Identification of Modal Parameters of an Elastic Rotor With Oil Film Bearings

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Investigations of the dynamic behavior of structures have become increasingly important in the design process of mechanical systems. To have a better understanding of the dynamic behavior of a structure, the knowledge of the modal parameters is very important. The powerful method of experimental modal analysis has been used to measure modal parameters in many mechanical engineering problems. But the method was mainly applied to nonrotating structures. This presentation shows improvements of the classical modal analysis for a successful application in rotating machinery with nonconservative effects. An example is given, investigating the modal parameters of an elastic rotor with oil film bearings.

Introduction

The dynamic behavior of many rotating machines e.g., turbines, compressors, pumps is influenced by stiffness and damping characteristics of nonconservative effects like oil film forces, forces in seals etc. It is important to know that besides the forced unbalance vibrations also unstable vibrations may occur, caused by the abovementioned self-exciting mechanisms [1]. To have a better understanding of the vibration behavior of rotating systems, the knowledge of the modal parameters—eigenvalues and eigenvectors—is very valuable.

Calculation procedures exist, for example, the finite element method, to determine the modal parameters in design stage [2]. But the input data for the calculation are not exactly known in any case and the predicted eigenvalues and eigenvectors may be different from those of the real machine in operation. Therefore mechanical engineers also try to find the modal parameters of built rotating machines or test rig rotors by measurements [3].

In the past years the powerful method of experimental modal analysis has been used to measure modal parameters in many engineering problems [4]. The method was mainly applied to nonrotating structures without destabilizing effects. A successful application of the method in rotating machines requires some improvements of the classical experimental modal analysis. In [5] such improvements were described and a very simple rigid rotor in oil film bearings was investigated. In this paper a more complicated application is given, dealing with an elastic test rig rotor in oil film bearings. The essential eigenvalues and the corresponding eigenvectors (natural modes) could be found by experimental modal analysis.

Contributed by the Technical Committee on Vibration and Sound and presented at the Design and Production Engineering Technical Conference, Dearborn, Mich., September 11–14, 1983 of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, June 6, 1983. Paper No. 83-DET-11.

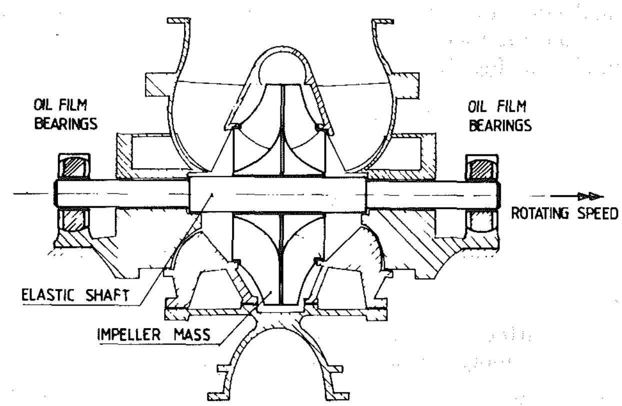

Fig. 1 Turbopump rotor

Natural Vibrations of an Elastic Rotor in Journal Bearings

Mechanical Model. Figure 1 shows a typical rotating machine, a turbopump rotor, consisting of an elastic shaft with an impeller mass in the middle of the shaft. The rotor is running in oil film journal bearings. To measure modal parameters of such a rotating shaft is not easy, because there are only a few points along the rotor to excite the rotor and to measure the system response. A systematic investigation of the vibration behavior of the machine during operation with all effects (oil film forces, hydraulic forces) is difficult to realize. In a first step we consider a simpler rotor system similar to that of the turbopump rotor. Figure 2 shows this test rig rotor with an elastic shaft, a disk, and two oil film bearings. The modal parameters of this elastic system can be measured in a systematic manner. All of the rotor locations can be excited and the displacements can be picked up at all locations. However, it has to be noted that not all of the effects of the real machine can be investigated with the test rig rotor.

