*Research Article*

# Existence of Positive Periodic Solutions for a Class of $n$-Species Competition Systems with Impulses

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By using the fixed point theorem on cone, some sufficient conditions are obtained on the existence of positive periodic solutions for a class of $n$-species competition systems with impulses. Meanwhile, we point out that the conclusion of (Yan, 2009) is incorrect.

## 1. Introduction

In recent years, the problem of periodic solutions of the ecological species competition systems has always been one of the active areas of research and has attracted much attention. For instance, the traditional Lotka-Volterra competition system is a rudimentary model on mathematical ecology which can be expressed as follows:

$$\dot{x}_i(t) = x_i(t) \left[ r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j \right], \quad i = 1, 2, \ldots, n. \tag{1.1}$$

Owing to its theoretical and practical significance, the systems have been studied extensively by many researchers. And many excellent results which concerned with persistence, extinction, global attractivity of periodic solutions, or almost periodic solutions have been obtained.

However, the Lotka-Volterra competition systems ignore many important factors, such as the age structure of a population or the effect of toxic substances. So, more complicated

competition systems are needed. In 1973, Ayala and Gilpin proposed several competition systems. One of the systems is the following competition system:

$$\dot{N}_i(t) = r_i N_i \left[ 1 - \left( \frac{N_i}{K_i} \right)^{\theta_i} - \sum_{j=1, j \neq i}^{n} \alpha_{ij} \frac{N_j}{K_j} \right], \quad i = 1, 2, \ldots, n, \tag{1.2}$$

where $N_i$ is the population density of the $i$th species; $r_i$ is the intrinsic exponential growth rate of the $i$th species; $K_i$ is the environmental carrying capacity of species $i$ in the absence of competition; $\theta_{ij}$ provides a nonlinear measure of interspecific interference, $\alpha_{ij}$ provides a nonlinear measure of interspecific interference.

On the other hand, in the study of species competition systems, the effect of some impulsive factors has been neglected, which exists widely in the real world. For example, the harvesting or stocking occur at fixed time, natural disaster such as fire or flood happen unexpectedly, and some species usually migrate seasonally. Consequently, such processes experience short-time rapid change which can be described by impulses. Therefore, it is important to study the existence of the periodic solutions of competitive systems with impulse perturbation (see [1–7] and the references therein).

For example, by using the method of coincidence degree, Wang [1] considered the existence of periodic solutions for the following $n$-species Gilpin-Ayala impulsive competition system:

$$\dot{x}_i(t) = x_i(t) \left[ r_i(t) - \sum_{j=1}^{n} a_{ij}(t) x_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n} b_{ij}(t) x_j^{\alpha_{ij}} \left( t - \tau_{ij}(t) \right) - \sum_{j=1}^{n} c_{ij}(t) x_i^{\alpha_{ii}}(t) x_j^{\alpha_{ij}}(t) \right], \quad t \neq t_k;$$

$$\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = p_k^i x_i(t_k), \quad k = 1, 2, 3, \ldots,$$

$$\tag{1.3}$$

where the constant $p_k^i$ satisfied $-1 < p_k^i < 0$, $i = 1, 2, \ldots, n$. What is more, [1] also obtained several results for the persistence and global attractivity of the periodic solution of the model.

In [2], Yan applied the Krasnoselskii fixed point theorem to investigate the following n-species competition system:

$$\dot{y}_i(t) = y_i(t) \left[ r_i(t) - \sum_{j=1}^{n} a_{ij}(t) y_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n} b_{ij}(t) y_j^{\beta_{ij}} \left( t - \tau_{ij}(t) \right) \right.$$

$$\left. - \sum_{j=1}^{n} c_{ij}(t) \int_{-\sigma_{ij}}^{0} K_{ij}(\xi) y_i^{\gamma_{ij}}(t + \xi) y_j^{\delta_{ij}}(t + \xi) d\xi \right], \quad i = 1, 2, \ldots, n, \tag{1.4}$$

where the constants $\alpha_{ij}, \beta_{ij}, \gamma_{ij} \geq 1$, $i, j = 1, 2, \ldots, n$. He obtained a necessary and sufficient condition for the existence of periodic solutions of system (1.4). Unfortunately, its last conclusion is wrong. Please see the remark in Section 3 of this paper.

