The Macroeconomics of Piketty

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Abstract

Since the early 2000s, research by Thomas Piketty, Emmanuel Saez, and their coathors has revolutionized our understanding of income and wealth inequality. In this paper, I highlight some of the key empirical facts from this research and comment on how they relate to macroeconomics and to economic theory more generally. Top inequality is tightly linked to Pareto distributions. The paper describes simple mechanisms that give rise to this Pareto inequality and considers the economic forces that influence top inequality over time and across countries.

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Since the early 2000s, research by Thomas Piketty and Emmanuel Saez (and their coathors, including Anthony Atkinson and Gabriel Zucman) has revolutionized our understanding of income and wealth inequality. The crucial departure point for this revolution is the extensive data they have used, based largely on administrative tax records, to study inequality at the very top of the income and wealth distributions. Piketty's *Capital in the Twenty-First Century* is the latest contribution in this line of work, especially with the new data it provides on capital and wealth.

In this paper, I highlight some of the key empirical facts from this research and comment on how they relate to macroeconomics and to economic theory more generally. Top inequality is tightly linked to Pareto distributions. The paper describes simple mechanisms that give rise to this Pareto inequality and considers the economic forces that influence top inequality over time and across countries.

To organize what follows, recall that GDP can be written as the sum of "labor income" and "capital income." This split highlights several kinds of inequality that we can explore. In particular, there is *within* inequality for each of these components: How much inequality is there within labor income? How much inequality among capital income — or, more appropriately here, among the wealth itself for which capital income is just the annual flow? And there is also *between* inequality related to the split of GDP between capital and labor. This between inequality takes on particular relevance given the "within" inequality fact that most wealth is held by a small fraction of the population. In the three main sections of this paper, I consider each of these concepts in turn. I first highlight some of the key facts related to each type on inequality. Then I explain how economic theory can help us understand and interpret the facts.

1. Labor Income Inequality

1.1. Basic Facts

One of the key papers documenting the rise in top income inequality is Piketty and Saez (2003), and it is appropriate to start with an updated graph from their paper.





Note: The figure shows the composition of the top 0.1 percent income share. Source: These data are taken from the "data-Fig4B" tab of the September 2013 update of the spreadsheet appendix to Piketty and Saez (2003).

Figure 1 shows the share of income going to the top 0.1 percent of families in the United States, along with the composition of this income. Piketty and Saez emphasize three key facts seen in this figure. First is the long U-shaped pattern to top inequality: high prior to the Great Depression, low and relatively steady between World War II and the mid-1970s, and then rising since then, ultimately achieving similar levels today to the high levels of top income inequality we saw in the 1910s and 1920s. Second is that fact that much of the decline in top inequality in the first half of the 20th century was associated with capital income. And third is the fact that much of the rise in the last several decades is associated with labor income, particularly if one includes "business income" in this category.

1.2. Theory

The next section of the paper will discuss wealth and capital income inequality. Here, I'd like to focus on labor income inequality. In particular, what are the eco-

nomic determinants of top labor income inequality, and why might they change over time and differ across countries?

At least since Pareto (1896) first discussed income heterogeneity in the context of his eponymous distribution, it has been appreciated that incomes at the top are well characterized by a power law. That is, apart from a proportionality factor to normalize units, $\Pr[\text{Income} > y] = y^{-1/\eta}$. It is easy to show in this case that the fraction of income going to the top p percentiles equals $(\frac{100}{p})^{\eta-1}$. With $\eta = 1/2$, the share of income going to the top 1 percent is $100^{-1/2} = .10$, or 10 percent, while if $\eta = 2/3$, this share is $100^{-1/3} \approx 0.22$, or 22 percent. They key parameter in this distribution is η , and an increase in η leads to a rise in top inequality. Hence this parameter is naturally called a measure of Pareto inequality. A theory of top income inequality, then, needs to explain two things: (i) why the income distribution at the top is Pareto and (ii) what economic forces determine η .