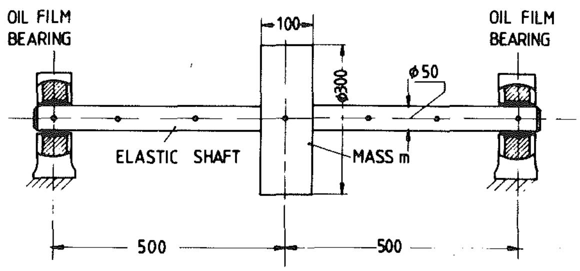

Fig. 2 Test rig rotor

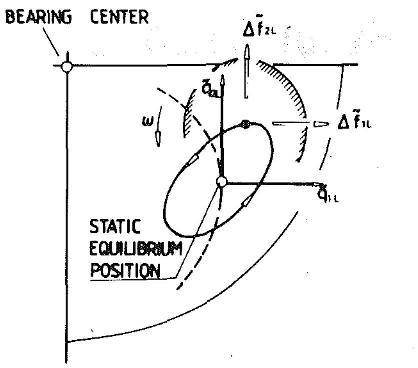

Fig. 3 Vibrations of the journal

From the theoretical considerations it is known, that for small vibrations of the journal around a static equilibrium position, the following force motion relationship for the oil film is true (Fig. 3)

$$\begin{bmatrix} \Delta\tilde{f}_{1L} \\ \Delta\tilde{f}_{2L} \end{bmatrix} = -\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \dot{\tilde{q}}_{1L} \\ \dot{\tilde{q}}_{2L} \end{bmatrix} - \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \tilde{q}_{1L} \\ \tilde{q}_{2L} \end{bmatrix} \tag{1}$$

with

k_{ik} stiffness coefficients of the bearings
b_{ik} damping coefficients of the bearings

The stiffness and damping coefficients depend on the Sommerfeld number, and on the rotational speed, respectively. Besides anisotropy, the stiffness cross coupling terms are unequal in general. This unsymmetry is the reason for self-excited bending vibrations of the shaft.

Equations of Motion. The equations of motion for the simple shaft (Fig. 2) express the equilibrium of inertia, damping, stiffness, and external forces

$$\mathbf{M}\,\ddot{\tilde{\mathbf{q}}}+\mathbf{D}\,\dot{\tilde{\mathbf{q}}}+\mathbf{K}\,\tilde{\mathbf{q}}=\tilde{\mathbf{f}}(t) \tag{2}$$

$\mathbf{M}$ mass matrix
$\mathbf{D}$ damping matrix
$\mathbf{K}$ stiffness matrix
$\tilde{\mathbf{q}}$ vector of displacements
$\tilde{\mathbf{f}}$ vector of external forces

The matrices **D** and **K** contain stiffness and damping terms from the bearings. They are nonsymmetric and depend on the running speed of the rotor.

Natural Vibrations, Eigenvalues, and Natural Modes. From the homogeneous equations of motion ($\tilde{\mathbf{f}}$ = 0) the natural vibrations can be calculated. Assuming a solution of the form

