

2D Shape Transformation Using 3D Blending

G. Pasko¹, A. Pasko^{2,1}, M. Ikeda¹, and T. Kunii^{2,1}

¹ IT Institute, Kanazawa Institute of Technology, Tokyo, Japan

E-mail: {gpasko, ikeda}@iti.kanazawa-it.ac.jp

² Hosei University, Tokyo, Japan

E-mail: {pasko, kunii}@k.hosei.ac.jp

Abstract

Computer animation is one of the key components of a multimedia document or presentation. Shape transformation between objects of different topology and positions is an open modeling problem in computer animation. We propose a new approach to solving this problem for two given 2D shapes. The key steps of the proposed algorithm are: dimension increase by converting input 2D shapes into half-cylinders in 3D space, bounded blending with added material between the half-cylinders, and making cross-sections for getting frames of the animation. We use the bounded blending set operations defined using R-functions and displacement functions with the localized area of influence applied to the functionally defined 3D half-cylinders. The proposed approach is general enough to handle input shapes with arbitrary topology defined as polygons with holes and disjoint components, set-theoretic objects, or analytical implicit curves. The obtained unusual amoeba-like behavior of the 2D shape combines metamorphosis with the non-linear movement on the plane.

1. Introduction

Shape transformation is one of the important operations in animation and free-form modeling. It includes simple linear transformations (translation, scaling, rotation), non-linear transformations such as free-form transformations and other non-linear space mappings, and metamorphosis or morphing (transformation of one given shape into another). Approaches to 2D metamorphosis are well developed for two given simple polygons. They use physically-based methods [S92, S93], star-skeleton representation [SR95], warping and distance field interpolation [CLS96], wavelet-based [ZH00] and surface reconstruction methods [SSBT01]. The existing approaches are based on one or several of the following assumptions: equivalent topology (mainly topological disks are considered), polygonal shape representation, shape alignment (shapes have common coordinate origin and significantly overlap in most of the case studies), possibility of shape matching (establishing of shape vertex-vertex, control points or other features correspondence), the resulting transformation should be close to the motion of an articulated figure.

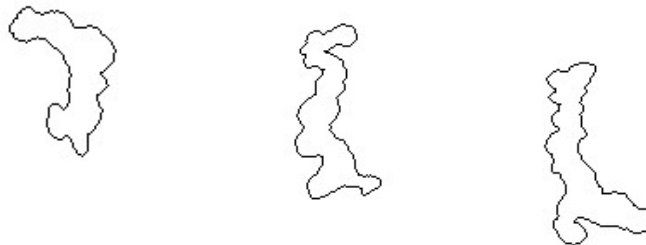


Figure 1. Three steps of biological amoeba motion.

The specific aspects of the shape transformation problem considered in this paper are the following:

- initial 2D shapes can have arbitrary topology not corresponding to each other;
- the shapes can be defined as 2D polygons, implicit curves, or constructive 2D solids;
- the shapes are not aligned or overlap, and can occupy different positions on the plane;

- there is no correspondence established between the boundary points or other shape features;
- a combined transformation is considered including metamorphosis and non-linear motion similar to the behavior of a biological amoeba illustrated in Fig. 1.

In this work, we propose a new method of shape transformation including non-linear motion and metamorphosis. We use implicit curves [B97] for simple shapes without holes and self-intersections, and 2D FRep solids [PASS95] in more general cases. Both these models use real-valued functions of two variables $F(x,y)$ to represent 2D shapes. The boundary of such shape is an implicit curve $F(x,y)=0$. A 2D FRep solid is constructed using primitives and operations, and can be defined by the inequality $F(x,y)\geq 0$, where the function F is evaluated at the given point by a procedure traversing an FRep constructive tree. A given polygonal shape can be converted automatically in an FRep solid as it is discussed below. Function-based metamorphosis [FPA00] using the linear (for two initial solids) or bilinear (for four initial solids) function interpolation can be applied to transform function-based shapes, but it can produce poor results for not aligned objects with different topology. We propose an alternative way based on increasing the object dimension, function-based blending, and consequent cross-sections as it is explained in the following section. Short animations produced as illustrations of the proposed approach can be found at URL

<http://www.k.hosei.ac.jp/~pasko/2DtransWeb/>

The HyperFun modeling language [A99] and supporting software tools were used to produce these animations and most of the illustrations in the paper.

