Industrial Trent Combustor—Combustion Noise Characteristics

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Thermoacoustic resonance is a difficult technical problem that is experienced by almost all lean-premixed combustors. The Industrial Trent combustor is a novel dry-low-emissions (DLE) combustor design, which incorporates three stages of lean premixed fuel injection in series. The three stages in series allow independent control of two stages—the third stage receives the balance of fuel to maintain the desired power level—at all power conditions. Thus, primary zone and secondary zone temperatures can be independently controlled. This paper examines how the flexibility offered by a 3-stage lean premixed combustion system permits the implementation of a successful combustion noise avoidance strategy at all power conditions and at all ambient conditions. This is because at a given engine condition (power level and day temperature) a characteristic ''noise map'' can be generated on the engine, independently of the engine running condition. The variable distribution of heat release along the length of the combustor provides an effective mechanism to control the amplitude of longitudinal resonance modes of the combustor. This approach has allowed the Industrial Trent combustion engineers to thoroughly ''map out'' all longitudinal combustor acoustic modes and design a fuel schedule that can navigate around regions of combustor thermoacoustic resonance. Noise mapping results are presented in detail, together with the development of noise prediction methods (frequency and amplitude) that have allowed the noise characteristics of the engine to be established over the entire operating envelope of the engine. [S0742-4795(00)00802-4]

1 Introduction

Recent years have seen the emergence of combustion noise resonance as a central technical problem in the design of Dry Low Emissions (DLE) combustors. Despite the variety of technical approaches used in the design of DLE combustors (annular versus can combustors, parallel versus series staging of fuel and/or air, various flame stabilization strategies, etc.) the problem has been experienced by almost all gas turbine manufacturers.

The practical combustion noise problem can be summarized by the following fundamental question: given a set of combustor operating conditions, will the combustor exhibit a resonant acoustic mode and if so, what will be the frequency and amplitude of the pressure oscillations?

Most theoretical attempts at answering the above question have focused on the identification of the unstable modes of the combustor, using linear stability analysis. The recent work of Hubbard et al. [1] is a good example. Typically, the conservation equations are linearized for small amplitudes, so that a dispersion relation that predicts the linear growth of the unstable modes can be obtained. The result of this type of analysis is a prediction of the frequencies of the unstable mode(s), but nothing is obtained in terms of the resulting limit-cycle amplitude, or the effect of the engine cycle on the onset of the instability. Furthermore, linear stability models provide limited predictive capability, since a key ingredient to the approach is some information about the dynamic response of the flame when subjected to acoustic perturbations. A so-called ''flame model'' is necessary and usually needs to be obtained experimentally [2].

Yet, to the development engineer, the maximum amplitude of the instability is a key parameter that needs to be quantified, together with a reliable prediction of the engine conditions at which the resonance will occur. Under certain conditions (if not most conditions), a linear stability analysis will predict more than one unstable mode in the combustor. In these circumstances, the result is of no immediate practical value since it will fail to identify which of the unstable modes will be selected by the combustion system.

Work on liquid rocket motor instability has long established that, at first order, the frequency of the resonance always corresponds to a natural acoustic mode of the combustor [3]. Thus, the possible frequencies of the unstable modes are easily predicted with reasonable accuracy. On the other hand, Chu [4] established the relationship between a fluctuation in the rate of heat release and the amplitude of the resulting pressure wave. Considering a control volume that encloses a region of heat release inside an infinite tube, and a perturbation in the rate of energy release inside the control volume, Chu [4] obtained the following result:

$$q'=\frac{\Delta w}{w}=\frac{1}{\gamma M_1}\frac{\left(1+\left(\frac{c_2}{c_1}\right)\right)}{\left(\frac{T_2}{T_1}-1\right)}\left[\frac{\left(1+\left(\frac{\Delta p_1}{p_1}\right)\right)\left(\frac{\Delta p_1}{p_1}\right)}{\sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{\Delta p_1}{p_1}\right)+1}}\right] \tag{1}$$

where q' is a fluctuation in the rate of heat release (percent), w is the rate of energy released per unit area, Δw is a *finite* perturbation in the rate of heat release, M_1 is the incoming Mach number, γ is the ratio of specific heats, Δp is the amplitude of the generated pressure wave and c, T, and p denote the sound speed, temperature, and pressure, respectively. The indices 1 and 2 refer to the unburned and burned states, respectively.