$$\tilde{\mathbf{q}}(t)=\varphi\,e^{\lambda t} \tag{3}$$

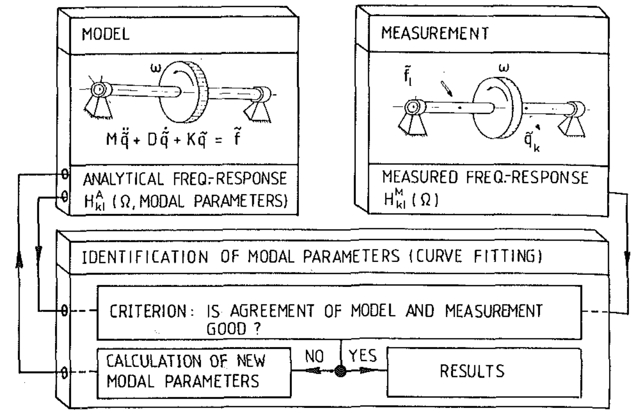

Fig. 4 Identification of modal parameters

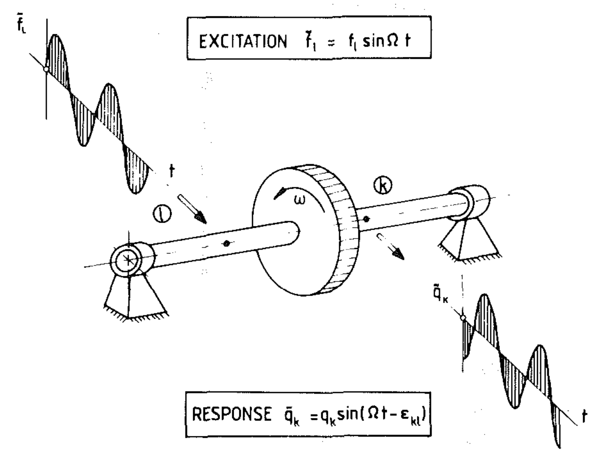

Fig. 5 Input and output signals at the rotor

we obtain a quadratic eigenvalue problem

$$\lambda^2\mathbf{M}+\lambda\,\mathbf{D}+\mathbf{K})\varphi=\mathbf{0} \tag{4}$$

with 2N eigenvalues λ_j and corresponding eigenvectors φ_j. Eigenvalues as well as eigenvectors in the most cases occur in conjugate complex pairs

$$\begin{aligned} &\text{Eigenvalues:} \quad \lambda_j=\alpha_j+i\omega_j; \quad \bar{\lambda}_j=\alpha_j-i\omega_j \\ &\text{Eigenvectors:} \quad \varphi_j=\mathbf{s}_j+i\mathbf{t}_j; \quad \bar{\varphi}_j=\mathbf{s}_j-i\mathbf{t}_j \end{aligned} \tag{5}$$

We consider only the part of the solution, which belongs to a conjugate complex pair

$$\tilde{\mathbf{q}}_j(t)=B_j e^{\alpha_j t}\{\mathbf{s}_j\sin(\omega_j t+\gamma_j)+\mathbf{t}_j\cos(\omega_j t+\gamma_j)\} \tag{6}$$

ω_j is the circular natural frequency of this part and α_j the damping constant. If the damping constant $\alpha_j > 0$ the natural vibrations increase, for $\alpha_j < 0$ the natural vibrations decrease.

We define the expression in parantheses { } of equation (6) as natural mode. Opposite to conservative systems there is no constant modal shape, proportions and relative phasing in general vary from point to point at the shaft. The natural modes of nonconservative systems represent time dependent curves in space. The plane motion of one point of the shaft is an elliptical orbit.

If we transpose the matrices **M, D, K,** we obtain the so called left-hand eigenvalue problem

$$(\lambda^2\mathbf{M}^T+\lambda\,\mathbf{D}^T+\mathbf{K}^T)\,\psi=\mathbf{0} \tag{7}$$

which has the same eigenvalues λ but different eigenvectors ψ. Both eigenvector sets are needed for an expansion of the frequency response functions in terms of the modal parameters.

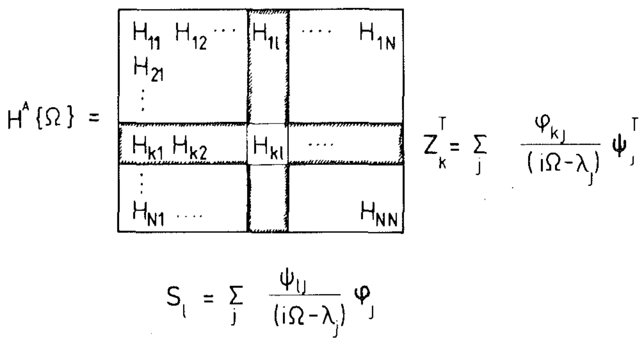

Fig. 6 Matrix of frequency response functions

Identification of Modal Parameters of Rotors

It is well known that experimental modal analysis has been often used in many mechanical engineering problems to identify modal parameters of nonrotating elastic systems. Application of the method in rotating machines with non-conservative effects requires consideration of some important differences, for example

- the nonsymmetry of the system matrices **K, D**
- the dependence of the modal parameters from the operating speed
- the necessity to excite the rotor and to measure the response during rotation

In consideration of these differences the modal analysis procedure is also available for rotating structures (Fig. 4). The procedure consists of the following steps:

Between a number of measurement points frequency response functions $H_{kl}(\Omega)$ are measured at the real structure, by exciting the system at locations l and measuring the response at locations k.

