Closing Pandora's Box: Additional Insights on Inclination Bias Using a Random Walk Approach

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Abstract

A fundamental working assumption in paleomagnetic studies is that the Earth's magnetic field averages to a geocentric axial dipole (GAD) when sufficiently sampled. One of the main tools for evaluating the GAD hypothesis in pre-Cenozoic times is based on the distribution of inclination values. Recent studies of inclination-only data show a bias towards low inclination and a number of alternative explanations were forwarded to explain this bias. The inclination only analysis relies on the fact that the planet has been adequately sampled in a spatially and/or temporally random manner. A recent paper argued that the inclination only studies might misrepresent the field because the extant global paleomagnetic database does not provide an adequate sampling of the field. In this study, we examine other sources of bias in the database. We find that the apparent contributions of quadrupolar and octupolar fields may depend upon the binning procedure used. For example, the Cenozoic database can be favorably compared to GAD when assigned to temporal bins based on geologic periods, but is decidedly non-GAD when averaged on a finer temporal scale. We also demonstrate that the Paleozoic inclination distribution may result from a regional sampling bias and we quantitatively assess the probability that the Precambrian global paleomagnetic dataset sufficiently integrates the time-averaged Earth's magnetic field. Our analysis suggests that the extant inclination database contains myriad forms of bias and may not represent the Earth's magnetic field. Unfortunately, the analysis cannot rule out the existence of persistent nondipolar fields. The global paleomagnetic database does indeed show a rather consistent bias towards low-inclination values (median inclination is 40° versus 49° for the GAD). Models of the earth's magnetic field and the thermal evolution of the planet may yield additional clues regarding its GAD or non-GAD nature.

Introduction

Neil Opdyke's storied career in paleomagnetism currently spans six decades. His pioneering work on magnetostratigraphy, the Earth's reversal record and the nature of the magnetic field forms the basis for many modern studies of the Earth's magnetic field. Paramount among the assumptions in paleomagnetic studies is that the Earth's magnetic field is reduced to a geocentric axial dipole (GAD) when sufficiently sampled. An early test of the GAD assumption was conducted by Opdyke and Henry (1969). Thev concluded that for the most recent 2 million years, the Earth's magnetic field resembled a Subsequent examinations of the recent field (McElhinny et al., 1996; GAD field. Hatakeyama and Kono, 2002) also support the GAD assumption for the past 5 million years. Tests of the GAD assumption probing deeper into geologic time have produced disparate results. Evans (1976) used inclination-only data from paleomagnetic studies and concluded that the frequency distribution of those data were indistinguishable from that of an expected GAD field. Subsequent inclination-only studies by *Piper and Grant* (1989) and Kent and Smethurst (1998) suggested that there were periods in earth history when the magnetic field differed significantly from GAD. Deviations from the GAD field for Paleozoic and Mesozoic times were also supported by recent studies by Torsvik and Van der Voo (2002) and Van der Voo and Torsvik (2001). Hollerbach and Jones (1995) argued that the size of the inner core has a stabilizing effect on the geodynamo and a smaller inner core might result in persistent higher harmonic fields (e.g. quadrupolar and octupolar) in the Paleozoic and Precambrian. Bloxham (2000) tested the effects of a smaller inner core (0.25 present-day) and found that a smaller sized core produced insignificant deviations from the GAD model. Instead, Bloxham (2000) argued that the large octupolar component inferred from inclination-only data arises from the periodic effects of lateral heat transfer across the core-mantle boundary. A number of other, non-geodynamo causes for the observed low-inclination bias have been proposed and were discussed by Kent and Smethurst (1998).

A successful inclination-only analysis relies either on sufficiently distributed sampling sites or that the sampling sites become randomized via continental drift. *Meert et al.* (2003) recently challenged the sensitivity of the inclination-only method on resolving the GAD field through the use of a random walk model. The random walk

model, assumes a GAD planet and generates inclination data for well-distributed sites on randomly drifting continents. *Meert et al. (2003)* concluded that the current paleomagnetic database does not represent a sufficiently random sample and therefore the non-GAD features observed in previous studies are simply due to the effects of poor spatial-temporal coverage in the extant database. Here we examine several other flaws in conducting inclination-only analyses and extend our random walk models to look at very small sample sizes.