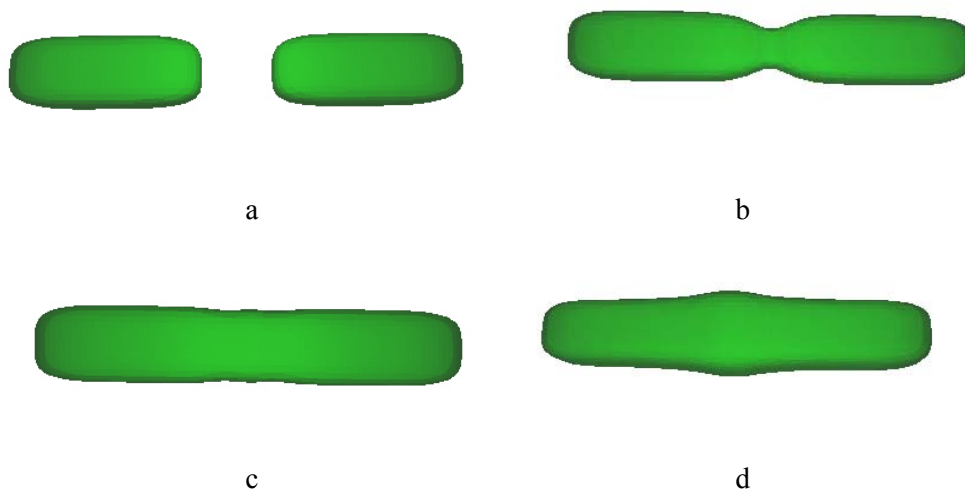


Figure 2. Blending union of two disjoint solids (a) with added material results in a single solid (b-d) with the different shapes of the blend depending on parameters.

2. Shape transformation using higher-dimensional blending

A blending operation in 3D shape modeling generates smooth transition between two surfaces. Note that sometimes the term “blending” is used to designate metamorphosis of 2D shapes, but we use it here in the way traditional to geometric and solid modeling. Blending versions of set-theoretic operations (intersection, union, and difference) on solids approximate exact results of these operations by rounding sharp edges and vertices. Such operations are usually used in computer-aided design for modeling fillets and chamfers. In the case of blending union of two disjoint solids with added material, a single resulting solid with a smooth surface can be obtained as it is shown in Fig. 2. This property of the blending union operation is the basis of our approach to the shape transformation.

