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FURTHER APPLICATION OF STOCHASTIC RESONANCE FOR ENERGY HARVESTING

Dongxu Su
Institute of Industrial Science
University of Tokyo
Tokyo, Japan 153-8904
sudx@iis.u-tokyo.ac.jp

Kimihiko Nakano
Institute of Industrial Science
University of Tokyo
Tokyo, Japan 153-8904
knakano@iis.u-tokyo.ac.jp

Honggang Hu
Institute of Industrial Science
University of Tokyo
Tokyo, Japan 153-8904
huhg@iis.u-tokyo.ac.jp

Matthew P Cartmell
Department of Mechanical Engineering
University of Sheffield
Sheffield, UK S1 3JD
m.cartmell@sheffield.ac.uk

Masanori Ohori
Institute of Industrial Science
University of Tokyo
Tokyo, Japan 153-8904
ohori@iis.u-tokyo.ac.jp

Rencheng Zheng
Institute of Industrial Science
University of Tokyo
Tokyo, Japan 153-8904
topzrc@iis.u-tokyo.ac.jp

ABSTRACT

In addition to the wide range of applications of stochastic resonance in the field of signal processing, the phenomenon has also been investigated as an effective tool for enhancing vibrational energy harvesting. This paper proposes a hypothetical method for achieving stochastic resonance and increasing the available energy from external ambient vibration. In order to illustrate this proposal, a bistable mechanical system is proposed to study the feasibility by theoretical analysis. The amount of available energy and the energy consumed to produce the small-scale additional force is analyzed through numerical simulations. It is shown that the proposed method can significantly enhance the harvested vibrational energy.

NOMENCLATURE

$\mathbf{B}$ Magnetic flux density.

$\mathbf{r}_c$ Location vector .

$\mathbf{r}_s$ Location vector of the permanent magnet.

$\mathbf{r}_{c/s}$ Vector to the point of interest.

$|\mathbf{r}_{c/s}|$ Distance between the permanent magnets

μ_0 Permeability of free space.

v_s Volume of the source magnet.

v_s Volume of the source magnet.

v_c Volume of the target magnet.

v Volume of magnets.

$\mathbf{m}_s$ Magnetic moment of the source magnet.

$\mathbf{m}_c$ Magnetic moment of the target magnet.

$\mathbf{M}_s$ Magnetization amplitudes of the source magnet.

$\mathbf{M}_c$ Magnetization amplitudes of the target magnet.

$\mathbf{M}_f$ Magnetization amplitudes of the top fixed magnet.

k_c Stiffness of the cantilever.

F Restoring force.

h Distance between the magnetic end mass and the permanent magnets in the x-direction.

d Distance between the magnetic end mass and the permanent magnets in the y-direction.

m Magnetic end mass of the bistable mechanism.

c Damping of the cantilever.

$\bar{c}$ Non-dimensional damping of the cantilever.

k Linear stiffness of the bistable mechanism.

k_3 Nonlinear stiffness of the bistable mechanism.

x Displacement of the magnetic end mass.

t Time.

N External excitation.

δ Non-dimensional displacement of the magnetic end mass.

τ Non-dimensional time.

Q Non-dimensional external excitation.

γ Non-dimensional additional excitation.

γ_a Non-dimensional constant force.

C Constant value.

U Non-dimensional potential.

P Non-dimensional net power.

E Non-dimensional net energy.

INTRODUCTION

Vibrational energy has become a potential power supply source for small-scale electronics over the past few years. In the existing literature most researchers have focused on methods for increasing the bandwidth of vibration based devices. These techniques cover resonance frequency tuning, multimodal energy harvesting, frequency up-conversion, and nonlinear oscillations [1-7].

Stochastic resonance, as a method which is widely adopted in the field of signals analysis, is also one way to improve the performance for a nonlinear bistable oscillator with two potential wells. Daqaq [8] demonstrated that under Gaussian white noise excitations, bistabilities in the potential do not provide any enhancement over the traditional linear resonant generators which have a single-well potential. However, McInnes *et al*. firstly and theoretically exploited the possibility of enhancing the performance of a bistable mechanism with stochastic resonance. It was found that by adding periodic modulating forcing to a vibrationally excited bistable mechanism, the power available from the device could be enhanced over that of without periodic forcing [9]. On this basis, the possibility that stochastic resonance can occur in vibrating systems has been proved through experimental study by Hu *et al* [10]. Here the small-scale periodic forcing should match the transition frequency between the two potential wells known as the Kramer's rate [11]. However, the noise density, as a necessary parameter for obtaining the value of the Kramers rate, is difficult to measure in practical situations.