The economics literature in recent years includes a number of papers that ask related questions. For example, Gabaix (1999) studies the so-called Zipf's Law for city populations: why is the size distribution of cities Pareto, and why is the inequality parameter very close to 1? Luttmer (2007) asks the analogous question for firms: why is the size distribution of firms in the United States Pareto with an inequality parameter very close to 1? Here, the questions are slightly different: why is the income distribution Pareto, and why does the inequality parameter change over time and differ across countries? Interestingly, it turns out that there is a lot more inequality among city populations or firm employment than there is among incomes (where $\eta \approx 0.6$ in the United States today), and the size distribution of cities and firms is surprisingly stable when compared to the sharp rise in top income inequality in the U.S.

From this recent economics literature as well as from an earlier literature that it builds on, we learn that the basic mechanism for generating a Pareto distribution is surprisingly simple: *exponential growth that occurs for an exponentially-distributed amount of time leads to a Pareto distribution*.¹

¹Excellent introductions to Pareto models can be found in Mitzenmacher (2004), Gabaix (2009), Benhabib (2014), and Moll (2012b). Benhabib traces the history of Pareto-generating mechanisms

To see how this works, we first require some heterogeneity. Suppose people are exponentially distributed across some variable x, which could denote age or experience or talent. For example, $\Pr[Age > x] = e^{-\delta x}$, where δ denotes the death rate in the population. Next, we need to explain how income varies with age in the population. A natural assumption is exponential growth: suppose income rises exponentially with age (or experience or talent) at rate μ : Income = $e^{\mu x}$. In this case, the log of income is just proportional to age, so the log of income obeys an exponential distribution with parameter δ/μ .

Next, we use an interesting property: if the log of income is exponential, then the level of income obeys a Pareto distribution:²

$$\Pr[\text{Income} > y] = y^{-\delta/\mu}.$$

Recall that the Pareto inequality measure is just the inverse of the exponent in this equation, which gives

$$\eta_{\rm income} = \frac{\mu}{\delta}.$$
 (1)

The Pareto exponent is increasing with μ , the rate at which incomes grow with age and decreasing in the death rate δ . Intuitively, the lower is the death rate, the longer some lucky people in the economy can benefit from exponential growth, which widens Pareto inequality. Similarly, faster exponential growth across age (a higher return to experience?) also widens inequality.

Jones and Kim (2014) build a model along these lines in which both μ and δ are endogenous variables. In their setup, x is related to the length of time since a researcher first discovers a new idea, thereby becoming an entrepreneur. The entrepreneur's effort affects the growth rate μ , and δ is an endogenous rate of creative destruction by which one entrepreneur is displaced by another. Technological changes that make a given amount of entrepreneurial effort more effective, such as the world wide web, will increase top income inequality. Conversely, exposing for-

and attributes the earliest instance of a simple model like that outlined here to Cantelli (1921).

²This derivation is explained in more detail in a companion paper Jones (2014), available at http://www.stanford.edu/~chadj/SimplePareto.pdf.

merly closed domestic markets to international competition may increase creative destruction and reduce top income inequality. Finally, the model also incorporates an important additional role for luck: the richest people are those who not only avoid the destruction shock for long periods but also who benefit from the best idiosyncratic shocks. Both effort and luck play important roles at the top, and models like this combined with data on the stochastic income process of top earners can allow us to quantify the roles of luck, technology, and effort.

2. Wealth Inequality

2.1. Basic Facts

Piketty's new book focuses on what turns out to be a more difficult subject, capital. It is more difficult both because data on wealth are more difficult to obtain and conceptually in that the models are inherently more complicated because wealth accumulates gradually over time. Income data are "readily" (in comparison only!) available from tax authorities, while wealth data are gathered less reliably. For example, common sources include estate taxation, which affects an individual infrequently, or surveys, in which wealthy people may be reluctant to share the details of their holdings. With extensive effort, Piketty assembles the wealth inequality data shown in Figure 2, and several findings stand out immediately.