Clearly, Chu's result has no link whatsoever to any of the possible resonant frequencies associated to a tube (combustor) of finite dimensions. Interestingly enough (and contrary to what is sometimes observed in DLE combustors) the above relationship predicts that the pressure amplitude *decreases* as the temperature ratio is decreased. The above relationship, although rigorously correct, does not incorporate the feedback mechanism necessary for the prediction of an unstable condition.

The necessary condition for combustion instability is of course the well-known Rayleigh criterion, which says that the pressure waves need to be in phase with the heat release [5]. The distribu-

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tion of heat release along the length of the tube is therefore an important consideration on whether or not instability will occur.

One key difference between lean, premixed combustors and conventional combustors is the distribution of heat release within the combustor volume. In DLE combustors (using lean premixed technology), heat release occurs abruptly across a flame front whereas in traditional combustors the heat release is *smeared* across a much wider region. The abrupt heat release at the flame front in a premix system allows for the occurrence of a narrow range of time delays between fuel injection and heat release. This makes it possible for a large fraction of the heat release to satisfy Rayleigh's criterion. In a diffusion flame type combustor, there is a large collection of time delays between heat release and fuel injection and only a small fraction of the heat release can satisfy Rayleigh's criterion.

It is interesting to note that thermoacoustic oscillations are routinely observed in cryogenic systems [6] where they are the result of large temperature gradients along the length of tubes of finite length. Theoretical analysis of this phenomenon shows that the thermoacoustic instability in this case is strongly affected by the distribution of temperature along the length of the tube.

In this paper, it will be shown that the frequency and amplitude of the resonant axial modes in a DLE combustor are strongly affected by the axial distribution of heat release and temperature inside the combustor. The Industrial Trent combustor is a 3-stage lean premixed combustor. This design has allowed us to identify the frequencies and amplitudes of all the resonant modes of the combustor, independently of engine running conditions.

After a brief introduction to the 3 axial stage concept, noise-mapping results are presented. Correlations for each of the unstable modes are then presented, followed by a discussion of the predictive capabilities of the approach.

2 Axial Staging of Heat Release

A cross-section of the Industrial Trent combustor is shown in Fig. 1. The combustor consists of three premixing channels, which are respectively referred to as the primary, secondary, and tertiary premixers. The primary premix system is the only premix system that is self-stabilized. That is, the primary system can be operated alone, whereas the secondary and tertiary systems cannot. The secondary and tertiary premixed streams are ignited by the upstream stages, as they mix inside the combustor. If the primary stage flames out, the whole combustor flames out.

Flame temperatures associated to each of the premix stages can be calculated on the basis of ''cold flow'' effective areas (which provide combustor air splits) and measured fuel flows to each of the fuel stages.

The practical implication of the design is that the secondary and tertiary stages can be operated at much lower flame temperatures than what is normally required for flame stabilization. The result is a large turndown ratio in the achievable fuel-air ratios of the secondary and tertiary premixers.

Part of the secondary fuel-air mixture is entrained into the primary zone, to mix with the primary premix stream. Once the flame temperature associated with the secondary premix stream reaches a certain level, this entrainment effect will result in an improvement in the weak extinction limit of the primary system. This is because it is the average temperature resulting from the mixing of the two streams that governs the weak extinction of the primary zone. The improvement in primary zone stability resulting from secondary stream entrainment is shown in Fig. 2. The amount of secondary fuel-air mixture entrained into the primary zone of the combustor can be estimated from the slope of the line above 1400 K in Fig. 2.

At a given power level, the total amount of fuel inside the combustor is prescribed. The combustion engineer is left with the choice of the allocation of the total fuel between the three premix stages. The possible fuel splits are limited by a series of constraints: the primary weak extinction temperature, the maximum temperature for any of the three stages due to NOx reasons, and a minimum temperature of the last stage (tertiary) due to CO requirements. The secondary stage does not really have a minimum temperature, provided the tertiary stage is hot enough to accomplish CO burnout. There are no practical limits imposed by combustor metal temperatures or by limitations on the combustor temperature exit profile.

