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STUDY OF NATURAL CONVECTION IN INCLINED SQUARE ENCLOSURES WITH UNIFORM HEAT GENERATION

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ABSTRACT

Two-dimensional laminar natural convection in an inclined square enclosure with uniform internal heat generation is studied here. The steady-state solutions are obtained for inclination angles of $45^0, 30^0$ *and* 15^0 *and at Rayleigh number of* 1.5×10^5*. For these cases, the two counter-rotating rolls of fluid are present in the cavity. Streamlines, isotherms and heat transfer for these results are compared with the existing experimental results and are found to be in reasonably good agreement. It is found that the location of maximum non-dimensional temperature in the inclined cavity is higher than that for pure conduction case. The maximum non-dimensional temperature in the cavity decreases as the Rayleigh number increases. For* $Ra > 5 \times 10^4$*, the maximum non-dimensional temperature in inclined cavity is almost independent of the inclination angle. It is also observed that the local Nusselt number at the top wall is greater than the pure conduction solution, whereas that for bottom wall it is lower than the Nusselt number for pure conduction. The effect of Rayleigh number and inclination angle on the local Nusselt number and modified local Nusselt number are also studied. For horizontal cavity, at Rayleigh number greater than or equal to* 5×10^4*, periodic solutions are obtained. In this case, two unstable secondary rolls are present near the center of top wall, in addition to the primary rolls. The secondary rolls are dissipated and recreated during one period of oscillation.*

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NOMENCLATURE

C	Heat capacity
L	Dimension of square cavity
Ra	Rayleigh Number
t	Time
u	Velocity component in x-direction
v	Velocity component in y-direction
μ	Viscosity of fluid
ρ_0	Density of fluid
γ	Inclination angle

INTRODUCTION

Buoyancy driven flows in differentially heated cavities have been studied extensively in many experimental and computational works. Natural convection in closed cavities with internal heat generation is important in various fields like-geophysics, astronomy, and post accident heat removal from nuclear reactors and storage of nuclear waste. Natural convection markedly enhances the rate of heat transfer as compared to a purely conductive mechanism. Buoyancy driven flows in differentially heated cavities have been studied extensively in many experimental and computational works. The problem of enclosures with internal heat generation is treated by relatively few investigators.

Two-dimensional steady state laminar natural convection problem in differentially heated rectangular cavity is solved numerically by de Vahl Davis [1] with vorticity-stream function approach. Markatos and Preicleous [2] have studied laminar and turbulent natural convection in a differentially heated square cavity. The range of Rayleigh numbers considered in this study is from 10^3 to 10^6. Two-dimensional steady state governing equations in primitive variables are solved using finite volume approach. Upwind differencing is used for convective terms.

The benchmark numerical solution for the natural convection of air in a square cavity with differentially heated side walls and adiabatic top and bottom walls is provided by de Vahl Davis [3]. The benchmark results are also provided for four different Rayleigh numbers 10^3, 10^4, 10^5 and 10^6 [4]. Streamlines, isotherms and Nusselt number data are provided.

The problem of natural convection in enclosures with uniform internal heat generation has been studied both experimentally and numerically. Lee and Goldstein [5] have investigated this problem experimentally. They have used an enclosure of square cross section $3.81cm \times 3.81cm$ and 25.4cm long. All the four walls forming the square cross section of the cavity are isothermal. The test fluid used is distilled water with NaCl added to increase the electrical conductivity of the water. The salinity of water is less than 0.01 molar, so that the thermo-physical properties of the fluid are close to that of pure water. The results are presented for four inclination angles 0^0, 15^0, 30^0 and 45^0. For each inclination angle, four Rayleigh numbers 10^4, 5×10^4, 10^5 and 1.5×10^5 are considered.

Therefore, there is a definite need to understand the flow in enclosures with internal heat generation. Existing literature suggests that limited study is conducted to understand the natural convection in inclined cavity. Hence, in this work attention is focused in the numerical study of natural convection processes in inclined square cavity for a range of Rayleigh number.

PHYSICAL PROBLEM

Figure 1 shows a square cavity with uniform internal heat generation. The cavity is sufficiently long in z direction for the motion to be assumed two dimensional. The cavity is inclined to the horizontal at an angle γ. It is further assumed that motion is laminar. All the four walls of the cavity are assumed to be isothermal. The heat generation causes density changes within the fluid, which leads to buoyancy-driven flow.