Analytical frequency response functions can be calculated in dependence of the eigenvalues, left-hand and right-hand eigenvectors.

Finally the analytical functions are fitted to the measured functions by variation of the modal parameters, resulting in a set of identified modal parameters.

Analytical Frequency Response Functions. If the rotor is excited in a certain point l by means of a harmonic force function $\tilde{f}_l$ with exciter freuqency Ω and the displacement $\tilde{q}_k$ is measured in another point k, the response behavior of the rotor can be characterized by the complex frequency response function (Fig. 5)

$$H_{kl}(\Omega) = \frac{q_k e^{i(\Omega t - \epsilon_{kl})}}{f_l e^{i\Omega t}} = \frac{q_k}{f_l} e^{-i\epsilon_{kl}} \tag{8}$$

respectively by the ratio of the amplitudes q_k/f_l and the phase ϵ_{kl} between the two signals. Both are frequency dependent functions.

In [5] it is shown that the complex frequency response functions can be expressed in terms of the eigenvalues λ_j and the corresponding eigenvectors φ_j, ψ_j.

$$H_{kl}(\Omega) = \frac{\tilde{q}_k}{\tilde{f}_l} = \frac{q_k}{f_l} e^{-i\epsilon_{kl}} = \sum_{j=1}^{2N} \frac{\varphi_{kj}\psi_{lj}}{i\Omega - \lambda_j} \tag{9}$$

For a rotor with N degrees of freedom $N \times N$ frequency response functions exist, assembled in the matrix $\mathbf{H}^A$ (Fig. 6). It is important to note that each row $\mathbf{Z}_k$ contains all of the left eigenvectors ψ_j and each column $\mathbf{S}_l$ contains all of the right eigenvectors φ_j.

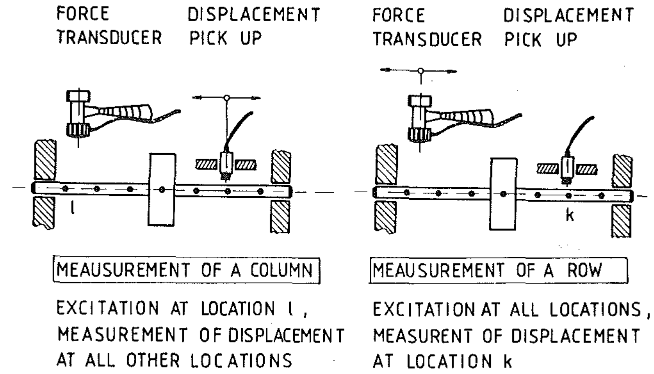

Fig. 7 Excitation and response locations

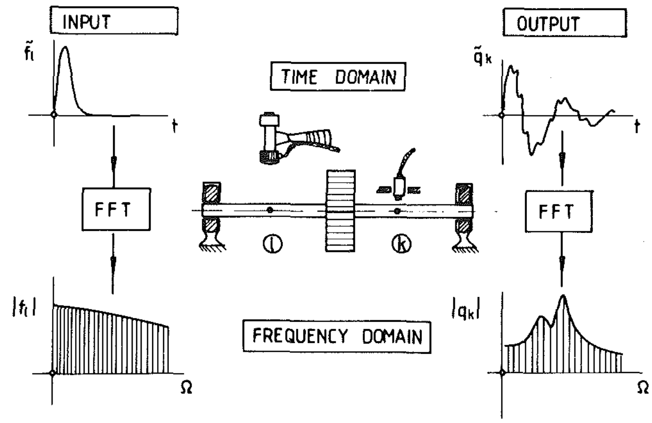

Fig. 8 Measurement of frequency response functions

$$\mathbf{Z}_k{}^T = \frac{\varphi_{k1}}{i\Omega - \lambda_1}\psi_1{}^T + \frac{\varphi_{k2}}{i\Omega - \lambda_2}\psi_2{}^T + \ldots \tag{10}$$

$$\mathbf{S}_l = \frac{\psi_{l1}}{i\Omega - \lambda_1}\varphi_1 + \frac{\psi_{l2}}{i\Omega - \lambda_2}\varphi_2 + \ldots \tag{11}$$