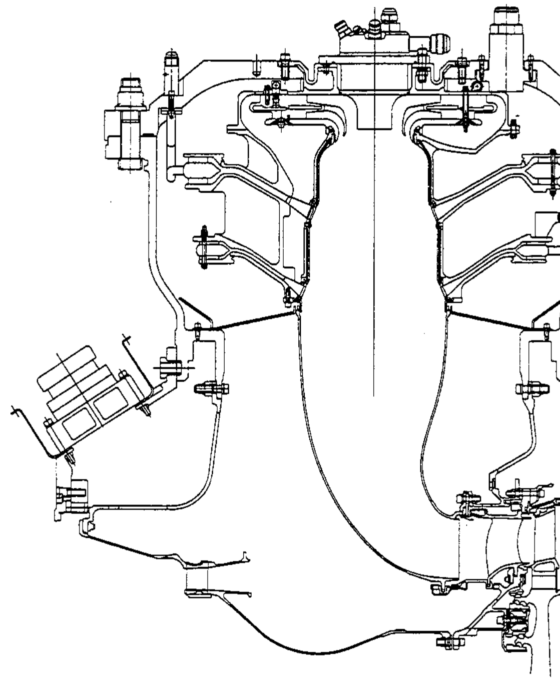

Fig. 1 Industrial Trent combustor

At a given power level, the ensemble of possible ways to allocate the fuel inside the combustor actually defines an operating envelope, whose axes are best defined in terms of premixer (or zone) temperatures. For instance, once a primary temperature is chosen (say, 1750 K), a secondary temperature (say, from 1200 K to 2000 K) can then be chosen. The amount of fuel to be allocated to the tertiary then falls out from the total amount of fuel required by the engine. Note that no fuel at all to the tertiary is also an option.

Figure 3 shows typical possible operating envelopes for the Trent combustor at different power levels. The temperatures of the *y*-axis and *x*-axis are the primary and secondary flame temperatures, from which a reference temperature was subtracted. Hence, the *bias* of the primary and secondary temperatures from a reference temperature is used to represent the results.

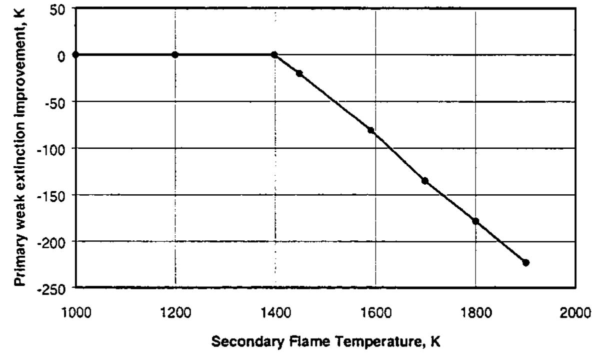

Fig. 2 Primary weak extinction characteristics

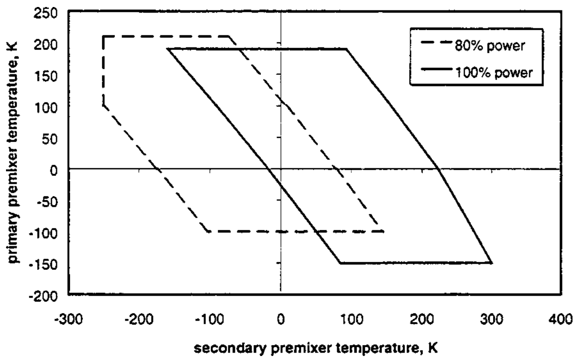

Fig. 3 Typical operating envelopes for the industrial Trent combustor (ISO day)

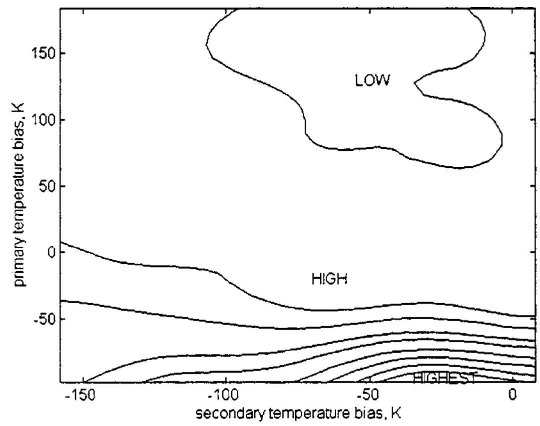

Fig. 5 Noise amplitude contour map near 80 percent power

3 "Noise Mapping"

In order to assess the noise and emissions characteristics of the engine, a detailed mapping exercise was undertaken at a variety of power levels and ambient conditions, and on different engines.

Depending on the ambient conditions, the relationship between combustor inlet and outlet temperatures can vary quite significantly. Thus, at different engine operating conditions, the allocation of fuel between primary, secondary, and tertiary that gives the best combination of NOx, CO, and noise will vary. The objective of the development testing of the combustion system was the definition of a combustor fuel schedule. The *fuel schedule* is essentially a look-up table, which is a function of power level and ambient temperature that the control system can use to split the total fuel between the three stages.