In this paper, a theory for a new method is proposed in this paper to achieve stochastic resonance by adding a small-scale excitation.

METHODOLOGY

Energy Harvesting Device

A conceptually bistable mechanism is designed to study the vibrational energy harvesting using the theory of stochastic resonance [11]. As illustrated in Fig. 1, the mechanism consists of a piezoelectric beam with a magnetic end mass, two identical fixed permanent magnets and two electromagnets, in conjunction with a small permanent magnet.

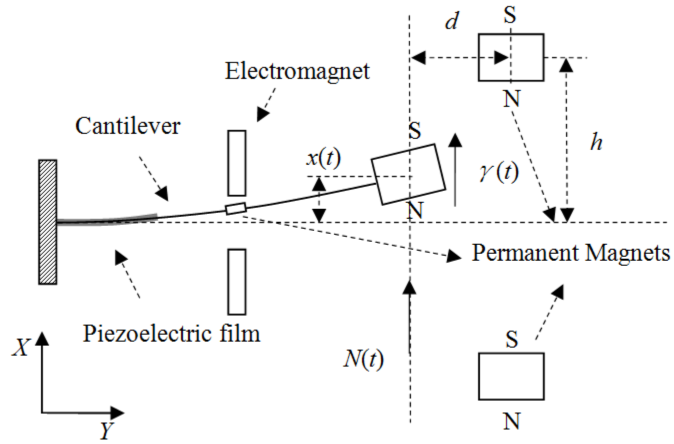

FIGURE 1. SCHEMATIC OF THE BISTABLE VIBRATIONAL ENERGY HARVESTER

In order to study the interaction between the magnetic end mass and the fixed magnets, a dipole model is used here [12]. The magnetic flux density at the location $\mathbf{r}_c$ due to a magnet located at $\mathbf{r}_s$ can be defined by

$$\mathbf{B} = -\frac{\mu_0}{4\pi}\nabla\frac{\mathbf{m}_s\cdot\mathbf{r}_{c/s}}{\left|\mathbf{r}_{c/s}\right|^3} \tag{1}$$

where $\mu_0 = 4\pi\times10^{-7}$, $\mathbf{r}_{c/s} = \mathbf{r}_c - \mathbf{r}_s$, $\left|\mathbf{r}_{c/s}\right|$ is the distance between two magnets. The magnetic moment of the source magnet $\mathbf{m}_s = \mathbf{M}_s v_s$. The potential energy of the magnet at $\mathbf{r}_c$ in the field generated by the magnet at $\mathbf{r}_s$ is defined by

$$U = -\mathbf{m}_c\cdot\mathbf{B} \tag{2}$$

where $\mathbf{m}_c = \mathbf{M}_c v_c$. It is assumed that the magnetic end mass and the fixed magnets have the same volume v. The magnetisation amplitudes of the magnetic end mass and the top fixed magnet are defined by $\mathbf{M}_b = (M_{bx}, M_{by})$ and $\mathbf{M}_f = (M_{fx}, M_{fy})$. Using Eq. (1) and Eq. (2), to take into account the restoring energy of the cantilever, the potential energy of the magnetic end mass can be expressed as

$$U = \frac{\mu_0 v^2}{4\pi}\left(\Phi(h+x)+\Phi(h-x)\right)+\frac{1}{2}k_c x^2 \tag{3}$$

where

$$\Phi(x) = -\frac{3\left(dM_{by}+xM_{bx}\right)\left(dM_{fy}+xM_{fx}\right)}{\left(d^2+x^2\right)^{5/2}} + \frac{M_{fy}M_{by}+M_{fx}M_{bx}}{\left(d^2+x^2\right)^{3/2}} \tag{4}$$

The restoring forces can be obtained from the opposite signed value of the derivative of the potential energy with respect to x

$$F(x) = \frac{\mu_0 v^2}{4\pi}\left(\Theta(h+x)-\Theta(h-x)\right)-k_c x \tag{5}$$

where

$$\Theta(x) = \frac{3\left(M_{by}\left(dM_{fx}+xM_{fy}\right)+M_{bx}\left(dM_{fy}+3xM_{fx}\right)\right)}{\left(d^2+x^2\right)^{5/2}} - \frac{15x\left(xM_{fx}+dM_{fy}\right)\left(M_{bx}+dM_{by}\right)}{\left(d^2+x^2\right)^{7/2}} \tag{6}$$

The values of d and h can be tunable to adjust the parameters of the mechanism. This system is excited by external ambient vibration $N(t)$, and the small-scale additional excitation is applied to the cantilever by the electromagnets.