Motivated by [1, 2], in this paper, we investigate the following impulsive $n$-species competition system:

$$\dot{y}_i(t) = y_i(t)\left[ r_i(t) - \sum_{j=1}^{n} a_{ij}(t) y_j^{\alpha_{ij}}(t) - \sum_{j=1}^{n} b_{ij}(t) y_j^{\beta_{ij}}(t - \tau_{ij}(t)) \right.$$

$$\left. - \sum_{j=1}^{n} c_{ij}(t) \int_{-\sigma_{ij}}^{0} K_{ij}(\xi) y_i^{\gamma_{ij}}(t + \xi) y_j^{\delta_{ij}}(t + \xi) d\xi \right], \quad t \neq t_k, \ i = 1, 2, \ldots, n,$$

$$\Delta y_i(t_k) = y_i(t_k^+) - y_i(t_k^-) = I_{ik}(y_1(t_k), \ y_2(t_k), \ldots, y_n(t_k)), \quad k = 1, 2, \ldots, m,$$

$$(1.5)$$

where $y_i(t)$ is the population density of the $i$th species at time $t$; $r_i(t)$ is the intrinsic exponential growth rate of the $i$th species at time $t$; $\tau_{ij}(t)$ is the time delay; $\sigma_{ij}$ is a positive constant; $a_{ij}(t), b_{ij}(t), c_{ij}(t)$ $(i \neq j)$ measure the amount of competition between the species $Y_i$ and $Y_j$; $\alpha_{ij}, \ \beta_{ij}, \ \delta_{ij}$ $(i \neq j)$ provide a nonlinear measure of interspecific interference; $y_i(t_k^+)$ $(y_i(t_k^-))$ is the left (right) limits of $y_i(t)$ at $t = t_k$, $i, j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, m$.

The main features of the present paper are as follows. The Gilpin-Ayala species competition system (1.5) has impulsive effects. As is known to us, there were few papers to study such system. Finally, we point out that the conclusion of [2] is incorrect.

For an $\omega$-periodic function $u(t) \in C(\mathbb{R}, \mathbb{R})$, let $\bar{u} = 1/\omega \int_0^\omega u(t)dt$. Throughout this paper, assume the following conditions hold.

(H1) $r_i, a_{ij}, b_{ij}, c_{ij}, \tau_{ij}$ are continuous $\omega$-periodic functions, and $\bar{r}_i > 0$, $a_{ij}(t)$, $b_{ij}(t)$, $c_{ij}(t) \geq 0$, $i, j = 1, 2, \ldots, n$, and there exists $i_0$ $(1 \leq i_0 \leq n)$ such that $\min_{1 \leq j \leq n}(\bar{a}_{i_0 j} + \bar{b}_{i_0 j}) > 0$.

(H2) $K_{ij} \in C([-\sigma_{ij}, 0], \mathbb{R})$, $K_{ij} \geq 0$, $\sigma_{ij}$ is a positive constant, and $\int_{-\sigma_{ij}}^{0} K_{ij}(t)dt = 1$, $i, j = 1, 2, \ldots, n$.

(H3) $I_{ik} \in C(\mathbb{R}^n, [0, +\infty))$, and for $0 < t_1 < t_2 < \cdots < t_m < \omega$, there exists an positive integer $l > 0$ such that $t_{k+lm} = t_k + l\omega, I_{ik}(y_1, y_2, \ldots, y_n) = I_{i(k+lm)}(y_1, y_2, \ldots, y_n)$, where $(y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots, m$.

(H4) $\alpha_{ij}, \beta_{ij}, \delta_{ij} > 0$, $\gamma_{ij} \geq 0$ are constants, $i, j = 1, 2, \ldots, n$.

In order to prove our main result, now we state the fixed point theorem of cone expansion and compression.