Evaluation of Previous Models

Evans (1976) and Kent and Smethurst (1998) used a binning technique to help filter out spatial-temporal biases in their inclination analysis. Both previous studies used a spatial binning of 10° x 10° and temporal bins were based on geologic periods with the exception that Kent and Smethurst (1998) evaluated the entire Precambrian using 50 Ma intervals. We do not fault the rationale of using spatial-temporal binning; however, we note that the choice of breakpoints can greatly affect the perceived inclination bias. For example, Kent and Smethurst (1998) argued that the Cenozoic and Mesozoic inclination distributions were indistinguishable from GAD. Meert et al. (2003) noted an error in the chi-square statistic (χ^2) calculation resulting in a Mesozoic distribution that was significantly different from GAD above the 99% confidence level (see Table 1, χ^2 critical value 99%=20.09). However when the Mesozoic and Cenozoic distributions are added together, the resultant inclination distribution is also significantly different from GAD (Table 1, figure 1a,b) with a best fit when the quadrupolar contribution (G2) is ± 0.28 and the octupolar contribution (G3) is +0.1425. The best fit for the Phanerozoic inclination distribution (Table 1, Figure 1c,d) differs from GAD with a best fit when $G2=\pm 0.18$ and G3=+.1425. The Precambrian inclination distribution (Table 1, Figure 1e,f) gives a best fit when $G2=\pm0.14$ and G3=+0.232. If we combine all the inclination distributions, the resulting best fit is obtained when $G2=\pm 0.18$ and G3=+0.14 (Table 1, Figure 1g,h).

Bloxham (2000) examined the effects of an intermittent Y_2^0 pattern of lower mantle heat flux variation on the geomagnetic field. The assumption was that such a pattern would inhibit the emergence of a poloidal field in equatorial regions and lead to the expression of an octupolar contribution to the magnetic field. He examined unbinned inclination data for the Cenozoic+Mesozoic, the Paleozoic, and the Precambrian. He

concluded that the Mesozoic+Cenozoic distributions resembled the GAD because 250 Ma is too short a period to adequately average the Earth's magnetic field and detect these octupolar components. Although a Y_2^0 pattern of lower mantle convection may result in a predominantly octupolar contribution to the field, we identify several problems with the analysis of *Bloxham* (2000). *Bloxham* (2000) did not apply any statistical tests in an effort to distinguish if the Mesozoic and Cenozoic distributions were different from GAD. We used the updated global paleomagnetic database and applied the same selection criteria to the inclination data (n=3671 unbinned values) and obtained the distribution shown in Figure 2a. This distribution is significantly different from GAD (Table 1) using all 3 statistical parameters (see *Meert et al.*, 2003) and therefore, if these inclination values faithfully reflect the magnetic field, then the past 250 Ma also shows significant departures from GAD. Secondly, the Paleozoic (lasting 293 million years) is only slightly longer, and less well sampled (see below), than the combined Mesozoic+Cenozoic (250 Ma) and thus is less likely to provide adequate time averaging of the geomagnetic field. Lastly, Bloxham (2000) argues that quadrupolar terms average to zero in his model, yet our analysis of the inclination-only data (see Figure 1) would indicate that most inclination distributions are best modeled with a nonzero G2 term. Nevertheless, we cannot dismiss this possibility and note that when we combine all the binned inclination data from the database and plot it as a cumulative frequency curve (Figure 2b, median inclination 40°), a best fit is obtained to the *Bloxham* (2000) model when the Y_2^0 amplitude is ~17% of the superadiabatic heat flux.

Cenozoic Dataset

Kent and Smethurst (1998) demonstrated that the Cenozoic dataset, when binned by geological Period (Neogene and Paleogene), was indistinguishable from GAD. The reason for the GAD fit is best explained by the even distribution of sampling sites rather than the effects of randomization via continental drift. We note here that the similarity to GAD is also due to the binning method applied. Assuming that the GAD-like distribution arises solely from the even distribution of sites, we should be able to bin the data at a finer temporal scale and obtain a GAD-like distribution. Figure 3 shows the Cenozoic data binned at 5 Ma and 10 Ma intervals compared to GAD and the Neogene-Paleogene binning of *Kent and Smethurst (1998)*. The 10 Ma binning produces a total of 519 bins and the resulting distribution is significantly different than GAD above the 99% confidence level (χ^2 =51.93; N_{crit}=154; RMSEA=0.119). The 5 Ma binned distribution is nearly identical to the 10 Ma distribution; however the 5 Ma procedure produces 655 spatial-temporal bins and is also significantly different from GAD at well above the 99% confidence interval.