According to Eq. (6), by choosing the corresponding magnetisation amplitudes of the permanent magnets $M_{fx} = M_{bx} = -9\times 10^5\ A/m$, $M_{fy} = -M_{by} = -8\times 10^5\ A/m$ and with dimensions of length 30mm, width 10 mm and height 10 mm, and setting d =17 mm and h =52 mm, the magnetoelastic force, the magnetic force and cantilever elastic force versus the end mass displacement are shown in Fig. 2.

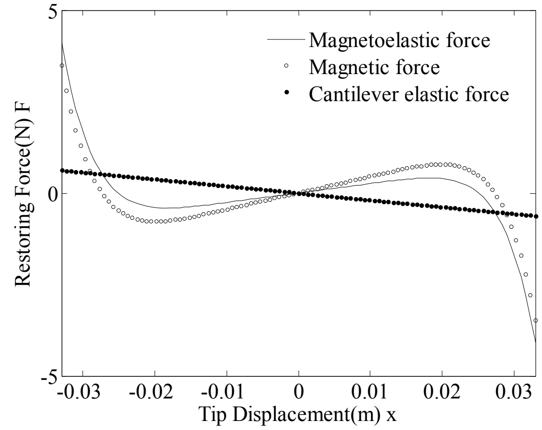

FIGURE 2. RESTORING FORCES VERSUS THE END MASS DISPLACEMENT

The magnetoelastic force function in Fig. 2 suggests that the mechanism can be governed by a Duffing equation with negative linear stiffness and a hardening cubic term

$$m\ddot{x}+c\dot{x}-kx+k_3x^3 = N(t) \tag{7}$$

With non-dimensional position coordinate $\delta = \sqrt{k_3/k}\,x$, non-dimensional time $\tau = t\sqrt{k/m}$, non-dimensional damping $\overline{c} = c/\sqrt{mk}$ and non-dimensional external excitation $Q(t) = \sqrt{k_3/k^3}\,N(t)$, the model in Eq. (7) can be re-stated as

$$\delta'' + \overline{c}\delta' - \delta + \delta^3 = Q(t) \tag{8}$$

where $(')$ indicates differentiation with respect to τ . The potential of the system can be defined by

$$U(\delta) = -\frac{1}{2}\delta^2 + \frac{1}{4}\delta^4 \tag{9}$$

Analysis of the Conditions for Stochastic Resonance

It is assumed that in addition to the system being excited by the ambient vibration force $Q(t)$, a small-scale excitation $\gamma(t)$ is introduced to change the shape of the system potential wells so that it is easier to overcome the potential well barrier and allow stochastic resonance. The dynamic model of the overall system is defined by

$$\delta'' + \overline{c}\delta' - \delta + \delta^3 - \gamma(t) = Q(t) \tag{10}$$

It can be seen that the potential of the system originally defined by Eq. (9) now includes a time dependent function

$$U(\delta,t) = -\frac{1}{2}\delta^2 + \frac{1}{4}\delta^4 - \gamma(t)\delta \tag{11}$$

The additional excitation $\gamma(t)$ can also be seen to express the asymmetry of the potential well. Considering firstly that a constant force γ_a is applied to the magnetic end mass, and writing the partial differential of the potential with respect to δ as

$$\frac{dU_a(\delta)}{d\delta} = -\delta + \delta^3 - \gamma_a = 0 \tag{12}$$

When Eq. (12) has only two different roots, the boundary value of the parameter γ_a can be given by

$$\gamma_a = \pm\frac{2\sqrt{3}}{9} \tag{13}$$

Figure 3 shows the potential of the system under the condition $\gamma_a = -2\sqrt{3}/9$. It means that the system retains the bistable state when the value of γ_a is less than $2\sqrt{3}/9$.