One row $\mathbf{Z}_k$ and one column $\mathbf{S}_l$ of the frequency response matrix $\mathbf{H}^A$ need to be measured in order to identify all of the modal parameters λ_j, φ_j, ψ_j of a rotor (Fig. 7). It is sufficient to measure only one column, if eigenvalues and right eigenvectors are required. For determination of eigenvalues λ_j without natural modes, the whole information is contained in one frequency response H_{kl} already. There are exceptions, if the points of excitation or response are identical with node points of the natural modes.

Measurements of the Frequency Response Functions. As pointed out, the modal parameters of rotating structures can be determined from a set of frequency response functions. The measurement of these frequency response functions is an important step of modal analysis. Several experimental methods exist for acquiring this information. Usually measurements are made by applying the force input artificially through some type of exciter. As mentioned previously, one could excite the system with harmonic forces and measure the system response (amplitude and phase). But sine testing is slow compared to other types of signals, since it only excites the structure at one frequency at a time. Other types of excitation, signals like impact and random forces are considered to be the most useful today. They have a broad-band characteristic in the frequency domain and measurements can be carried out in a relatively short time.

Basically frequency response functions may be computed directly from the ratio of the Fourier transforms of the measured output and input signals

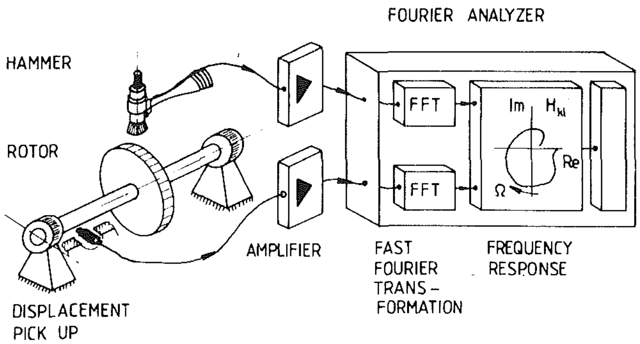

Fig. 9 Measuring device

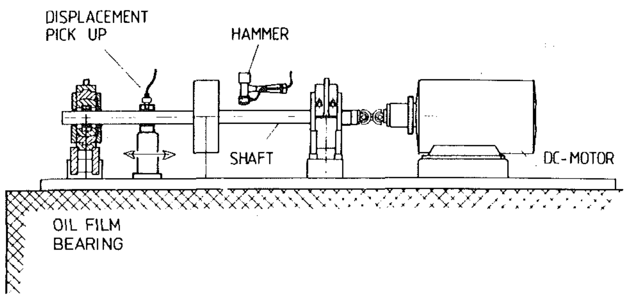

Fig. 10 Rotor test rig

$$H_{kl}^{M}(\Omega)=\frac{q_k(\Omega)}{f_l(\Omega)}=\frac{\sum\limits_{-\infty}^{\infty}\tilde{q}_k e^{-i\Omega t}dt}{\sum\limits_{-\infty}^{\infty}\tilde{f}_l e^{-i\Omega t}dt} \qquad (12)$$

This Fourier Transformation theorem permits replacement of the time consuming frequency by frequency excitation technique by the excitation with an arbitrary signal. The only requirements are, that the signals be Fourier transformable. Recent developments in the area of digital signal processing, especially progress of Fast Fourier Transform algorithms enabled the successful application of this technique.

The employed measurement method consists of applying an impact forcing function $\tilde{f}_l$ to a point l of the structure while picking up at the same time the displacement $\tilde{q}_k$ of point k (Fig. 8). The time signals are transformed to the frequency domain by means of the Fast Fourier Transformation and the ratio is calculated.