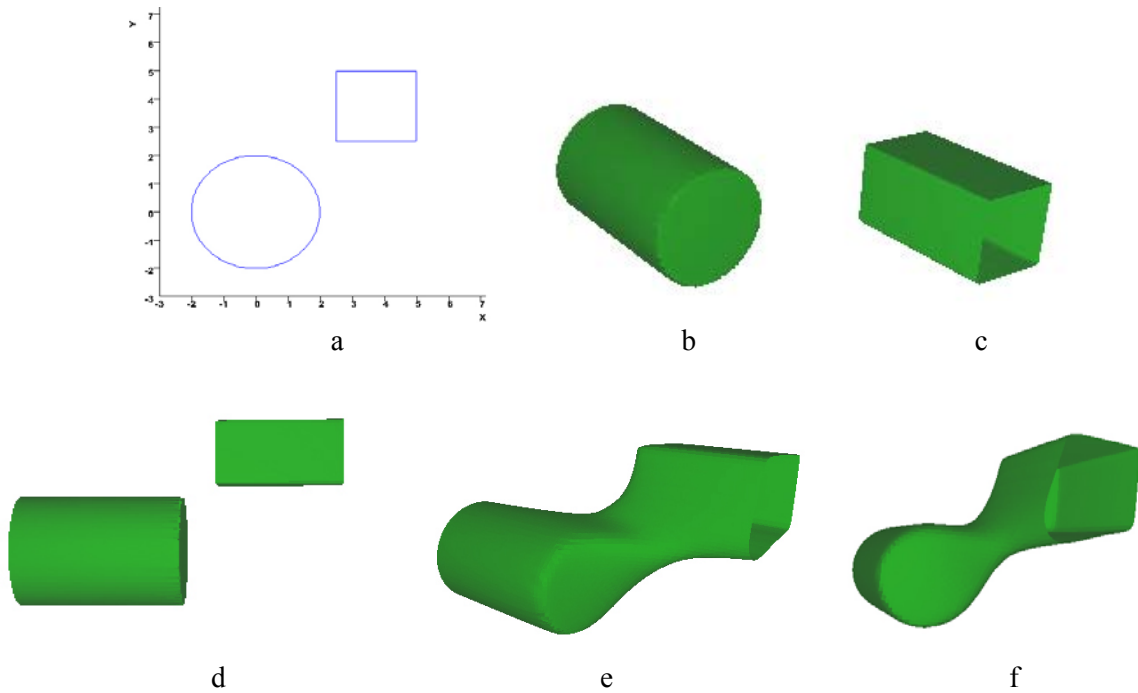


Figure 3. Steps of 2D shape transformation using 3D blending

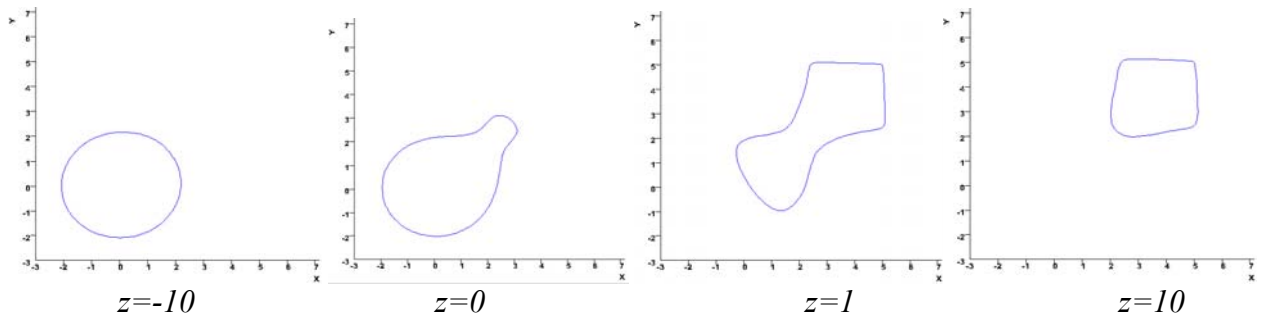


Figure 4. Frames of animation of the disk to square transformation using unbounded blending (initial shapes cannot be obtained, see $z=-10$ and $z=10$ cross-sections)

Let us formulate step by step the approach we propose and illustrate it by Figs. 3, 4:

- 1) two initial shapes are given on the xy -plane (a disk and a square in Fig. 3a);
- 2) each shape is considered as a cross-sections of a half-cylinder in 3D space (a cylinder bounded by a plane from one side) as it is shown in Figs. 3b and 3c;
- 3) the axes of both cylinders are parallel to some common straight line in 3D space, for example, to the coordinate z -axis, and the bounding planes of two half-cylinders are placed at some distance to give space for making the blend (Fig. 3d);
- 4) apply the added material blend operation (similar to one illustrated in Fig. 2) to the half-cylinders (Fig. 3e,f);
- 5) adjust parameters of the blend such that a satisfactory intermediate 2D shape is obtained in one or several 2D cross sections by planes orthogonal to z -axis (Fig. 4);
- 6) considering additional z -coordinate as time, make consequent orthogonal cross-sections along z -axis (Fig. 4) and combine them into 2D animation.

As it is shown in Fig. 4, the initial shapes cannot be obtained even in cross-sections with big absolute values of z -coordinate, if global blending is applied. The description of the function-based blending and its localized version solving the above problem are given in the following sections.

2.1 Function-based blending

The major requirements to blending operations are tangency of the blend surface with the initial surfaces; automatic clipping of unwanted parts of the blending surface; blending definition of the basic set-theoretic operations; and support of added and subtracted material blends. An analytical definition of blending satisfying these requirements was proposed in [PS94]. This definition covered added and subtracted material blends as well as symmetric and asymmetric blending.

An analytical definition of blending set-theoretic operations on solids with implicit surfaces was proposed in [PS94] on the basis of properties of R-functions [R87, PASS95], which provide exact analytical definitions of set-theoretic operations with C^1 and higher-level continuity. The following definition of the blending set-theoretic operation was proposed:

$$F_b(f_1, f_2) = R(f_1, f_2) + disp_b(f_1, f_2), \quad (1)$$

where $R(f_1, f_2)$ is an R-function corresponding to the type of the operation, the arguments of the operation $f_1(X)$ and $f_2(X)$ are defining functions of two initial solids, and $disp_b(f_1, f_2)$ is a Gaussian-type displacement function. The following expression for the displacement function was used:

$$disp_b(f_1, f_2) = \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}, \quad (2)$$

where a_0 , a_1 , and a_2 are parameters controlling the shape of the blend. The proposed definition is suitable for blending union, intersection, and difference, allows for generating added and subtracted material, symmetric and asymmetric blends. For example, the union operation can be exactly defined by the R-function as

$$R_{\text{int}}(f_1, f_2) = f_1 + f_2 + \sqrt{f_1^2 + f_2^2} \quad (3)$$

with the corresponding blending union described as

$$F_{b\text{int}} = f_1 + f_2 + \sqrt{f_1^2 + f_2^2} + \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}$$