Paleozoic Dataset

The Paleozoic inclination distributions of Kent and Smethurst (1998) and Piper and Grant (1989) both showed a low-latitude bias. Figure 4 (a-c) shows the spatialtemporal distribution of Paleozoic sampling sites. Most of the sampling sites are from North America and Europe with significantly fewer results from the former Gondwana elements. This low-inclination bias may have its origins in a strongly non-dipolar field or it may arise from sampling bias. Meert et al. (2003) argued for the latter explanation and both Bloxham (2000) and Kent and Smethurst (1998) argued for the former explanation. One way to test for sampling bias (in addition to those conducted by Meert et al., 2003) is to assume that we have faithfully sampled a GAD field in the Paleozoic and represent the motion of the continental blocks via their apparent polar wander paths (APWP's). We compiled Paleozoic APWP's from the published literature for Siberia, Baltica, Laurentia and Gondwana (Torsvik et al., 1996; Piper, 1987; Van der Voo, 1993; Smethurst et al., 1998). These apparent polar wander paths were then smoothed and divided into 20 Ma segments for the period from 550-250 Ma. Sampling sites on each of the continents were placed at 5-degree intervals and samples were collected every 20 Ma based on their predicted latitudes from the APWP's. Figure 5a shows the synthetic distribution of inclination data for Laurentia with a clear low-latitude bias. Figure 5b shows the combined Baltica-Laurentia distribution (curve NE) compared to the global compilation (curve KE) obtained by Kent and Smethurst (1998). A best fit (curve BF) to the synthetic distribution is obtained with a pure octupole (G3) contribution of 22.4%. The best fit to the Kent and Smethurst (1998) distribution required a G2=±0.11 and a G3=+0.28. Figure 5c shows the synthetic distribution for Gondwana and figure 5d shows the sum of all the synthetic data. Figure 5d indicates that if the continents mentioned were well sampled in the Paleozoic, the resultant inclination distribution would have a low-inclination bias. We also note that the cumulative curve in Figure 5d is heavily weighted by Gondwana results (because of the binning procedure) such that a true representation based on the actual numbers of sampling locations would show a stronger bias towards low inclinations. Based on this analysis and those conducted by *Meert et al.* (2003) we conclude that it is not possible to use inclination-only data in the Paleozoic to distinguish between sampling bias and contributions from non-dipole fields.

Precambrian Dataset

The Precambrian inclination-only distribution shows a significant departure from GAD (*Kent and Smethurst, 1998; Meert et al., 2003*). We analyzed the 2003 global paleomagnetic database according to the procedures outlined in *Kent and Smethurst (1998)* for the interval from 550-4000 Ma. The 1362 values resulted in 549 spatial-temporal bins. The resultant inclination distribution is not radically different from the previous study (fig 6) and shows a bias towards low inclinations. *Meert et al. (2003)* argued that the low-inclination bias might arise from incomplete sampling in the Precambrian. Figure 7 (a-c) shows the spatial-temporal bias in the Precambrian dataset. The study locations are concentrated in Europe and North America and 80% of the data are younger than 2000 Ma (median <1400 Ma; Fig 7b). Figure 7d shows the inclination distribution that would be expected for the present-day locations of these sites and demonstrates that plate motion must play an important role in producing a random distribution of sampling sites in the Precambrian.

Meert et al. (2003) argued that a sizeable dataset is necessary to adequately test the GAD hypothesis. The requirement placed on the inclination-only analysis is that the sampling must guarantee (with 95% confidence or better) that the field has been adequately sampled. A further condition is that this requirement is met for whatever temporal period is examined. For example, a small dataset collected for one particular time interval may result in a distribution that is indistinguishable from GAD. However, additional samples added during the next time interval may result in a non-GAD distribution. When there is a clear spatial bias to the data we require a sampling interval guaranteed to faithfully represent the average magnetic field. The Precambrian dataset sampled less than 5% of the available spatial-temporal bins available making it unlikely to generate the required random sample. *Meert et al.* (2003) demonstrated with

several examples that such a small dataset is unlikely to sufficiently test a GAD field, but the argument for very small sample sizes was not quantified in detail.

Here, we test small sample sizes as follows. The random-walk model was conducted on a GAD planet. Samples were collected every 50 Ma and plate direction changes were conducted every 75 Ma. Plate velocities ranged from 0 to 8 cm yr⁻¹. We ran the model with an increasing number of sampling sites starting with 11 distributed sites and ending with 39 distributed sampling sites. The 11 sites produce a total of 550 'bins' and these are comparable to the sample size used in the *Kent and Smethurst (1998)* evaluation. Each model was run for 2500 Ma and 100 iterations. The program compiled a listing of acceptable representations of the known GAD field using the χ^2 test, the N_{crit}+RMSEA combination or the χ^2 -RMSEA combination (see *Meert et al., 2003*). We define a GAD-like fit as being statistically indistinguishable from the GAD distribution using the critical values outlined in *Meert et al. (2003)*.