Shown in Figs. 4, 5, and 6 are the results of measurements of the observed noise amplitude at various power levels. Again, at a given power level, the primary and secondary temperatures can be controlled independently, while the tertiary takes the balance of the total fuel.

Each of the noise maps shown below is the result of an interpolation of 25 to 35 measurement points. More than 10 noise maps were obtained (different power level, different day condition, different engines) during development. Thus, more than 300 test cases were obtained. In all cases it was found that the resonant frequency of pressure fluctuations always corresponded to a natural acoustic mode of the combustor.

The amplitudes measured in the noise maps are the RMS value of the signal from piezoelectric transducers, filtered from 10 Hz to 2000 Hz. The detailed frequencies corresponding to the regions high noise will be discussed below.

What is seen from the contour maps is that at any power condition it is always possible to find regions of high noise and low noise. In other words, the RMS level of pressure fluctuations is clearly affected by the axial distribution of heat release inside the combustor (this is assuming that changing the fuel splits will change the distribution of heat release). In general, the noise amplitude appears to scale linearly with the combustor inlet pressure. Thus, the potential for structural damage is a lot higher at high engine pressure ratios. This represents a difficult challenge for the Trent, since the inlet pressure can reach a level of up to 40 atm.

Depending on the power level and on the ambient conditions, the regions of high noise will be located in a different region of the operating envelope. In Fig. 6, there are two regions of high noise; both located in the region of low primary temperatures. However, as the secondary temperature is varied, it becomes possible to find an optimum condition that will minimize the noise amplitude.

It turns out that the two distinct regions of high noise in Fig. 6 correspond to two different acoustic modes of the combustor. For the specific geometry of the Industrial Trent (i.e., L/D is relatively large) the modes that can be excited in the range of 10–2000 Hz are only longitudinal modes. The noise mapping results presented above can be somewhat condensed when plotted in terms of a non-dimensional amplitude ($\Delta p/P$) and a non-dimensional wavelength (L/λ). That is, the single sided RMS noise amplitude Δp is non-dimensionalized by the combustor inlet pressure P and the sound wavelength λ is non-dimensionalized by the length of the combustor L. This is shown in Fig. 7.

The data from Fig. 7 are obtained from two different engines, at five different power levels, and from a range of ambient condi-

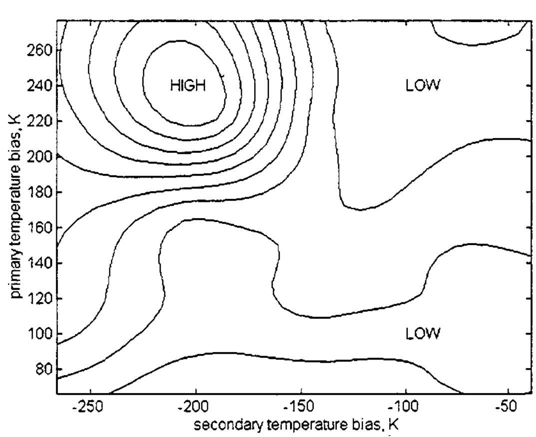

Fig. 4 Noise amplitude contour map near 50 percent power

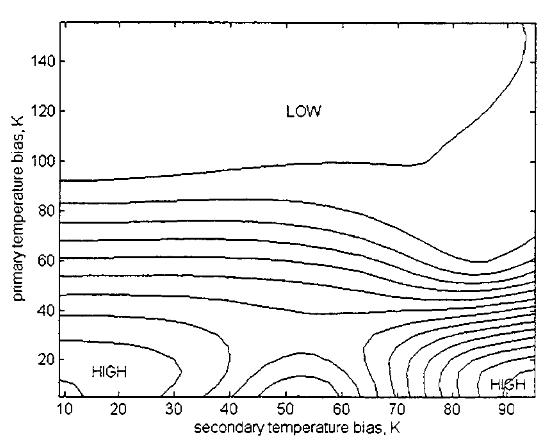

Fig. 6 Noise amplitude contour map near 100 percent power

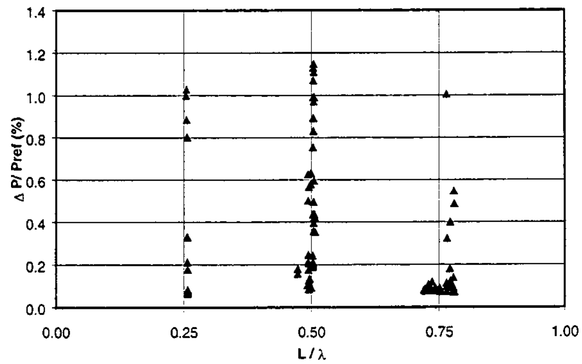

Fig. 7 Non-dimensional representation of the measured combustion noise amplitudes and frequencies