This blending union operation was used to blend the half-cylinders in Figs. 3,4. The exact initial shapes (a disk and a square) cannot be obtained at any cross-section, see, for example, $z=-10$ and $z=10$ cross-sections in Fig. 4. The proposed displacement function does not get zero value anywhere in the space. This is the reason of the main disadvantage of this definition - the blend has a global character and cannot be localized using its parameters. A different displacement function is needed, which would allow for the localization of the blend and respectively for defining an exact time interval for the shape transformation.

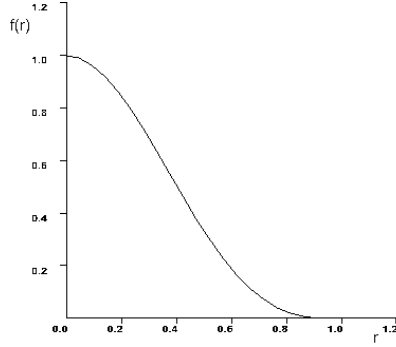


Figure 5. Displacement function for bounded blending.

2.2 Bounded blending

In this section, we apply a new formulation for blending operations with the blend bounded by an additional bounding solid as proposed in [PPIK02]. The definition (1) of a blending operation is used as the basis with the modifications introduced only for the displacement function. First, a displacement function with the local area of influence is selected, then, a generalized distance function for bounding blends using bounding solids is constructed.

Displacement function

At the first step of defining bounded blending, we select for the definition (1) a displacement function of one variable $disp_{bb}(r)$, which should satisfy the following conditions:

- 1) $disp_{bb}(r) \geq 0$ and $disp_{bb}(r)$ takes the maximal value for $r=0$;
- 2) $disp_{bb}(r) = 0, r \geq 1$
- 3) $\frac{\partial disp_{bb}}{\partial r} = 0, r = 1$ (the curve tangentially approaches axis x at $r=1$).

A plot of the desired displacement function is shown in Fig. 5. One can derive or find several analytical expressions for such function. In fact, this work has been done by many researchers for localizing individual components of skeletal implicit surfaces [B97]. One of the possible expressions, which we will use in the rest of the paper, is

$$disp_{bb}(r) = \begin{cases} \frac{(1-r^2)^3}{1+r^2}, & r < 1 \\ 0, & r \geq 1 \end{cases} \quad (4)$$

Here, r is a generalized distance, which is constructed using defining functions of the initial solids, as it will be shown in the next section.

Bounding solid

The introduction of an additional bounding solid can bring real local character to the blend. It is required that the blending surface exists only inside the bounding solid, and only original surfaces exist outside the bounding solid. To apply the bounded blending definition

$$F_{bb}(f_1, f_2, f_3) = R(f_1, f_2) + a_0 disp_{bb}(r) \quad (5)$$

with the displacement function (4), we have to provide the following interconnected properties:

- 1) The zero value of the displacement function $disp_{bb}(r)$ outside the bounding solid;
- 2) The maximal value of the displacement function $disp_{bb}(r)$ near the intersection points of two initial surfaces;
- 3) The zero value of the generalized distance function r at the intersection points of two initial surfaces;
- 4) The value 1 of the function r outside the bounding solid.

The following definitions satisfy the above requirements:

$$disp_{bb}(r) = \begin{cases} \frac{(1-r^2)^3}{1+r^2}, & r < 1 \\ 0, & r \geq 1 \end{cases} \quad \text{with } r^2 = \begin{cases} \frac{r_1^2}{r_1^2 + r_2^2}, & r_2 > 0 \\ 1, & r_2 = 0 \end{cases}, \quad (8)$$

where

$$r_1^2(f_1, f_2) = \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2,$$

and

$$r_2^2(f_3) = \begin{cases} \left(\frac{f_3}{a_3}\right)^2, & f_3 > 0 \\ 0, & f_3 \leq 0 \end{cases},$$

with numerical parameters a_1 and a_2 controlling the blend symmetry, and a_3 allowing the user to interactively control the influence of the function f_3 on the overall shape of the blend. This definition of the function r and the definition (4) of the displacement function $disp_{bb}(r)$ are not unique and can be changed, if it is necessary in particular applications.