First, wealth inequality is much greater than income inequality. The top 1 percent of families possess around 35 or 40 percent of wealth in the United States in 2010, versus around 17 percent of income. Put another way, the income cutoff for the top 1 percent is about \$330,000 — in the ballpark of the top salaries for academics. In contrast, according to the latest data from Saez and Zucman (2014), the wealth cutoff for the top 1 percent is an astonishing \$4 million! Note that both groups include about 1.5 million families.

Second, wealth inequality in France and the United Kingdom is dramatically lower today than it was in at any time between 1810 and 1960. The share of wealth going to the top 1 percent is around 25 or 30 percent today, versus peaks in 1910 of



Figure 2: Wealth Inequality

Note: The figure shows the share of aggregate wealth held by the richest 1 percent of the population. Source: Supplementary Table S10.1 for Chapter 10 of Piketty (2014), http://piketty.pse.ens.fr/en/capital21c2.

60 percent or more. Two world wars, the Great Depression, the rise of progressive taxation — some combination of these and other events led to an astonishing drop in wealth inequality both there and in the United States between 1910 and 1965.

Third, wealth inequality has increased during the last 50 years, but the increase seems small in comparison to the declines just discussed. An important caveat to this statement applies to the United States: the data shown are those used by Piketty in his book, but Saez and Zucman (2014) have recently assembled what they believe to be superior data in the United States, and these data show a rise to a 40 percent wealth share for the top 1 percent by 2010, much closer to the earlier U.S. peak in the first part of the 20th century.

2.2. Theory

A substantial and growing body of economic theory seeks to understand the determinants of wealth inequality.³ Pareto inequality in wealth readily emerges through the same mechanism we discussed in the context of income inequality: exponential growth that occurs over an exponentially-distributed amount of time. In the case of wealth inequality, this exponential growth is fundamentally tied to the interest rate, r: in a standard asset accumulation equation, the return on wealth is a key determinant of the growth rate of an individual's wealth. On the other hand, this growth in an individual's wealth occurs against a backdrop of economic growth in the overall economy. To get a variable that will exhibit a stationary distribution, one must normalize an individual's wealth level by aggregate wealth (or income) in the economy. If aggregate wealth grows at rate g, the normalized wealth of an individual then grows at rate r - g. This logic underlies the key r - g term for wealth inequality that makes a frequent appearance in Piketty's book. Of course, r and gare endogenous variables in general equilibrium so — as we will see — one must be careful in thinking about how they might vary independently.

To be more specific, imagine an economy of heterogeneous people. The details of the model we describe next are given in a companion paper, Jones (2014).⁴ but the logic is actually quite easy to follow. To keep it simple, assume there is no labor income and that individuals consume a constant fraction α of their wealth. As discussed above, wealth earns a basic return r. However, wealth is also subject to a wealth tax: a fraction τ is taken by the government every period. With this setup, the individual's wealth grows exponentially at a constant rate $r - \tau - \alpha$. Next, assume that aggregate wealth per person (e.g. capital per person) grows at rate g. The individual's share of aggregate wealth per person then grows exponentially at rate $r - g - \tau - \alpha > 0$. This is the basic "exponential growth" part of the requirement for a Pareto distribution.

³References include Wold and Whittle (1957), Stiglitz (1969), Huggett (1996), Quadrini (2000), Castaneda, Diaz-Gimenez and Rios-Rull (2003), Benhabib and Bisin (2006), Cagetti and Nardi (2006), Nirei (2009), Benhabib, Bisin and Zhu (2011), Moll (2012a), Piketty and Saez (2012), Aoki and Nirei (2013), Moll (2014), and Piketty and Zucman (2014).

⁴See http://www.stanford.edu/~chadj/SimplePareto.pdf.