tions and combustor fuel splits. Yet, all results are clearly grouped into three well-defined acoustic modes. At a given condition, there may not be any resonant mode, although there will always be a dominant frequency. In such a case, the level of pressure fluctuations is of the order of 0.1–0.2 percent of the reference pressure. This represents the ''no noise'' cases. When resonance sets in, it will always select the frequency of one of the natural acoustic modes. Depending on the operating conditions, the fuel split at which resonance appears and the frequency and amplitude of the selected mode will vary.

Each of the resonant modes identified in Fig. 7 appears to correspond to a specific pattern of heat release and temperature distribution along the length of the combustor. This is illustrated in Fig. 8. Because the Industrial Trent incorporates three fuel stages in series, the choice of fuel splits will in principle affect the axial distribution in heat release and gas temperature along the length of the combustor. These distributions were not experimentally mea-

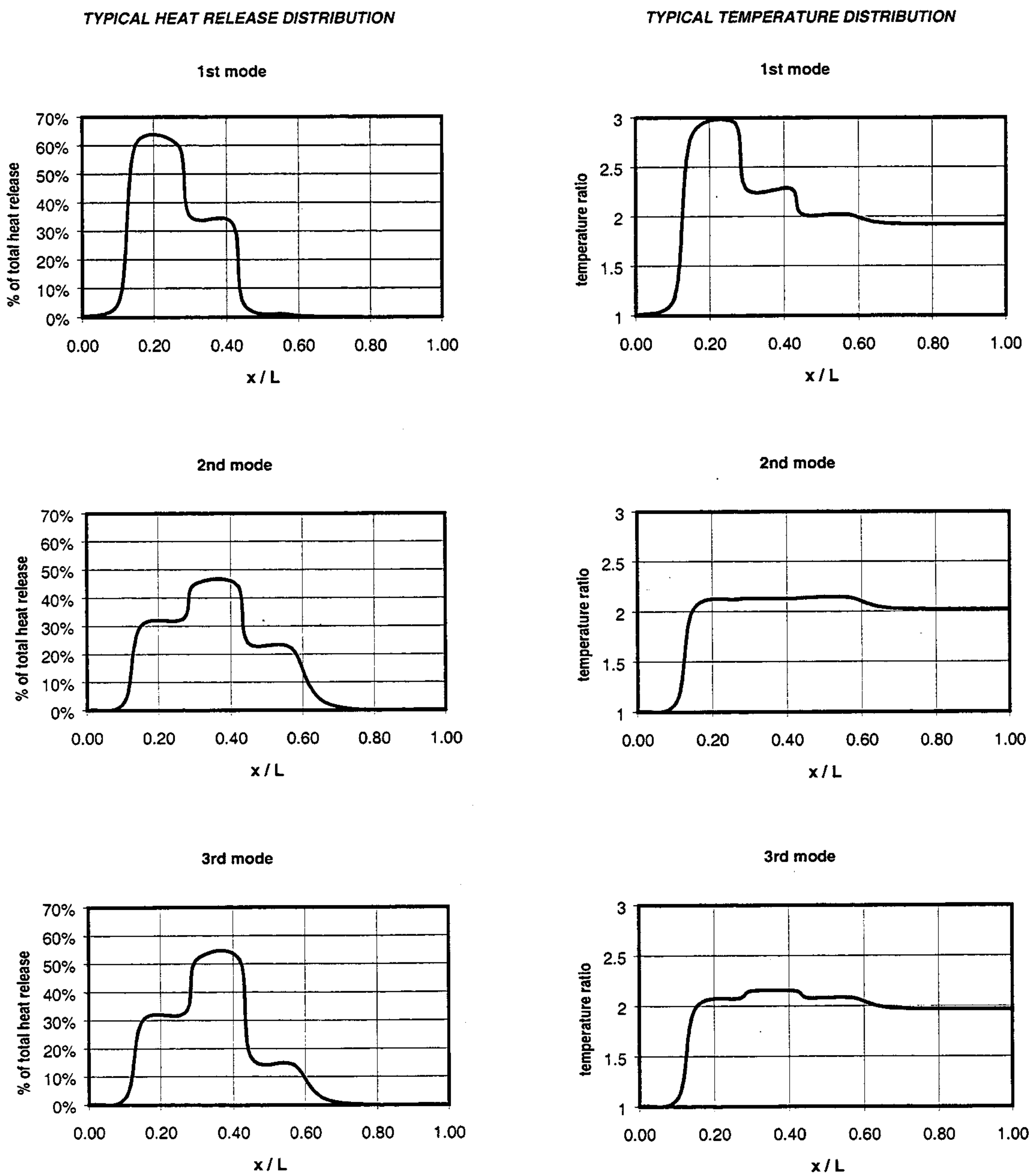

Fig. 8 Typical (calculated) heat release and temperature distribution along the length of the combustor for each of the unstable longitudinal acoustic modes. $x/L=1$ corresponds to the combustor exit.

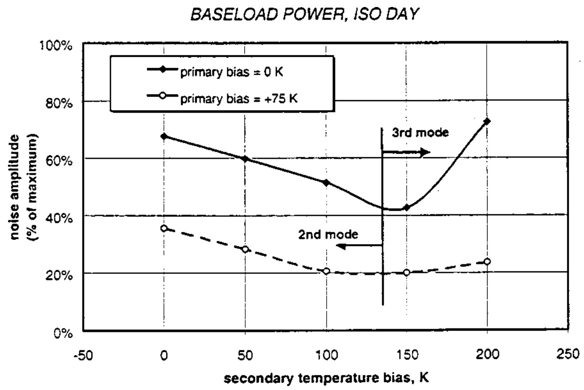

Fig. 9 Change in acoustic mode and noise amplitude as the combustor fuel split is varied at a steady engine operating condition

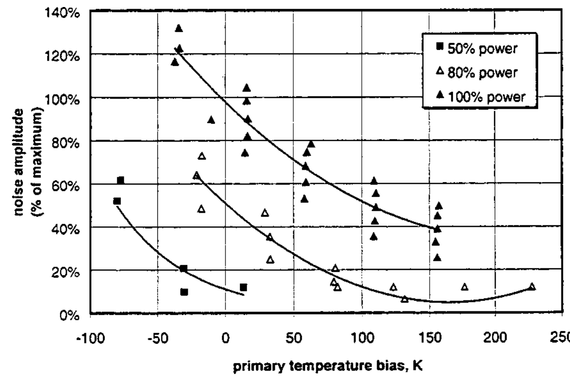

Fig. 10 Amplitude characteristics of the 2nd unstable mode ($L/\lambda=0.5$). The data in this figure are obtained at engine power levels ranging from 50 percent to 100 percent power.

sured but were instead calculated, with the assumption that the combustor air splits are not changed as the fuel split is varied.