Shape transformation using bounded blending

Let us apply the described above bounded blending union to get smooth transitions between two half-cylinders for the shape transformation (Figs. 6,7). The half-cylinders are bounded by the planes $z=0$ and $z=l$ to make the gap $[0,1]$ along z -axis between them. The bounding solid for the blend in this case is an infinite slab orthogonal to z -axis and defined by the function f_3 as an intersection of two halfspaces with the definitions $z \geq -10$ and $z \leq 10$. As it can be seen from Fig. 6, the blending displacement from the exact union of two half-cylinders takes zero value at the boundaries of the bounding solid (planes $z=-10$ and $z=10$). This results in the exact initial 2D shapes obtained at the cross-sections outside the bounding solid: the disk for $z \leq -10$ and the square for $z \geq 10$ (see Fig. 7 and compare with Fig. 4). The parameters $a_0 - a_3$ of the bounded blend influence the blend shape and respectively the shape of the intermediate cross-sections. The selection of the boundary values for the blend mainly depends on the overall size of the initial shapes and their positions. The key point here is that whatever interval is selected for the bounded blend along z -axis in 3D space, it can always be scaled to match the required time interval for the shape transformation on a 2D plane.



Figure 6. Bounded blending of two half-cylinders with a disk and a square as cross-sections.

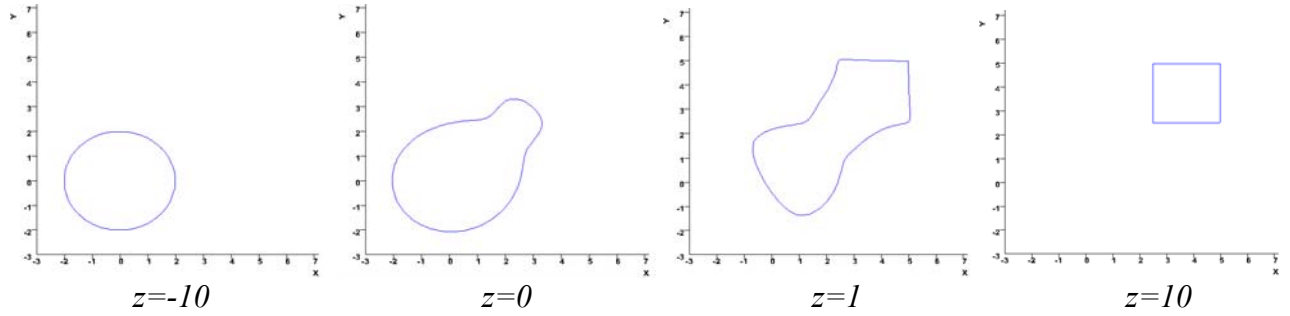


Figure 7. Frames of animation of the disk to the square transformation using bounded blending (exact initial cross-sections at the boundaries of the blend: cross-sections with $z=-10$ and $z=10$).

3. Transformation of 2D polygonal shapes

In this section, we illustrate application of the proposed approach to the transformation of two polygonal shapes of arbitrary topology. An arbitrary 2D polygon (convex or concave) can be represented by a real function $f(x,y)$ taking zero value at polygon edges [R74, PSS96]. The polygon-to-function conversion problem is stated as follows. A two-dimensional simple polygon is bounded by a finite set of segments. The segments are the edges and their extremes are the vertices of the polygon. A polygon is simple if there is no pair of nonadjacent edges sharing a point, and convex if its interior is a convex set. The polygon-to-function conversion algorithm using a monotone set-theoretic expression with R-functions was proposed in [R74], implemented and tested in [PSS96]. The conversion satisfies the following requirements:

- It provides an exact polygon boundary description as the zero set of a real function;
- No points with zero function value exist inside or outside of the polygon;
- It allows for the processing of any arbitrary simple polygon without any additional information.

In the case of a polygon with holes, each hole is processed as a separate polygon, and then the set-theoretic subtraction is applied between the initial polygon and the hole model. In the case of several disjoint components, each component is processed as a separate polygon, and then the set-theoretic union is applied to all components.

