM OF THE SQUARED ERRORS OF THE RECONSTRUCTED SIGNAL

freq. comp.	0	0-1st pair	0-2nd pair	0-3rd pair
error	0.0325	0.0017	0.0001	0

the reference signal using just a few low frequency components. Fig. 2 compares the reconstructed signals using $0-$1st, $0-$2nd and $0-$3rd pair of frequency components near the zero frequency component respectively. Table I shows the sum of the squared errors between the actual reference and re-constructed signals using a limited number of frequencies. It is obvious that only a small error arises if the reference signal is approximated using the $0-$3rd pair of frequency components near the zero frequency component. Hence $0-$3rd pair of frequency components are taken as the dominant frequency components for this example.

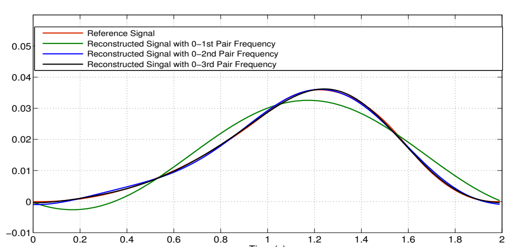


Fig. 2. Reference signal and its reconstructions using a limited number of frequencies.

C. Controller Design and Simulation Results

Using (6), The pole polynomial in the z transfer-function of the frequency sampling filter model for $l=3$

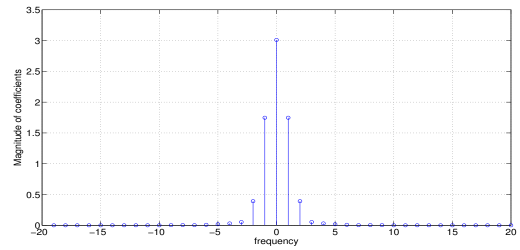


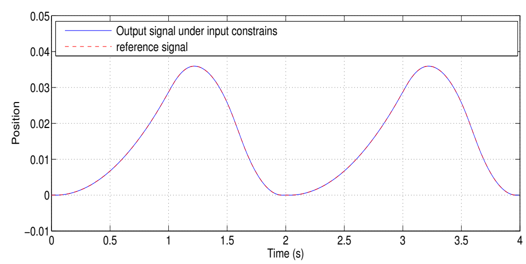
Fig. 3. Magnitudes of the coefficients in the frequency sampling filter.

is

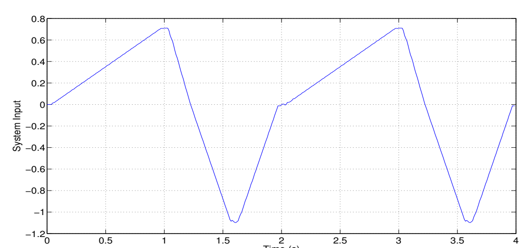
$$\begin{aligned} D(z) &= 1-6.70z^{-1}+20.93z^{-2}-34.86z^{-3}+34.86z^{-4} \\ &\quad -20.93z^{-5}+6.70z^{-6}-z^{-7}, \end{aligned} \tag{24}$$

and these, together with the state-space realization of the plant dynamics, now enable the construction of the augmented state space model (13). The filtered control signal $u_s(k)$ then can be optimized using the receding horizon principle described in Section IV.

Figure 4(a) shows the perfect tracking of the reference signal by the controlled plant and (as expected) Fig. 4(b) shows the repetitive nature of input signal.



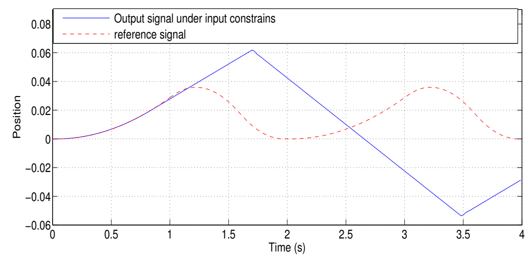
(a) System output



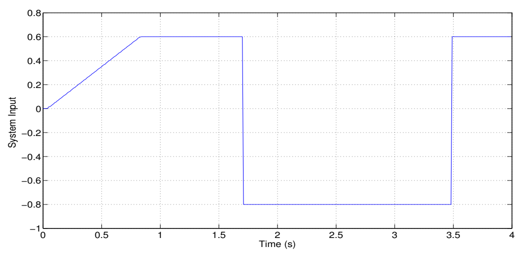
(b) Input signal

Fig. 4. System simulation results (without input constraints).

In practice, there are circumstances where the input of gantry robot could be saturated due, for example, to limitations of electrical or mechanical hardware and this could degrade the tracking performance when the input signal hits its limit. For example, in the case when $u^{max} = 0.6$ and $u^{min} = -0.8$, Fig. 5(a) shows that the system goes unstable if the input signal is directly saturated according to $u^{min} \leq u(k) \leq u^{max}$.

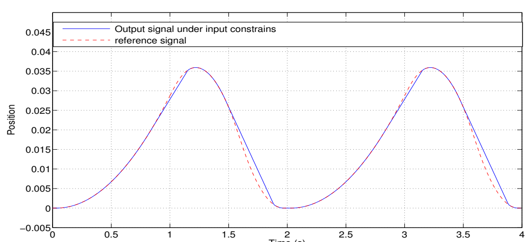


(a) System output under input constraints.

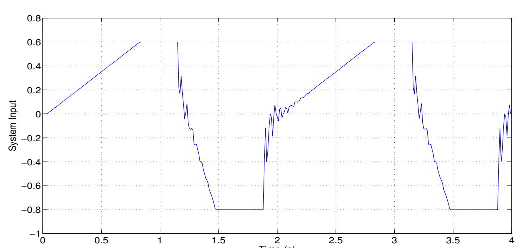


(b) Input signal under constraints.

Fig. 5. Simulated system response under input constraints.



(a) System output under input constraints.



(b) Input signal under constraints.

Fig. 6. Simulated system response under input constraints with the controller in place.

Figure 6(a) shows that if (21) and (22) are used in conjunction with optimization of the cost function, then tracking performance can be maintained. The tracking mismatch here occurs when the input signal hits the limits and is saturated.

VI. Conclusions

In this paper, a predictive repetitive control algorithm using frequency decomposition has been developed. This algorithm is developed from a state-space model formed by augmenting the plant state-space model with the dominant frequency components from the frequency sampling filter model. A model predictive control algorithm based on the receding horizon principle is then employed to optimize the filtered input to the plant. Simulation results based a gantry robot model shows that the algorithm can achieve i) perfect tracking of the periodic reference signal, and ii) superior tracking performance when the input is subject to constraints. One of the next areas for this work is to experimentally verify this algorithm on the gantry robot.

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