The first mode is usually observed at low power when the tertiary is not lit and hence it corresponds to a heat release distribution concentrated near the ''head'' of the combustor. For the second and third mode, all three stages are in operation and the energy release is distributed along a wider region of the combustor. The heat release distribution of the second and third mode are of a similar ''topology,'' i.e., most of the energy is released within the secondary zone. However, the resulting temperature distributions are different, as clearly seen from Fig. 8.

What is the relationship between the information shown in Fig. 7 and the noise maps of Figs. 4–6? A partial answer is presented in Fig. 9. This is effectively the equivalent of a slice through the noise map of Fig. 6. As the secondary bias is increased, the noise amplitude decreases until it reaches a minimum. Once the minimum is reached, the frequency of the resonance switches from the second to the third acoustic mode. It can also be seen that the overall noise levels can be reduced if the primary bias is increased.

Figure 9 is a good illustration of the flexibility offered by the 3-stage design. At a fixed engine running condition, a small change in the primary and/or secondary temperature can reduce the combustion noise amplitude by 50 percent and/or select the frequency at which resonance takes place.

The ability to ''select'' the frequency of the resonance is a unique feature of the system, which offers a significant advantage to reduce the vulnerability of the combustor against combustion noise. The combustor hardware will usually be more susceptible to certain frequencies, depending on the structural modes of the mechanical design. It is usually possible to ''design out'' some of the natural structural modes, but not all. Being able to also select the frequency of the resonance offers additional margins for mechanical integrity.