Next, we need heterogeneity. We obtain heterogeneity in the simplest possible fashion: assume that each person faces a constant probability of death, \bar{d} , each period. Because Piketty emphasizes the role played by changing rates of population growth, we'll also include population growth, assumed to occur at rate \bar{n} . Each new person born in this economy inherits the same amount of wealth and the aggregate inheritance is simply equal to the aggregate wealth of the people who die each period. It is straightforward to show that the steady-state distribution of this birth-death process is an exponential distribution, where the age distribution is $\Pr[\text{Age} > x] = e^{-(\bar{n}+\bar{d})x}$. That is, the age distribution is governed by the (gross) birth rate, $\bar{n} + \bar{d}$. This is intuitive once one appreciates that a fraction $\bar{n} + \bar{d}$ of new people are added to the economy each instant.

We now have exponential growth occuring over an exponentially-distributed amount of time. The simple model we presented in the context of the income distribution suggested that the Pareto inequality measure equals the ratio of the "growth rate" to the "exponential distribution parameter" and that logic also holds for this model of the wealth distribution. In particular, (normalized) wealth has a steadystate distribution that is Pareto with

$$\eta_{\text{wealth}} = \frac{r - g - \tau - \alpha}{\bar{n} + \bar{d}}.$$
(2)

An equation like this is at the heart of many of Piketty's statements about wealth inequality, for example as measured by the share of wealth going to the top 1 percent. An increase in r - g will increase wealth inequality: people who are lucky to live a long time — or are part of a long-lived dynasty — will accumulate greater stocks of wealth. Also, a higher wealth tax will lower wealth inequality. In richer frameworks that include stochastic returns to wealth, the super-rich are also those who benefit from a lucky run of good returns, and a higher variance of returns will increase wealth inequality.

Can this simple model explain why wealth inequality was so high historically in France and the United Kingdom relative to today? Or why wealth inequality was historically much higher in Europe than in the United States? Notice that if g is low

or if \bar{n} is low — both of which applied in the 19th century — wealth inequality will be higher.

Piketty summarizes the logic underlying models like this with characteristic eloquence: "[I]n stagnant societies, wealth accumulated in the past takes on considerable importance" (p. 232). On the role of population growth, for example, Piketty notes that an increase means that inherited wealth gets divided up by more offspring, reducing inequality. Similarly, a decline in population growth will concentrate wealth. A related effect occurs when the economy's per capita growth rate rises. In this case, inherited wealth fades in value relative to new wealth generated by economic growth. Silicon Valley in recent decades is perhaps an example worth considering. Reflections of these stories can be seen in the math of equation (2).

2.3. General Equilibrium

Whether or not changes in the parameters of models in this genre can explain the large changes in wealth inequality that we see in the data is an open question. However, one cautionary note deserves mention: the comparative statics just provided ignore the important point that arguably all the parameters of that equation are endogenous. As one simple example, recall that a standard Euler equation for consumption with log utility delivers the result that $r - g - \tau = \rho$, where ρ is the rate of time preference. That is, changes in g or τ get mirrored by changes in the interest rate itself, potentially leaving wealth inequality unchanged. To take another example, the fraction of wealth that is consumed, α , will naturally depend on the rate of time preference and the death rate in the economy.

In fact, if tax revenues are used to pay for government services that enter utility in an additively separable fashion, the formula for wealth inequality in this simple model reduces to $\eta_{\text{wealth}} = \frac{\bar{n}}{\bar{n}+d}$; see Jones (2014) for the details. Remarkably, there is an invariance to wealth taxes and the effect of population growth on wealth can actually go in the opposite direction. The intuition for this result is interesting: while in partial equilibrium, the growth rate of normalized wealth is $r - g - \tau - \alpha$, in general equilibrium, the key source of heterogeneity is population growth. Newborns in this economy inherit the wealth of the people who die. Because of population growth, there are more newborns than people who die, so newborns inherit less than the average amount of wealth per capita. This ties the distribution of wealth across ages at a point in time to population growth in general equilibrium.