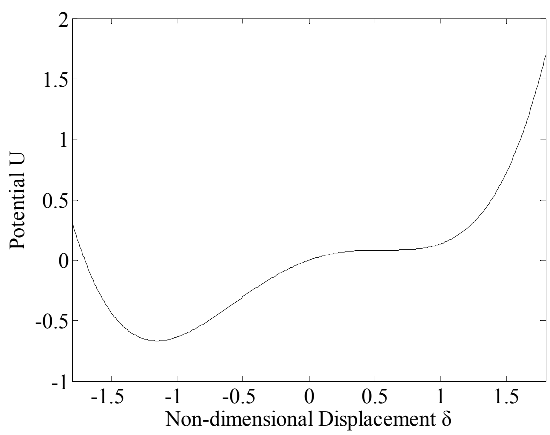

FIGURE 3. POTENTIAL VERSUS NON-DIMENSIONAL DISPLACEMENT WHEN $\gamma_a = -2\sqrt{3}/9$

In the case of the time-variant force it is assumed that the additional excitation $\gamma(t)$ can usefully be defined as follows

$$\gamma(t) = C\,\mathrm{sgn}(\delta') \tag{14}$$

where C is a positive constant value.

There are two kinds of physical meaning for $\gamma(t)$. The first and intuitive explanation is of a follower force which is applied directly to the mass of the bistable system so that it helps the mass jump between the symmetrical potential-wells. The other one is that $\gamma(t)$ can be used to modulate the shape of the potential wells, leading to asymmetry. Figure 4 illustrates the change of the potential wells under the additional force $\gamma(t)$. This means that the potential wells change between the two different asymmetric shapes. On considering that the mass starts to move from the point $P1$ then the arrows are in this direction and it is assumed that the value of $\gamma(t)$ equals 0.16 from $P1$ to $P2$, and then the mass reaches point $P2$, setting $\gamma(t)$ equal to 0.16, the mass jumps to the point $P3$, the potential of the mass

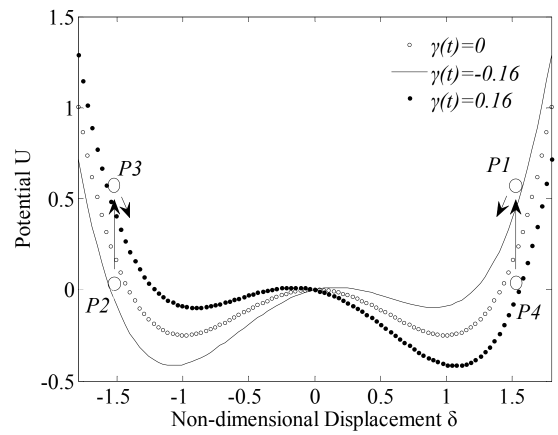

FIGURE 4. POTENTIAL VERSUS NON-DIMENSIONAL DISPLACEMENT OF $x(t)$

changing instantaneously while the displacement undergoes no change. Then, following the dotted line, it moves to point $P4$. Setting $\gamma(t)$ equals to -0.16, the mass finally returns to the start point $P1$. It is expected that the absolute value of the composition of the additional excitation and the noise will be less than $2\sqrt{3}/9$, otherwise the systems enters monostability. The additional excitation has to increase to overcome the potential barriers. In practice, mainly because of the mechanical damping loss, any excess in the input energy cannot be completely dissipated by the electrical damper, and it is necessary to subtract the energy provided by the additional excitation in order to calculate the net energy, so it will decrease the net energy that can be harvested from the external ambient vibration. However, it is impossible constantly to maintain the total below that threshold because the noise excitation varies randomly with time. Therefore, a suitable value of constant C should be chosen to give optimal conditions.

EHANCEMENT OF ENERGY HARVESTING BY STOCHASTIC RESONANCE

The energy dissipated by the damper can be controlled to maximise the capability of the proposed method to achieve stochastic resonance and to enhance the harvested energy. Equation (10) can be written as

$$\delta'\delta'' - \delta'\delta + \delta'\delta^3 = -\bar{c}\delta'^2 + \delta'\gamma(t) + \delta' Q(t) \qquad (15)$$

Then Eq. (15) can be rewritten as follows

$$\begin{aligned} &\frac{d}{d\tau}\left(\frac{1}{2}\delta'^2 - \frac{1}{2}\delta^2 + \frac{1}{4}\delta^4\right) + \bar{c}\delta'^2 \\ &= \delta'\gamma(t) + \delta' Q(t) \end{aligned} \qquad (16)$$