Let us present a short experimental animation showing a transformation between two arbitrary polygonal shapes with holes and disjoint components (Fig. 8). The idea of this test came from the “Dancing Buddhas” project [G01] devoted to multimedia augmenting of famous Buddhist texts. The problem in this test is to get a new type of transformation of a Buddha shape into a Chinese character. First, two polygonal shapes were obtained from the images: the Buddha shape (see the image in Fig. 8 left) consists of the main concave polygon (49 segments) and two simple polygonal holes, the Chinese character (see the image in Fig. 8 right) consists of two disjoint components, one of which is a simple concave polygon (left part of the character) and another (right part of the character) is a concave polygonal shape (12 segments) with five holes. As one can see, the initial shapes have quite different topology, they also were placed on the plane without overlapping. We applied the proposed

algorithm using bounded blending in 3D space and obtained an animation (see some of the frames in Fig. 9). The transformation is quite smooth except some acceleration within the interval $[0,1]$ along z -axis, where the most drastic transformations occur. The 3D shape of the blend and respectively the behavior of the 2D shape during the transformation can be controlled by the blend parameters, the length of the gap between the half-cylinders along z -axis, and the positions of the planes bounding the blend.



Figure 8. Initial Buddha shape (left) and a Chinese character (right).

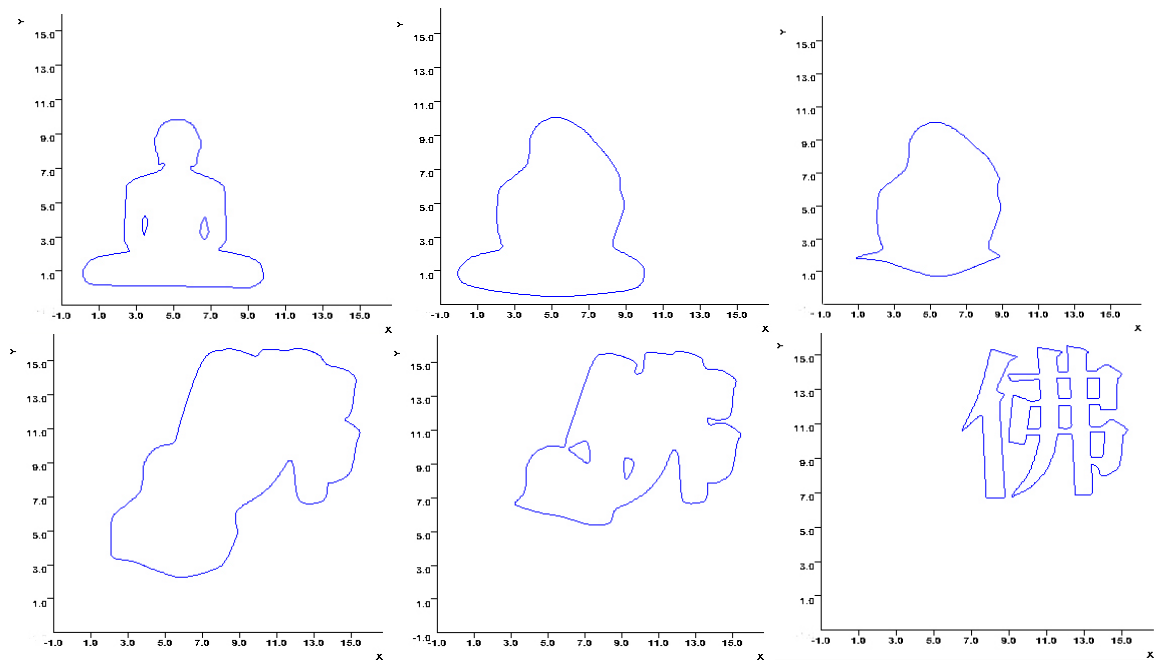


Figure 9. Frames of the animation: transformation of a Buddha shape into a Chinese character.

4. Conclusions

We introduced a new approach and analytical formulations for the 2D shape transformation on the basis of the dimension increase, bounded blending between 3D objects, and cross-sectioning the blend area for getting frames of the animation. It is shown that the blending operation with such simple bounds as two planes satisfies the condition of the localization of the shape transformation at some predefined time interval. The proposed approach can handle non-overlapping 2D shapes with arbitrary topology defined by input polygons, set-theoretic or analytical expression. The obtained behavior of the 2D shape during the transformation process does not imitate the motion of an articulated figure, but rather has amorphous or amoeba-like character including non-linear motion and metamorphosis. The extension of this approach to the case of 3D input shapes and bounded blending of 4D space-time objects will be the topic of our further research efforts in this direction. The 2D animations illustrating the proposed approach can be found at URL <http://www.k.hosei.ac.jp/~pasko/2DtransWeb/>

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