**Lemma 1.1** (see [4]). *Let $E$ be a Banach space, and let $P$ be a cone in $E$. Assume that $\Omega_1$, $\Omega_2$ are open subsets of $E$ with $0 \in \Omega_1$, $\overline{\Omega}_1 \subseteq \Omega_2$. Let $A : P \cap (\overline{\Omega}_2 \setminus \Omega_1) \to P$ be a completely continuous operator such that one of the following two conditions is satisfied:*

(i) *$Ax \ngeq for \ x \in P \cap \partial\Omega_1$; $Ax \nleq for \ x \in P \cap \partial\Omega_2$,*
(ii) *$Ax \ngeq for \ x \in P \cap \partial\Omega_2$; $Ax \nleq for \ x \in P \cap \partial\Omega_1$,*

*Then, $A$ has a fixed point in $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$.*

The organization of this paper is as follows. In the next section, we introduce some lemmas and notations. In Section 3, the main result will be stated and proved on the existence of periodic solutions of system (1.5).

## 2. Preliminaries

Let $PC(\mathbb{R}, \mathbb{R}^n) = \{y(t) = (y_1(t), y_2(t), \cdots y_n(t)) : \mathbb{R} \to \mathbb{R}^n \mid y_i(t)$ be continuous at $t \neq t_k$, left continuous at $t = t_k$, and the right limit $y_i(t_k^+)$ exists for $i = 1, 2, \ldots, n, \ k = 1, 2, \ldots, m\}$. Evidently,

$$E = \{y(t) = (y_1(t), y_2(t), \ldots y_n(t)) \in PC(\mathbb{R}, \mathbb{R}^n) \mid y_i(t) = y_i(t + \omega), \ i = 1, 2, \ldots, n\} \quad (2.1)$$

is a Banach space with the norm $\|y\| = \sum_{i=1}^n |y_i|_0$, where $|y_i|_0 = \max_{t \in [0, \omega]} |y_i(t)|$.

Define an operator $T : E \to E$ by $(Ty)(t) = ((Ty)_1(t), (Ty)_1(t), \ldots, (Ty)_n(t))$, where

$$(Ty)_i(t) = \int_t^{t+\omega} G_i(t, s) y_i(s) \left[ \sum_{j=1}^n a_{ij}(s) y_j^{\alpha_{ij}}(s) + \sum_{j=1}^n b_{ij}(s) y_j^{\beta_{ij}}(s - \tau_{ij}(s)) \right.$$

$$\left. + \sum_{j=1}^n c_{ij}(s) \int_{-\sigma_{ij}}^0 K_{ij}(\xi) y_i^{\gamma_{ij}}(s + \xi) y_j^{\delta_{ij}}(s + \xi) d\xi \right] ds \quad (2.2)$$

$$+ \sum_{k=1}^m G_i(t, t_{k+q_k m}) I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)),$$

where $G_i(t, s) = \exp(-\int_t^s r_i(v) dv)/(1 - \exp(-\omega \bar{r}_i))$, $t \leq s \leq t + \omega$, $i = 1, 2, \ldots, n$, and $t_{k+q_k m} = t_k + q_k \omega \in [t, t + \omega]$, where $q_k$ is a positive integer, $k = 1, 2, \ldots, m$.

It is obvious that the functions $G_i(t, s)$, $i = 1, 2, \ldots, n$ have the following properties.

(i)  $G_i(t, s) > 0$ for $(t, s) \in \mathbb{R}^2$, and $G_i(t, s) = G_i(t + \omega, s + \omega)$.

(ii) $A \leq G_i(t, s) \leq B$ for $(t, s) \in \mathbb{R}^2$, where $A = \min_{1 \leq i \leq n} \{\exp(-\omega \bar{r}_i)/(1 - \exp(-\omega \bar{r}_i))\}$, $B = \min_{1 \leq i \leq n} \{\exp(\omega \bar{r}_i)/(1 - \exp(-\omega \bar{r}_i))\}$.

Now, we choose a set defined by

$$P = \{y(t) = (y_1(t), y_2(t), \ldots y_n(t)) \in E \mid y_i(t) \geq \sigma |y_i|_0, \ t \in [0, \omega], \ i = 1, 2, \ldots, n\}, \quad (2.3)$$

where $\sigma = A/B$. Clearly, $P$ is a cone in $E$.