Equation (16) describes the conversion of energy in the bistable condition. The rate of change of the kinetic energy and the potential of the system equals the instantaneous external excitation and the additional excitation input [9]. In the left hand terms of Eq. (16), $\delta'^2/2$ is the kinetic energy of the mass and cantilever, $U(\delta) = -\delta^2/2 + \delta^4/4$ is the potential of the mechanism, and $\bar{c}\delta'^2$ represents the instantaneous energy dissipated by the damper. The energy input of the additional small amplitude excitation is expressed by $\delta'\gamma(t)$ which is always non-negative because $\gamma(t)$ is a force that follows the magnetic end mass. $\delta' Q(t)$ is the input energy from the external ambient vibration. In order to estimate the net harvested power of the mechanism under stochastic resonance, the power provided by the additional excitation $\gamma(t)$ should be subtracted from the total harvested power. Therefore the instantaneous net power can be defined by

$$P(t) = \bar{c}\delta'^2 - \gamma(t)\delta' \qquad (17)$$

The net energy during the non-dimensional time period τ_0 is given by

$$E = \int_0^{\tau_0} (\bar{c}\delta'^2 - \gamma(t)\delta')d\tau \qquad (18)$$

The parameter values of $\bar{c} = 0.24$ and $C = 0.19$ are chosen to compare the responses of the mechanism under conditions with and without the small-scale additional excitation. The external ambient force $N(t)$ is provided by Gaussian White Noise with a noise density of 0.054.

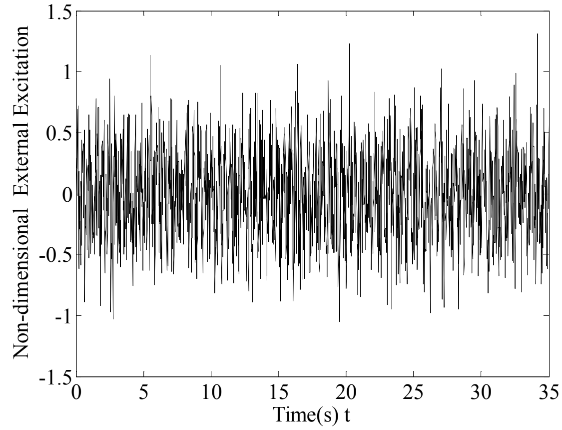

FIGURE 5. NON-DIMENSIONAL TOTAL EXCITATION VERSUS TIME

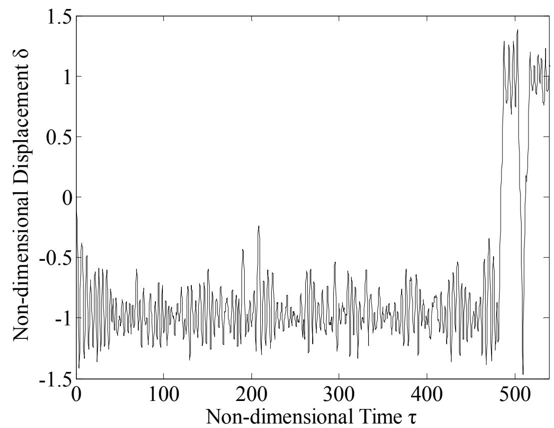

FIGURE 6. NON-DIMENSIONAL DISPLACEMENT OF $x(t)$ VERSUS NON-DIMENSIONAL TIME

Figure 5 shows the time domain of the composition of forces when both the external excitation $Q(t)$ and $\gamma(t)$ are applied to the mechanism. The corresponding non-dimensional displacement of the mass is shown in Fig. 7. Compared with the responses which are shown in Fig. 6, it can be seen that the mass fluctuates between the two potential wells in a state of stochastic resonance with large amplitude displacements. Therefore the energy harvested by the damper is enhanced greatly. Figures 8 and 9 show the harvested instantaneous net power under conditions with and without the additional excitation. The net integrated energy from both cases is shown in Fig. 10. Although the total energy consumed by providing an additional excitation will reduce the total energy harvested, it is found that under the forcing of $\gamma(t)$, the harvested net energy from the external ambient force is significantly increased compared with the energy generated from that of only ambient vibration.