4 Correlations and Predictions of Noise

The objective of the noise mapping test was to allow the definition of a fuel schedule for the engine. In other words, what is the best fuel split between primary, secondary, and tertiary for a given power level (on a given ambient condition) which will avoid combustion noise?

To answer this question it was necessary to develop an empirical correlation for each of the possible resonant modes of the combustor. This way, not only the amplitude, but also the frequency of the combustion resonance could be predicted. The assumption that was made was that of *a principle of superposition*. That is, the unstable mode selected by the combustor is simply the one that is predicted to have the highest amplitude.

The behavior of each of the unstable modes was quite different. For instance, the dependence of the amplitude of the second longitudinal mode ($L/\lambda=0.5$) on combustor fueling conditions and engine power is shown in Fig. 10. The noise amplitude (in psi-RMS) is non-dimensionalized by the maximum noise amplitude acceptable to the requirements of durability of the hardware.

The amplitude of the second mode always decreases as the primary bias is increased. What appears to be large scatter at a given primary bias is actually the effect of the secondary bias being changed. Even though the noise amplitude is non-dimensionalized by the combustor reference pressure, all the curves do not collapse into one. Thus, the 2nd mode depends to a certain degree on the total energy being released inside the combustor.

The 3rd unstable mode ($L/\lambda=0.75$) has amplitude characteristics which are completely different from what was observed with the 2nd mode. As seen from Fig. 11, whenever the dominant frequency of the spectrum was that of the 3rd mode, it was possible to collapse all of the observations onto a single curve. It appears that a necessary condition for the appearance of the 3rd mode is that the secondary zone temperature must exceed the primary zone temperature. This condition can be achieved in a number of ways, and at different power levels.

Under most combustor operating conditions, the primary zone temperature is slightly above the secondary zone temperature. This is because the primary zone needs to keep a small margin above weak extinction, whereas the secondary doesn't. However, at high power the primary does have a large margin above weak extinction (see Fig. 2) so it becomes possible to have a condition where the secondary zone temperature exceeds the primary zone

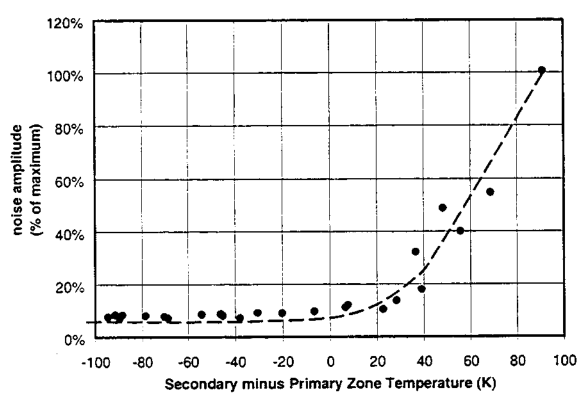

Fig. 11 Amplitude characteristics of the 3rd unstable mode ($L/\lambda=0.75$). The data in this figure are obtained at engine power levels ranging from 30 percent to 100 percent power.

temperature. Although this is a necessary condition for the existence of the 3rd mode, it is not a sufficient one. Note that the data points shown in Fig. 11 all correspond to engine conditions where the dominant frequency in the frequency spectrum corresponded to that of the 3rd mode.

Depending on the power level and day temperature, an unstable mode may or may not manifest itself. It is quite possible (depending on combustor operating conditions) to have situations where neither the 2nd nor the 3rd mode (nor the 1st mode) are present. In these cases, the background noise level in the combustor is comparable to what would be measured in say, a diffusion flame combustor.

Having obtained some indications of the behavior of the unstable modes, an attempt was made to predict the amplitude of each of the unstable modes using empirical correlations. As mentioned in the Introduction, the work by Chu [4], although not linked to any mechanism of resonance, is ''exact'' in terms of establishing a nonlinear relationship between heat release fluctuations and the amplitude of the resulting pressure waves. This is because the conservation equations for a control volume enclosing a region of heat release are solved analytically.

If one makes the assumption that a given level of pressure fluctuations have a one-to-one correspondence with a level in heat release fluctuations, then it is expected from Eq. (1) that the nondimensional amplitude of the noise is predominantly a function of the temperature ratio across the region of heat release. In the case of a three-stage (axially staged) combustor there are three regions of heat release, and hence there are at least three important temperature ratios that will affect noise amplitude. It might also be argued that the overall heat release in the combustor (and the corresponding temperature ratio from combustor inlet to combustor exit) might also be a controlling parameter.