For the sake of convenience, we define an operator $F : P \to E$ by $(Fy)(t) = ((Fy)_1(t), (Fy)_2(t), \ldots, (Fy)_n(t))$, where

$$(Fy)_i(t) = y_i(t) \left[ \sum_{j=1}^n a_{ij}(t) y_j^{\alpha_{ij}}(t) + \sum_{j=1}^n b_{ij}(t) y_j^{\beta_{ij}}(t - \tau_{ij}(t)) \right.$$

$$\left. + \sum_{j=1}^n c_{ij}(t) \int_{-\sigma_{ij}}^0 K_{ij}(\xi) y_i^{\gamma_{ij}}(t + \xi) y_j^{\delta_{ij}}(t + \xi) d\xi \right], \quad i = 1, 2, \ldots, n. \quad (2.4)$$

**Lemma 2.1.** *The operator $T : P \to P$ is completely continuous.*

*Proof.* First, it is easy to see $T : PC(\mathbb{R}, \mathbb{R}^n) \rightarrow PC(\mathbb{R}, \mathbb{R}^n)$. Next, since

$$
\begin{aligned}
(Ty)_i(t + \omega) & \\
&= \int_{t+\omega}^{t+2\omega} G_i(t + \omega, s)(Fy)_i(s)ds + \sum_{k=1}^{m} G_i(t + \omega, t_{k+q_km} + \omega) I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \\
&= \int_t^{t+\omega} G_i(t + \omega, v + \omega)(Fy)_i(v + \omega)dv + \sum_{k=1}^{m} G_i(t, t_{k+q_km}) I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \\
&= \int_t^{t+\omega} G_i(t, v)(Fy)_i(v)dv + \sum_{k=1}^{m} G_i(t, t_{k+q_km}) I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \\
&= (Ty)_i(t),
\end{aligned}
\tag{2.5}
$$

we have $Ty \in E$.

Observe that $A \leq G_i(t, s) \leq B$, $i = 1, 2, \ldots, n$, for all $s \in [t, t+\omega]$. Hence, we obtain that, for $y \in P$,

$$
\left| (Ty)_i \right|_0 \leq B \int_0^{\omega} (Fy)_i(s)ds + B \sum_{k=1}^{m} I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)),
$$

$$
(Ty)_i(t) \geq A \int_0^{\omega} (Fy)_i(s)ds + A \sum_{k=1}^{m} I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \geq \frac{A}{B} \left| (Ty)_i \right|_0 = \sigma \left| (Ty)_i \right|_0.
\tag{2.6}
$$

Thus, $Ty \in P$, that is, $T(P) \subset P$.

Obviously, the operator $T$ is continuous. Next, we show that $T$ is compact. Let $S \subset E$ be a bounded subset; that is, there exists $d > 0$ such that $|y_i|_0 \leq d$, $i = 1, 2, \ldots, n$ for all $y \in S$. From the continuity of $F, I_k$, $k = 1, 2, \ldots, m$, we have, for all $y \in S$,

$$
\begin{aligned}
\left| (Ty)_i \right|_0 &\leq B \int_0^{\omega} (Fy)_i(s)ds + B \sum_{k=1}^{m} I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \\
&\leq B\omega d \sum_{j=1}^{n} \left( \overline{a}_{ij} d^{\alpha_{ij}} + \overline{b}_{ij} d^{\beta_{ij}} + \overline{c}_{ij} d^{\gamma_{ij}+\delta_{ij}} \right) + Bm E_i =: D_i,
\end{aligned}
\tag{2.7}
$$

where $E_i = \max_{y \in S} |I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k))|$, $i = 1, 2, \ldots, n$.

Therefore, $\|Ty\| = \sum_{i=1}^{n} |(Ty)_i|_0 \leq \sum_{i=1}^{n} D_i =: D$, which implies that $T(S)$ is uniformly bounded.