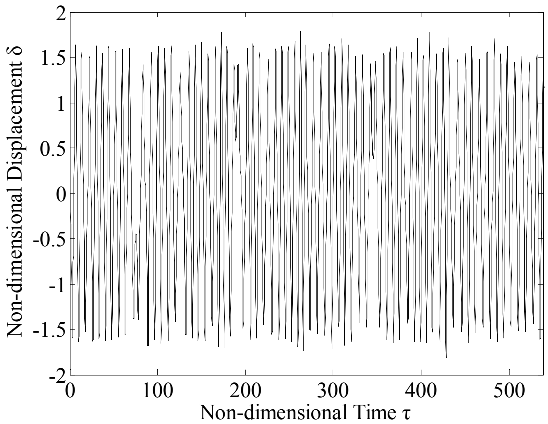

FIGURE 7. NON-DIMENSIONAL DISPLACEMENT OF $x(t)$ VERSUS NON-DIMENSIONAL TIME UNDER FORCING OF ADDITIONAL EXCITATION

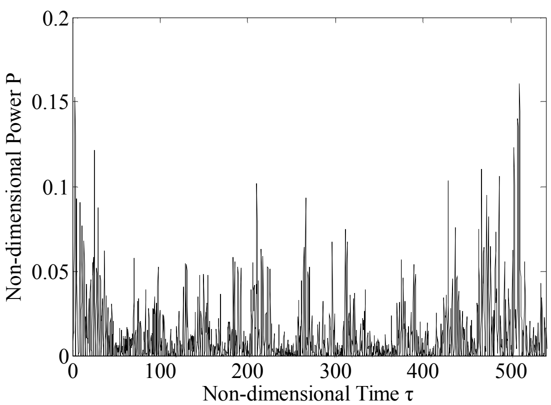

FIGURE 8. NON-DIMENSIONAL POWER VERSUS NON-DIMENSIONAL TIME

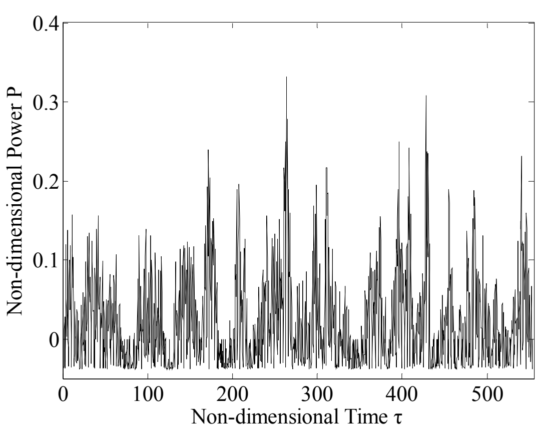

FIGURE 9. NON-DIMENSIONAL POWER VERSUS NON-DIMENSIONAL TIME UNDER FORCING OF SMALL-SCALE ADDITIONAL EXCITATION

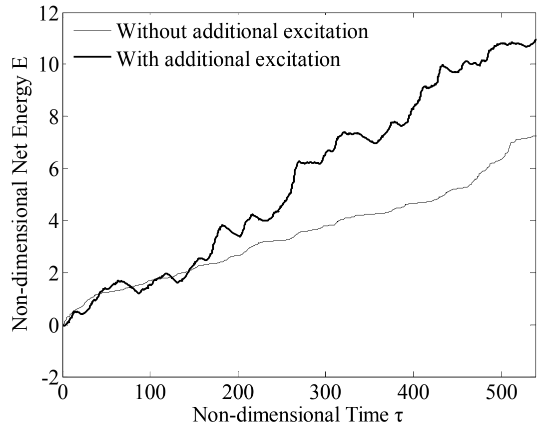

FIGURE 10. NON-DIMENSIONAL NET ENERGY VERSUS NON-DIMENSIONAL TIME

Unlike the approach for enhancing the available energy by adding a weak periodic excitation, this new method does not require an estimation of the frequency, however the specific amplitude of the parameter of $\gamma(t)$ must be determined.

CONCLUSIONS

A bistable nonlinear harvester system has been designed to investigate an enhanced condition for stochastic resonance. By means of theoretical analysis and simulation it has been shown that the method proposed here can enhance the availability of harvestable energy from the external ambient vibration. This is a different approach to the method of achieving stochastic resonance with an additional periodic force, and it is not necessary to estimate the forcing frequency according to the noise density using the Kramer's rate. It is therefore easier to apply. However, further experimental investigation should be carried out to provide support for the simulation results.

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