The frequency content and the amplitude of the force signal can be influenced by selection of the hammer mass, the flexibility of the impact cap and the impact velocity. With a short impulse the energy is distributed in a wide frequency range, with a long impulse in lower frequencies.

Figure 9 shows the measurement equipment. The hammer excites the rotating shaft and the force is measured by means of a force transducer or an accelerometer. The displacements of the shaft can be picked up by capacitive or inductive pickups. Force and displacement signals are amplified and after analog-digital-conversion and Fast-Fourier-Transformation the frequency response functions can be calculated. They are stored on a magnetic tape for further treatment.

Modal Parameter Estimation. Different sophisticated techniques are available to estimate modal parameters from frequency response data. Which one is the most suited in a special case depends on the degree of modal coupling.

At any given frequency Ω the frequency response represents the sum of all the modes of vibration which have been excited (equation (9)). Normally the contribution of a particular

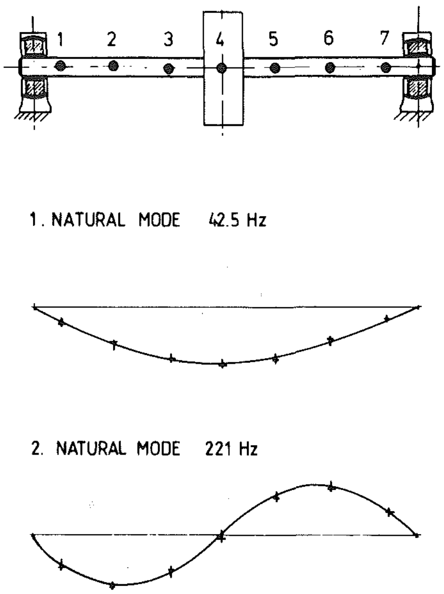

Fig. 11 Natural modes of the shaft with rigid bearings

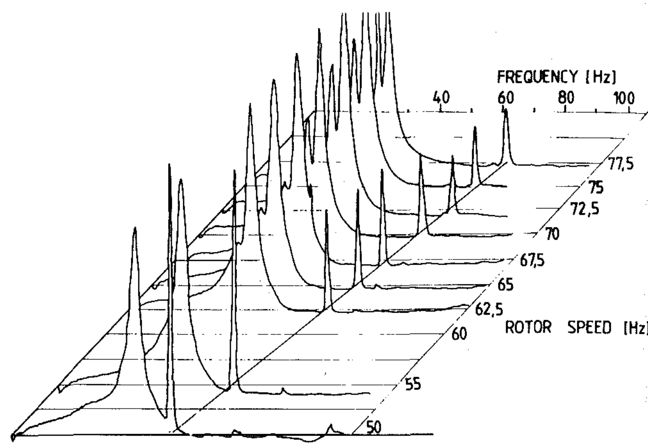

Fig. 12 Frequency spectrum of the test rig rotor

mode is the greatest near its natural frequency. When modal coupling is light, the frequency response can be considered in the vicinity of a resonance as if it were a single degree of freedom system. The simplest approach to determine modal parameters of such systems with well separated modes is to pick up natural frequencies and amplitudes at the resonance locations.

The investigated rotor bearing system has heavy modal coupling. In this case a more sophisticated technique to extract modal parameters is a multidegree of freedom curve fit, based on equation (9). Within a limited frequency range the goal of the procedure is to find the complex modal parameters λ_j, φ_j. The basic idea consists of finding a best fit between the measured response plot and the generated plot from the analytical expression (9). A well known method for doing this is to use a least squares procedure. The criterion is to minimize the error function

$$E=\sum_{p}^{P}\{RE(H_{kl}^{M}(\Omega_p))-RE(H_{kl}^{A}(\Omega_p,\lambda_j,\varphi_j))\}^2 + \sum_{p}^{P}\{IM(H_{kl}^{M}(\Omega_p))-IM(H_{kl}^{A}(\Omega_p,\lambda_j,\varphi_j))\}^2 \qquad (13)$$