For these reasons, the assumed functional form of the empirical correlations that were used for each of the unstable modes was

$$\frac{\Delta p}{p}=K_1+K_2P_3^A\sum_{i=1}^{N}\left[\frac{(T_i-T_3)}{k_i}\right]^{\alpha_i}, \tag{2}$$

where K_1, K_2, A, k_i, and α_i are all arbitrary constants. The summation index i, which runs from 1 to N, refer to each of the three combustion zones of the combustor, in addition to the overall heat release inside the combustor. The arbitrary constants were obtained from linear regression of engine data for each of the unstable modes.

What were obtained then are three independent empirical correlations for each of the longitudinal resonant modes. The assumption that was then made was that the mode observed would be the one having the highest amplitude of the three, given the combustor operating conditions. Note that this approach permits prediction of not only the amplitude but also the frequency of the combustion resonance.

The capabilities of the correlations were tested on development engines, where fuel schedule changes were made so as to deliberately create noise during engine acceleration to baseload. Two sets of results from two different engines at two different ambient conditions (+20°C and −5°C) are shown in Fig. 12.

When a combustion resonance sets in, it sets in abruptly. This is manifested by a sharp ''kink'' in the curves in Fig. 12. Note that the empirical correlations capture this feature quite well. In other words, the empirical correlations developed are able to predict relatively well the boundaries beyond which combustion noise will be encountered.

The occurrence of combustion noise is dependent on the ambient conditions. This is because a given power level on the engine is achieved quite differently depending on the day conditions. The combustor inlet pressure and temperature and combustor exit temperature will be quite different at say, 40 MW, if the ambient temperature is at 20°C or −5°C. Furthermore, it should be kept in mind that the *fuel schedule* will also be different, depending on power level and day conditions.

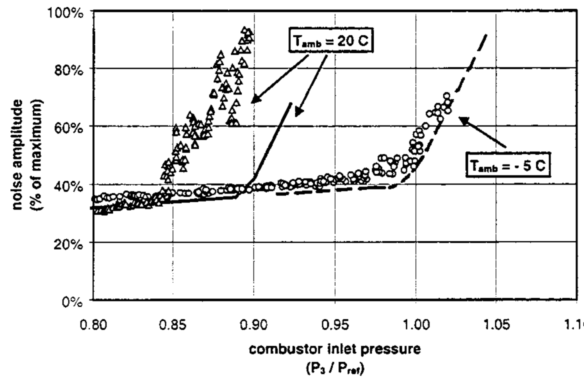

Fig. 12 Comparison between predictions and engine results during slow engine acceleration to baseload, at two different ambient conditions. The lines represent the prediction, whereas the symbols represent the engine data.

Nonetheless, Fig. 12 indicates that the correlations are able to capture relatively well the effects of engine cycle, ambient conditions and changes in fuel schedule (or equivalently, variations in combustor fuel splits). The correlations were able to reproduce the noise mapping results (e.g., Figs. 4, 5, and 6) within approximately 10 percent accuracy (i.e. the standard error of the estimate was 10 percent of the maximum allowable pressure amplitude inside the combustor).

One of the most difficult tasks in DLE combustor design, particularly for the entry-in-service of the engine, is to be able to predict the conditions under which the combustor will experience thermoacoustic resonance. The empirical correlations developed during development of the Industrial Trent allowed such predictions to be made so that the fuel schedule could be designed to avoid regions of high combustion noise across the entire operating range of the engine.

5 Conclusions

Axial staging of the heat release inside the Industrial Trent combustor allows a wide turndown of flame temperatures for the purpose of emissions control, but this also permits a direct influence on the amplitudes and frequency of combustion thermoacoustic resonance. At a fixed engine condition, the variation in fuel splits among the three stages effectively allows a direct control of combustion noise. The large flexibility of the three axial stages is such that the noise avoidance strategy for the Industrial Trent is really one of an *adaptive fuel schedule* (i.e., variation of combustor fuel splits in response to measured noise amplitudes) rather than one of *active noise control* (e.g., fast modulation of a small amount of the combustor fuel flow to suppress the instability).

The ''noise mapping'' capability of the three axial stages made it possible to obtain reliable empirical correlations for the occurrence of the unstable acoustic modes of the combustor. Today, it is possible to predict how a fuel schedule change will affect combustion noise on the engine from −30°C to +30°C. Both the frequency and the limit-cycle amplitude of the resonance can be predicted with reasonable accuracy.

6 Acknowledgments

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