Meert et al. (2003) describes the sensitivity of the χ^2 test to small and large sample sizes. A small sample size will almost always be indistinguishable from the expected and large sample sizes will nearly always indicate a significant difference from the expected distribution. Our first sample run (using 10 distributed sites) showed that only 53% of the distributions were indistinguishable from GAD using the χ^2 test (Table As sample sites were added to the model, the number of GAD-like distributions 2). generally decreased (Table 2, Figure a) with only 22.7% acceptable fits when there are 39 distributed sites. However, we note an additional complexity in interpreting these results because the χ^2 values oscillate over the sampling interval and many of the acceptable fits are achieved at low-N (see Figure b). Therefore, we also looked at the percentage of GAD-like fits achieved at the end of the run and found that these also generally decreased with increasing sample sizes (Table 2). Lastly, we note that in no case did we achieve the required 95% using only the χ^2 test. In contrast, we found that the number of GAD-like fits based on the RMSEA+N_{crit} values increased in dramatic fashion with increasing N (from a low of 10% to 75% when n=39 sites; Figure a). Although none of these values reached the requisite 95% value, Meert et al. (2003) showed that when the number of samples is large and the runs are lengthy, the model accurately reflects the GAD world at above the 95% confidence level.

Conclusions

One of the main tools for evaluating the GAD hypothesis in pre-Cenozoic time is based on the distribution of inclination values. Recent studies of inclination-only data show a bias towards low inclination and a number of alternative explanations were forwarded to explain this bias. One of the assumptions made in the analysis is that the planet has been adequately sampled in a spatially and temporally random manner. In a recent paper, Meert et al. (2003) argued that the extant paleomagnetic database is not capable of adequately testing the GAD hypothesis. Here we have examined other sources of bias in the database. We found that the apparent contributions of quadrupolar and octupolar fields may depend upon the binning procedure used. For example, the Cenozoic database can be favorably compared to GAD when assigned to temporal bins based on geologic periods, but is decidedly non-GAD when averaged on a finer temporal scale. We also demonstrated that the Paleozoic inclination distribution may result from a regional sampling bias. We also quantitatively assess the probability that the Precambrian global paleomagnetic dataset might reflect integrated behavior of the Earth's magnetic field. Although GAD-like fits were obtained with small, randomly distributed sites, the probability of obtaining a good representation of the field was under 50%.

Unfortunately, our analysis cannot rule out the existence of persistent non-dipolar fields in geologic time. The global paleomagnetic database does indeed show a rather consistent bias towards low-inclination values (median inclination is 40° versus 49° for the GAD). Models of the earth's magnetic field and the thermal evolution of the planet may yield additional clues regarding its GAD or non-GAD nature.

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Table 1. Previous Results

Period	Bins or	χ_1^2	χ_2^2	N _{crit}	RMSEA	G2	<i>G3</i>
	Observations					±	+
Cenozoic (0-65 Ma)	253	3.63	9.19	426.3	0.07	NC	NC
Mesozoic (65-250 Ma)	342	7.18	24.50	216.9	0.10	0.28*	0.14*
Paleozoic (250-550 Ma)	352	32.23	<i>113.4</i> 8	48.9	0.21	0.11	0.28
Precambrian (550-3500 Ma)	531	20.39	108.76	76.6	0.17	0.14	0.23
All (0-3500 Ma)	1478		135.75	169.8	0.11	0.16	0.17
Phanerozoic	947		<i>54.33</i>	270.8	.091	0.18	0.14
Mesozoic+Cenozoic	595		28.72	321	.083	0.28	0.14
Mesozoic+Cenozoic-Bloxham	3671		603	95.4	.153	0.28*	0.22*

 χ_1^2 , as calculated by the original authors; χ_2^2 as calculated in this study (note they vary slightly from the numbers reported in Meert et al., 2003 due to a slightly refined best-fit program), N_{crit} =critical N-index or Hoelter Index, RMSEA = root mean square error of approximation, G2 and G3 are best-fit calculation to the observed binned distribution;NC=not calculated since the results are indistinguishable from GAD. *Best fit is significantly different than the observed distribution.

Table 2. Small Sample Runs

Run	Bins	χ^2	RMSEA and Ncrit	χ^2	
		$(all)^1$	$(end)^2$	$(end)^3$	
Prec11	551	53.4%	5.5%	42.0%	
Prec19	970	60.6%	23.8%	47.0%	
Prec29	1480	41.8%	62.0%	34.0%	
Prec39	2041	22.7%	75.0%	21.0%	

 1 Uses only the χ^{2} value in the analysis 2 Both the RMSEA and Ncrit values must reach critical levels of significance at the end of each run.