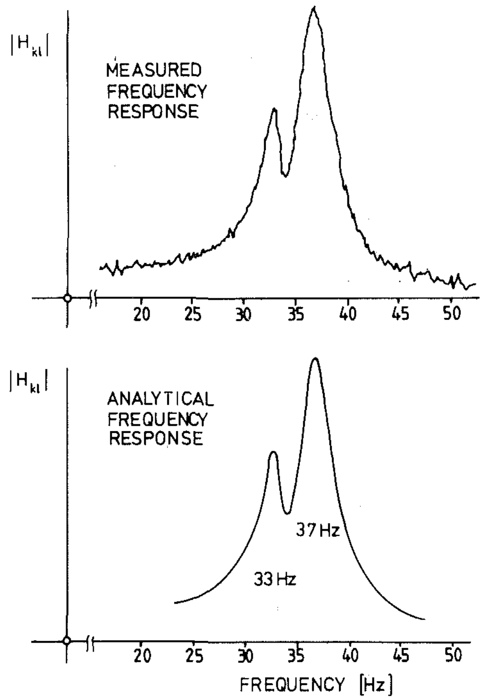

Fig. 13 Comparison of measured and calculated frequency response

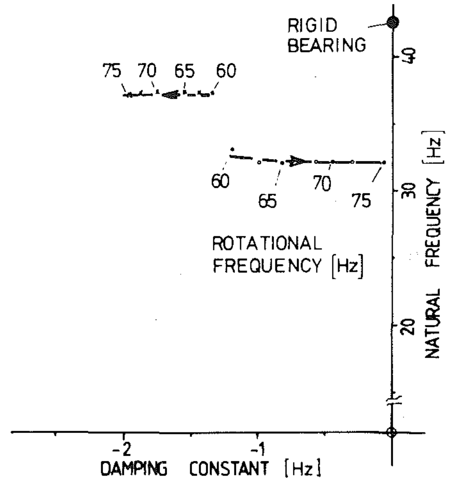

Fig. 14 Eigenvalues of the test rig rotor

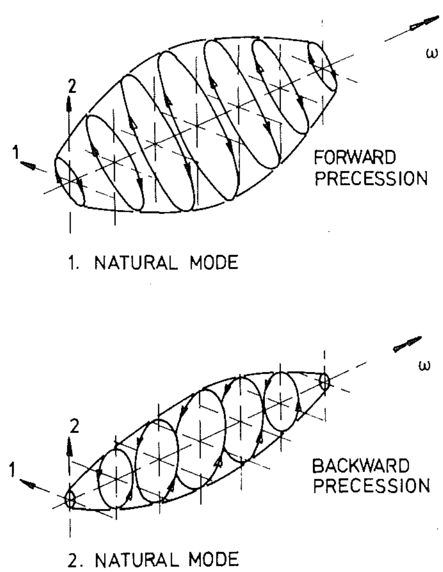

Fig. 15 Natural modes of the test rig rotor

H_{kl}^A analytical frequency response expression
H_{kl}^M measured frequency response

P is the number of frequency lines in the measured frequency response functions.

Differentiating E with respect to each unknown in turn and setting each result to zero, we obtain a number of equations for the unknown modal parameters. The equations are nonlinear in the eigenvalue parts, therefore they are solved by an iterative procedure (linearization). After each step the variation of the error function is controlled. At the beginning of the procedure a starting vector of the unknowns must be chosen. The process is repeated for further measured frequency curves.

Example—Modal Parameters of an Elastic Rotor in Journal Bearings

Rotor Test Rig. For testing the method, measurements were carried out at the test rig rotor shown in Fig. 2, respectively, Fig. 10. The rotor consists of a cylindrical shaft (diameter 50 mm, length 1000 mm) with a disk (mass 55 kg, diameter 300 mm, width 100 mm) at the center of the shaft. The shaft is driven by a d-c electric motor with control. The two oil film bearings are cylindrical bearings with a length-diameter ratio $B/D = 0.8$.