On the other hand, noticing that

$$
\begin{aligned}
\frac{d}{dt}(Ty)_i(t) &= r_i(t)(Ty)_i(t) + G_i(t, t+\omega)(Fy)_i(t+\omega) - G_i(t, t)(Fy)_i(t) \\
&= r_i(t)(Ty)_i(t) + [G_i(t, t+\omega) - G_i(t, t)](Fy)_i(t) \\
&= r_i(t)(Ty)_i(t) + (Fy)_i(t), \quad t \neq t_k, \ k = 1, 2, \ldots, m.
\end{aligned}
\tag{2.8}
$$

This guarantees that, for each $y \in S$, we have

$$
\left| \frac{d}{dt}(Ty)_i(t) \right| \leq r_i^M D_i + d \sum_{j=1}^{n} \left( a_{ij}^M d^{\alpha_{ij}} + b_{ij}^M d^{\beta_{ij}} + c_{ij}^M d^{\gamma_{ij}+\delta_{ij}} \right) =: \widetilde{D}_i \leq \widetilde{D} = \sum_{i=1}^{n} \widetilde{D}_i,
\tag{2.9}
$$

where $r_i^M = \max_{t\in[0,\omega]} r_i(t)$, $a_{ij}^M = \max_{t\in[0,\omega]} a_{ij}(t)$, $b_{ij}^M = \max_{t\in[0,\omega]} b_{ij}(t)$, $c_{ij}^M = \max_{t\in[0,\omega]} c_{ij}(t)$, $i = 1, 2, \ldots, n$.

Consequently, $T(S)$ is equicontinuous on $J_k$, $k = 0, 1, 2, \ldots, m$, where $J_0 = [0, t_1)$, $J_1 = [t_1, t_2), \ldots, J_{m-1} = [t_{m-1}, t_m)$, $J_m = [t_m, \omega)$. By the Ascoli-Arzela theorem, the function $T : P \to P$ is completely continuous from $P$ to $P$. $\square$

**Lemma 2.2.** *The system* (1.5) *has a positive $\omega$-periodic solution in $P$ if and only if $T$ has a fixed point in $P$.*

*Proof.* For $y \in P$ satisfying $Ty = y$, that is, $(Ty)_i(t) = y_i(t)$, $t \in [0, \omega]$, $i = 1, 2, \ldots, n$, it follows from (2.2) and (2.4) that

$$
\begin{aligned}
\dot{y}_i(t) &= \frac{d}{dt}(Ty)_i(t) \\
&= r_i(t)(Ty)_i(t) + G_i(t, t+\omega)(Fy)_i(t+\omega) - G(t, t)(Fy)_i(t) \\
&= r_i(t)(Ty)_i(t) + [G(t, t+\omega) - G(t, t)](Fy)_i(t) \\
&= r_i(t)(Ty)_i(t) + (Fy)_i(t), \quad t \neq t_k, \ k = 1, 2, \ldots, m.
\end{aligned}
\tag{2.10}
$$

And for $t = t_k$, $k = 1, 2, \ldots, m$,

$$
\Delta y_i(t_k) = y_i(t_k^+) - y_i(t_k^-) = I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)), \quad i = 1, 2, \ldots, n,
\tag{2.11}
$$

which implies that $y(t)$ is a positive $\omega$-periodic solution of (1.5).

Conversely, assume that $y \in P$ is an $\omega$-periodic solution of system (1.5). Then, the system (1.5) can be transformed into

$$\left(\dot{y}_i(t) + r(t)y_i(t)\right)e^{\int_0^t r(v)dv} = \left(Fy\right)_i(t)e^{\int_0^t r(v)dv}, \tag{2.12}$$

that is,

$$\left(y_i(t)e^{\int_0^t r(v)dv}\right)' = \left(Fy\right)_i(t)e^{\int_0^t r(v)dv}. \tag{2.13}$$