³Uses only the final χ^2 value in the analysis

Figure Legends

Figure 1: (a) Inclination distributions for the Cenozoic and Mesozoic based on the analysis of Kent and Smethurst (1998). The dashed line represents the best fit obtained when $G2 = \pm 0.28$ and G3 = +0.1425. (b) Distribution of G2 and G3 values that are statistically indistinguishable from the observed distribution for the Cenozoic and Mesozoic. (c) Inclination distributions for the Phanerozoic based on the analysis of Kent and Smethurst (1998). The dashed line represents the best fit obtained when $G2 = \pm 0.18$ and $G3 = \pm 0.1425$. (d) Distribution of G2 and G3 values that are statistically indistinguishable from the observed distribution for the Phanerozoic. (e) Inclination distributions for the Precambrian based on the analysis of Kent and Smethurst (1998). The dashed line represents the best fit obtained when $G2 = \pm 0.144$ and G3 = +0.232. (f) Distribution of G2 and G3 values that are statistically indistinguishable from the observed distribution for the Precambrian. (g) Inclination distributions for the entire database based on the analysis of Kent and Smethurst (1998). The dashed line represents the best fit obtained when $G2 = \pm 0.18$ and G3 = +0.1425. (b) Distribution of G2 and G3 values that are statistically indistinguishable from the observed distribution for the entire database.

Figure 2: (a) Frequency distribution for the unbinned Cenozoic+Mesozoic inclination data from the 2003 global paleomagnetic database. The distribution is significantly different from GAD. (b) Cumulative frequency of inclination data based on the best fit to the entire binned dataset of Kent and Smethurst (1998) in comparison to the expected GAD cumulative frequency curve. The best fit line closely approximates the curve obtained in the Bloxham (2000) model with a Y_2^0 amplitude of 17% of the superadiabatic heat flux.

Figure 3: The frequency distribution of the Cenozoic database compared to the GAD distribution. The Kent and Smethurst temporal bin (Neogene+Paleogene) produced a frequency that is indistinguishable from GAD. A finer temporal binning (either 5 Ma or 10 Ma) produces distributions that are significantly different from GAD.

Figure 4: (a) Spatial distribution of the Paleozoic database. (b) Temporal distribution of the Paleozoic paleomagnetic database shown as a cumulative frequency. The median age is 375 Ma and (c) The spatial-temporal distribution of the Paleozoic database.

Figure 5: (a) A synthetic inclination frequency distribution for Laurentian sites based on a smoothed apparent polar wander path with a sampling frequency of 20 Ma and a spatial binning of 5 degrees (b) A synthetic inclination frequency distribution for combined Baltica+Laurentian sites based on a smoothed apparent polar wander paths with a sampling frequency of 20 Ma and a spatial binning of 5 degrees. (c) A synthetic inclination frequency distribution for Gondwana sites based on a smoothed apparent polar wander path with a sampling frequency of 20 Ma and a spatial binning of 5 synthetic inclination frequency distribution degrees and (d) Α for

Siberia+Laurentia+Gondwana+Baltica sites based on a smoothed apparent polar wander path with a sampling frequency of 20 Ma and a spatial binning of 5 degrees.

Figure 6: Inclination frequency distribution of Precambrian data from the 2003 edition of the global paleomagnetic database versus the GAD model. The data were binned in 50 Ma temporal intervals and 10 degree spatial intervals. The total number of bins was 549 (from 1362 individual inclination values).

Figure 7: (a) Spatial distribution of the Precambrian database. (b) Temporal distribution of the Precambrian paleomagnetic database shown as a cumulative frequency. The median age is 1400 Ma and (c) The spatial-temporal distribution of the Precambrian database and (d) expected present-day inclination values of sampled Precambrian sites showing the sample bias inherent in this dataset.

Figure 8: (a) The percentage of GAD-like distributions based on the χ^2 values obtained at 50 Ma intervals compared to the number of binned data (gray solid line; see also Table 2), the percentage of GAD-like distributions based on the χ^2 values obtained at the last step of the simulation (2500 Ma; dashed line) and the percentage of GAD-like distributions based on the root mean square error of approximation and N_{crit} indices obtained at the last step of the simulation (2500 Ma; dark line). (b) Large graph shows the change in the χ^2 value at each 50 Ma step of one simulation. In this particular case, the resultant distribution is GAD-like only at the very beginning and very end of the run. The inset graph is given to demonstrate the variable drift rates generated by the random walk model.

