The bottom line is that care must be taken in using partial equilibrium solutions like (2). General equilibrium forces matter and can significantly alter the fundamental determinants of Pareto inequality. More generally, bequests, social mobility, progressive taxation, transition dynamics, and the role of both macroeconomic and microeconomic shocks need to be considered in a richer framework. The references cited earlier make progress on these fronts.

3. "Between" Inequality: Capital vs Labor

We next turn to "between" inequality: how is income to capital versus income to labor changing, and how is the wealth-income ratio changing? This type of inequality takes on particular importance given our previous fact about within inequality: the fact that most of the wealth is held by a small fraction of the population means that changes in the share of national income going to capital or in the aggregate capital-output ratio also contribute significantly to inequality. Whereas Pareto inequality describes how inequality at the top of the distribution is changing, this between inequality is more about inequality between the top 10 percent of the population (who hold around 3/4 of the wealth in the United States according to Saez and Zucman (2014)) and the bottom 90 percent.

3.1. Basic Facts

At least since Kaldor (1961), a key stylized fact of macroeconomics has been the relative stability of factor payments to capital as a share of GDP. Figure 3 shows the long historical time series for France, the United Kingdom, and the United States that Piketty has assembled. A surprising point emerges immediately: prior to World War II, the capital share exhibits a substantial negative trend, falling from around 40





Note: Capital shares (including land rents) for each decade are averages over the preceding ten years. Source: Supplementary tables for Chapter 6 of Piketty (2014), http://piketty.pse.ens.fr/en/capital21c2 for France and the U.K. The U.S. shares are taken from Piketty and Zucman (2014).

percent in the mid 1800s to below 30 percent. By comparison, the data since 1940 show some stability, though with a notable rise between 1980 and 2010. In Piketty's data, the labor share is simply one minus the capital share, so the corrsponding changes in labor's share of factor payments can be read from this same graph.⁵

Before delving too deeply into these numbers, it is worth appreciating another stylized fact documented by Piketty. Figure 4 shows the capital-output ratio for this same group of countries, back to 1870. The movements are once again striking. France and the United Kingdom exhibit a very high capital-output ratio around 7 in the late 1800s. This ratio falls sharply and suddenly with World War I, to around 3, before rising steadily after World War II to around 6 today. The destruction associated with the two World Wars and the subsequent transition dynamics as Europe recovers are an obvious interpretation of these facts. The capital-output ratio in the

⁵Recent papers studying the rise in the capital share in recent decades include Karabarbounis and Neiman (2013), Elsby, Hobijn and Şahin (2013), and Bridgman (2014).





United States appears relatively stable in comparison, though still showing a decline during the Great Depression and a rise from 3.5 to 4.5 in the postwar period.

Delving into the detailed data underlying these graphs — which Piketty generously and thoroughly provides — highlights an important feature of the data. By focusing on only two factors of production, capital and labor, Piketty includes land as a form of capital. Of course, the key difference between land and the rest of capital is that the former cannot be accumulated, while the latter can. For the purpose of understanding inequality between the top and the rest of the distribution, including land as a part of capital is eminently sensible. On the other hand, for connecting the data to macroeconomic theory, one must be careful.

For example, in the 18th and early 19th centuries, Piketty notes that rents paid to landlords averaged around 20 percent of national income. His capital income share for the United Kingdom before 1910 is taken from Allen (2007), with some adjustments, and shows a sharp decline in income from land rents (down to only 2

percent by 1910) which masks a rise in income from reproducible capital.