So, integrating the above equality from $t$ to $t + \omega$ and noticing that $y_i(t) = y_i(t + \omega)$, we have

$$(Ty)_i(t) = \int_t^{t+\omega} G_i(t,s)y_i(s)\left[\sum_{j=1}^n a_{ij}(s)y_j^{\alpha_{ij}}(s) + \sum_{j=1}^n b_{ij}(s)y_j^{\beta_{ij}}(s - \tau_{ij}(s))\right.$$

$$\left. + \sum_{j=1}^n c_{ij}(s)\int_{-\sigma_{ij}}^0 K_{ij}(\xi)y_i^{\gamma_{ij}}(s + \xi)y_j^{\delta_{ij}}(s + \xi)d\xi\right]ds \tag{2.14}$$

$$+ \sum_{k=1}^m G_i\left(t, t_{k+q_k m}\right)I_{ik}\left(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)\right),$$

where $G_i(t,s) = \exp\left(-\int_t^s r_i(v)dv\right)/(1 - \exp(-\omega\bar{r}_i))$, $t \le s \le t + \omega$, $i = 1, 2, \ldots, n$, and $t_{k+q_k m} = t_k + q_k\omega \in [t, t + \omega]$, where $q_k$ is a positive integer, $k = 1, 2, \ldots, m$, that is, $Ty = y$.

Therefore, $y \in P$ is a fixed point of the operator $T$. The proof of the Lemma is complete. $\square$

## 3. Main Results

**Theorem 3.1.** *Suppose (H1)–(H4) hold, and* $\lim_{|v| \to 0} I_{ik}(v)/|v| = 0$, $k = 1, 2, \ldots, m$, $i = 1, 2, \ldots, n$, *where* $v = (v_1, v_2, \ldots, v_n)$, $|v| = \min_{1 \le i \le n} |v_i|$. *Then system (1.5) has at least one positive $\omega$-periodic solution.*

*Proof.* Let $M_0 = \max_{1 \le i \le n}\{\sum_{j=1}^n (\bar{a}_{ij} + \bar{b}_{ij} + \bar{c}_{ij})\} > 0$. Choose $M_1 \ge M_0$ and $\varepsilon = 1/(B\omega M_1 + Bm) > 0$. Then, there exists $\delta > 0$ such that, for $0 < x < \delta$ and $0 < |v| < \delta$, we have

$$x^{\alpha_{ij}} < \varepsilon, \qquad x^{\beta_{ij}} < \varepsilon, \qquad x^{\gamma_{ij}+\delta_{ij}} < \varepsilon,$$

$$I_{ik}(v) < \varepsilon|v|, \quad k = 1, 2, \ldots, m, \ i, j = 1, 2, \ldots, n. \tag{3.1}$$

Choose $r < \delta$. Let $\Omega_1 = \{y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) \in E \mid |y_i|_0 < r, \ i = 1, 2, \ldots, n\}$. Now, we prove that

$$Ty \not\ge y, \quad \forall y \in P \cap \partial\Omega_1. \tag{3.2}$$

Suppose (3.2) does not hold. Then, there exists some $y \in P \cap \partial\Omega_1$ such that $Ty \geq y$. Since $y \in P \cap \partial\Omega_1$, we have $\sigma|y_i|_0 \leq y_i(t) \leq |y_i|_0$ for $t \in [0, \omega]$, $i = 1, 2, \ldots, n$. From (2.2) and (2.4), it follows that

$$
\begin{aligned}
(Ty_i)(t) &\leq B \int_t^{t+\omega} (Fy)_i(s)ds + B \sum_{k=1}^{m} I_{ik}(y_1(t_k), y_2(t_k), \ldots, y_n(t_k)) \\
&\leq B\omega|y_i|_0 \left( \sum_{i=1}^{n} \overline{a}_{ij} r^{\alpha_{ij}} + \overline{b}_{ij} r^{\beta_{ij}} + \overline{c}_{ij} r^{\gamma_{ij}+\delta_{ij}} \right) + Bm\varepsilon|y_i|_0 \\
&\leq (BM_0\omega\varepsilon + Bm\varepsilon)|y_i|_0 < |y_i|_0,
\end{aligned}
\tag{3.3}
$$

which implies $|(Ty)_i|_0 < |y_i|_0$, a contradiction. Hence, $Ty \ngeq y$ for $y \in P \cap \partial\Omega_1$.