Seven measurement points were established along the rotor axis. At each measurement point excitation by impact forces and measuring the system response is possible in vertical as well as horizontal direction. The hammer for pulse excitation has a mass of 1.5 kg. The displacement pickups are movable along the rotor. Measured quantities are the exciting force, the displacements of the shaft and the rotor speed.

Measurement Results. For the nonrotating shaft ($\omega = 0$) there is no oilfilm effect in the bearings and the system can be considered as a flexible shaft with rigid bearings. The results from experimental modal analysis are shown in Fig. 11. The natural frequency of the first bending mode is 42.5 Hz. The second bending mode has a comparatively high frequency of 221 Hz.

It is important to know the first natural frequency of the rigidly supported shaft. With this information the natural frequency of the shaft in oil film bearings can be estimated. The flexibility of the oil film lowers the natural frequency. Therefore it is to be expected, that the first bending natural frequency of the rotor in journal bearings is less than 42.5 Hz. Furthermore it is known that the instability onset speed is about two times the first natural frequency.

A number of frequency spectra from 0 to 100 Hz are shown in Fig. 12 for different rotating speeds between 50 Hz and 77.5 Hz. The frequency spectrum for each rotor speed was found by impacting the rotor at the disk and measuring and analyzing the response signal near the bearings.

For lower rotating speeds there is one peak at 37 Hz, which is identical with a system natural frequency. The second peak is caused by unbalance excitation. The frequency at this peak is the frequency of rotation. For higher speeds the frequency spectra show two natural frequencies between 30 Hz and 40 Hz and the unbalance response peak with decreasing amplitudes at increasing rotational frequencies. The growing peak at about 33 Hz shows that the rotor is running near the instability onset speed between 75 and 80 Hz.

Natural frequencies and damping constants can be determined approximately with the frequency spectrum. To get better results for system eigenvalues and eigenvectors, frequency response functions are used. For the described rotor in cylindrical bearings measurements were carried out in a speed range from 60 to 75 Hz. Because of the variation of eigenvalues and natural modes with rotor speed, the rotational frequency need to be constant during the measurements. For each rotor speed one column of the frequency response matrix was calculated from the corresponding measured time signals. In Fig. 13 the amplitude frequency characteristic of one of the frequency response functions is represented for a rotational speed of 70 Hz. The upper diagram shows the measured function, the lower the calculated function. There are two resonance peaks at 33 Hz and 37 Hz.

From measurements for different rotating speeds the two essential eigenvalues (natural frequencies and damping constants) were identified by the above described curve fitting procedure. Figure 14 shows in the complex plane the natural frequencies versus the damping constants, respectively imaginary parts of eigenvalues versus real parts. Parameter is the rotating speed of the shaft. There is only a little influence from the speed to the natural frequencies. The first natural frequency is about 33 Hz, the second 37 Hz.

It is important to note that the damping constant of the first eigenvalue tends to zero for increasing speeds. At the stability threshold speed $\sim$ 78 Hz the damping constant disappears. The rotor is unstable for higher speeds.

Besides the eigenvalues also the eigenvectors were identified. Figure 15 points out the two natural modes, as defined in equation (6), for a frequency of rotation of 70 Hz.

The natural modes represent time dependent curves in space. The plane motion of one point of the shaft is an elliptical orbit.

Forward precession appears in the first natural mode with a natural frequency of 33 Hz. This eigenvalue leads to instability. In the second natural mode with backward precession and a natural frequency of 37 Hz the damping constant increases with the rotational frequency.

Conclusions

In this paper an application of experimental modal analysis is given for rotating machines. It is shown, that the classical method has to be improved in the case of nonconservative rotors with unsymmetric matrices. In the special case of a simple elastic shaft with oil film bearings the complex eigenvalues, the natural modes and the instability onset speed could be determined. The measured results are in good correlation with theoretical values.

The application of the suggested procedure in practical cases is possible if suitable locations for excitation exist and the input energy is high enough to excite the essential natural vibrations.

Acknowledgment

This paper is dedicated to the 65th birthday of Prof. E. Krämer.

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