Similarly, much of the large swing in the European capital-output ratios shown in Figure 4 are due to land as well. Figures 3.1 and 3.2 in the book make this clear. For example, in 1700 in France, the value of land equals almost 500 percent of national income versus only 12 percent by 2010. And the rise since 1950 is to a great extent due to housing, which rises from 85 percent of national income in 1950 to 371 percent in 2010. Bonnet, Bono, Chapelle and Wasmer (2014) document this point in great detail, going further to show that the rise in recent decades is primarily due to a rise in housing prices rather than to a rise in the quantity of housing. In comparison to these large swings in the aggregate capital-output ratio in France and the U.K., the ratio of reproducible capital to output is much more stable. In fact, the value in 2010 is actually lower than the value in several decades in the 1800s for both France and the United Kingdom. I find this fact surprising and worthy of more consideration. Again, though: from the standpoint of changing inequality, the declining role of land and the rising role of housing is not necessarily crucial. However, if one wishes to use Piketty's long-run data to say something about the parameters of macroeconomic models, more care is required.

3.2. Theory

The macroeconomics of the capital-output ratio is arguably the best-known theory within all of macroeconomics, with its essential roots in the analysis of Solow (1956) and Swan (1956). The familiar formula for the steady-state capital-output ratio is $s/(n + g + \delta)$, where *s* is the (gross) investment share of GDP, *n* denotes population growth, *g* is the steady-state growth rate of income per person, and δ is the rate at which capital depreciates. Largely for expositional purposes, Piketty simplifies this formula to another that is mathematically equivalent: \tilde{s}/\tilde{g} , where $\tilde{g} = n + g$ and \tilde{s} now denotes the investment rate net of depreciation, $\tilde{s} = s - \delta K/Y$. This more elegant equation is helpful for a general audience and gets the qualitative comparative statics right: in particular, Piketty emphasizes that a slowdown in growth — whether in per capita terms or in population growth — will raise the capital-output

ratio in the long-run. Piketty occasionally uses the simple formula to make *quantitative* statements, e.g. if the growth rate falls in half, then the capital-ouput ratio will double (for example, see the discussion beginning on page 166). This statement is not correct and takes the simplification too far.⁶

It is plausible that some of the decline in the capital-output ratio in France and the United Kingdom since the late 1800s is due to a rise in n + g, and it is possible that a slowing growth rate of aggregate GDP in recent decades and in the future could contribute to a rise in the capital-output ratio. However, the magnitude of these effects are mitigated quantitatively by taking depreciation into account. This point is discussed in detail in Krusell and Smith (2014).

There is also some discussion in Piketty's book (e.g. starting on page 220) on the elasticity of substitution between capital and labor. For example, look back at Figures 3 and 4. That fact that the capital share and the capital-output ratio move together, at least broadly over the long swing of history, is taken as suggestive evidence that the elasticity of substitution between capital and labor is greater than one. Given the importance of land in both of these time series, however, I would be hesitant to make too much of this correlation. The state-of-the-art in the literature on this elasticity is inconclusive, with some papers arguing for an elasticity greater than one but others arguing for less than one.⁷

4. Conclusion

Through extensive data work, particularly with administrative tax records, Piketty and Saez and their coauthors have shifted our understanding of inequality in an important way. A substantial part of the rise in inequality in advanced countries of the world is due to increases within the top 1 percent and even the top 0.1 percent of the distribution. That is, to a much greater extent than we've appreciated before, the dynamics of top income and wealth inequality are crucial.

⁶In particular, it ignores the fact that \tilde{s} will change when the growth rate changes, via the $\delta K/Y$ term.

⁷In particular, see Karabarbounis and Neiman (2013) and Oberfield and Raval (2014).

Much of the data necessary to support these claims is actually publicly available. For example, the Statistics of Income division of the Internal Revenue Service makes available random samples of detailed tax records in their public use microdata files, dating back to the 1960s.⁸ Future research combining data like this with models of top income inequality is primed to shed light on this important phenomenon.

⁸For more information on these data, see http://users.nber.org/~taxsim/gdb/.

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