On the other hand, let $m_0 = \min_{1 \leq j \leq n}\{\overline{a}_{i_0 j}\sigma^{\alpha_{ij}} + \overline{b}_{i_0 j}\sigma^{\beta_{ij}}\}$, on the account of (H1), we know $m_0 > 0$. Choose $0 < m_1 \leq m_0$ and $M = 1/A\sigma\omega m_1 > 0$. Then, there exists $R_1 > 0$ such that, for $x > R_1$, we know that

$$
x^{\alpha_{ij}} > M, \quad x^{\beta_{ij}} > M, \quad i, j = 1, 2, \ldots, n.
\tag{3.4}
$$

Choose $R > max\{R_1, r\}$. Let $\Omega_2 = \{y(t) = (y_1(t), y_2(t), \ldots y_n(t)) \in E \mid |y_i|_0 < R, i = 1, 2, \ldots, n\}$. Then, $\partial\Omega_2 = \{y(t) = (y_1(t), y_2(t), \ldots y_n(t)) \in E \mid$ there exist some integers $j_0 (1 \leq j_0 \leq n)$ such that $|y_{j_0}|_0 = R; |y_i|_0 \leq R$ for $i \neq j_0\}$.

Next we show that

$$
Ty \nleq y, \quad \forall y \in P \cap \partial\Omega_2.
\tag{3.5}
$$

In fact, if there exists some $y \in P \cap \partial\Omega_2$ such that $Ty \leq y$ and since $y \in P \cap \partial\Omega_2$, we have $\sigma|y_i|_0 \leq y_i(t) \leq |y_i|_0$, $i = 1, 2, \ldots, n$ for $t \in [0, \omega]$, and there exists some $j_0$ such that $|y_{j_0}|_0 = R$. Therefore, this together with (H1) guarantees that, for $i_0$ $(1 \leq i_0 \leq n)$,

$$
\begin{aligned}
y_{i_0}(t) &\geq (Ty)_{i_0}(t) \geq A \int_t^{t+\omega} (Fy)_{i_0}(s)ds \\
&\geq A\sigma|y_{i_0}|_0 \int_t^{t+\omega} \left( a_{i_0 j_0}(s) y_{j_0}^{\alpha_{i_0 j_0}}(s) + b_{i_0 j_0}(s) y_{j_0}^{\beta_{i_0 j_0}}(s - \tau(s)) \right) ds \\
&\geq A\sigma|y_{i_0}|_0 \int_t^{t+\omega} \left( a_{i_0 j_0}(s)\sigma^{\alpha_{i_0 j_0}}|y_{j_0}|_0^{\alpha_{i_0 j_0}} + b_{i_0 j_0}(s)\sigma^{\beta_{i_0 j_0}}|y_{j_0}|_0^{\beta_{i_0 j_0}} \right) ds \\
&\geq A\sigma|y_{i_0}|_0 \omega M \left( \overline{a}_{i_0 j_0}\sigma^{\alpha_{i_0 j_0}} + \overline{b}_{i_0 j_0}\sigma^{\beta_{i_0 j_0}} \right) \\
&\geq A\sigma\omega M m_0 |y_{i_0}|_0 > |y_{i_0}|_0,
\end{aligned}
\tag{3.6}
$$

which is a contradiction. Thus, (3.5) is satisfied.

From all the above, the condition (i) of **Lemma 1.1** is satisfied. So the operator $T$ has a fixed point in $P \cap (\overline{\Omega}_2 \setminus \Omega_1)$. That is, system (1.5) has at least one positive periodic solution. $\square$

*Remark 3.2.* For any $R > 0$, if we let $\Omega = \{y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) \in E \mid |y_i|_0 < R, i = 1, 2, \ldots, n\}$, then $\partial\Omega = \{y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) \in E \mid \exists \ j_0(1 \leq j_0 \leq n), \ |y_{j_0}|_0 = R, |y_i|_0 \leq R, i \neq j_0\}$. However, in the proof of Theorem 1.1 of [2], it is regarded mistakenly as $\partial\Omega = \{y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) \in E \mid |y_i|_0 = R, i = 1, 2, \ldots, n\}$. Therefore, the proof of its sufficiency is not correct. So the result of [2] is incorrect.

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