Governing Equations

The Boussinesq approximated unsteady equations for the given problem are,

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

x-momentum equation:

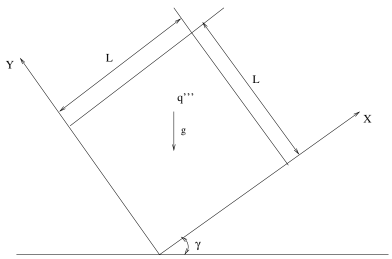

Figure 1: Schematic of cavity with internal generation

$$\rho_0\left(\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \rho_0[1-\beta(T-T_0)]g sin\gamma \quad (2)$$

y-momentum equation:

$$\rho_0\left(\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho_0[1-\beta(T-T_0)]g cos\gamma \quad (3)$$

Energy equation:

$$\rho_0 C\left(\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + q''' \quad (4)$$

At initial time $t = 0$, no slip condition holds at all four walls and temperature throughout the cavity is $T_0 = 0$, which can be expressed mathematically as

$$u = v = 0, T = T_0 = 0 \ \ at \ \ t = 0 \quad (5)$$

At times $t > 0$, no slip condition holds at walls and temperature is specified at wall, which can be written as,

$$u = v = 0, T = 0 \ \ for \ \ x = 0 \ \ and \ \ x = L \quad (6)$$

$$u = v = 0, T = 0 \ \ for \ \ y = 0 \ \ and \ \ y = L \quad (7)$$

Solution Procedure

The governing equations in non-dimensional form for the square cavity with uniform internal heat generation are solved using finite volume method. SIMPLE algorithm and a staggered grid is used for calculation. Upwind differencing scheme is used for discretization convective terms in the governing equations. Implicit scheme is used for discretization with respect to time. The grid used is a uniform grid with 50 points in both x and y directions. Discretized equations are solved using ADI method [6]. The parameters of the numerical solutions are chosen in such a way that a comparison with experiments by Lee and Goldstein [5]. Solutions are obtained for four different Rayleigh numbers - 10^4,5×10^4,10^5 and 1.5×10^5 and four different inclination angles - 0^0,15^0,30^0 and 45^0. Lee and Goldstein [5] did the experiments with distilled water with small amount of NaCl added to raise electrical conductivity of water. The thermo-physical properties of the fluid are very close to that of pure water. For comparison of results with these experiments, the Prandtl number is taken as 7.

It is to be noted that Rayleigh number is defined [7] in terms of the maximum temperature difference in a layer for one-dimensional pure conduction mode and the corresponding characteristic length as,

$$Ra = \frac{g\beta\left(\frac{q^{'''}L^2}{8k}\right)\left(\frac{L}{2}\right)^3}{\alpha\nu} \tag{8}$$

Results and Discussions

The unsteady governing equations are solved subject to initial and boundary conditions of the problem. For a given inclination angle, the steady state solutions are approached after an initial transient. The streamlines and isotherms, for inclination angles 15^0,30^0 and 45^0, obtained by present computation are now presented below.

It is observed in Fig. 2 that there are two strong counter-rotating rolls in the inclined cavity. The hot interior fluid starts to move up nearly parallel to the gravity vector, which divides the whole cross-section approximately in two halves. These two hot flows arrive at almost same position near the top wall. The flows then divide and move downward separately along the cold side walls. At inclination angle of $\gamma = 45^0$ the flow and temperature contours are symmetric about the vertical diagonal of the enclosure.

It is found from the isotherms that the position of the maximum non-dimensional temperature is higher than that in the pure conduction situation. This is because of the influence of natural convection. As the Rayleigh number increases, the position of the maximum non-dimensional temperature moves toward upper top corner. This indicates greater influence of natural convection at higher Rayleigh number, as shown in Fig. 3. With the increase in Rayleigh number, the wall boundary layers become more and more pronounced. Hence the streamline and isotherms near the walls become closely spaced. Figure 5 shows the plot of maximum non-dimensional temperature in the cavity against non-dimensional time for various Rayleigh numbers. The maximum dimensionless temperature in the cavity reduces as the Rayleigh number increases. This can be attributed to the stronger convective motion in the fluid at higher Rayleigh number which causes the higher heat transfer coefficient at the cold walls of the cavity.

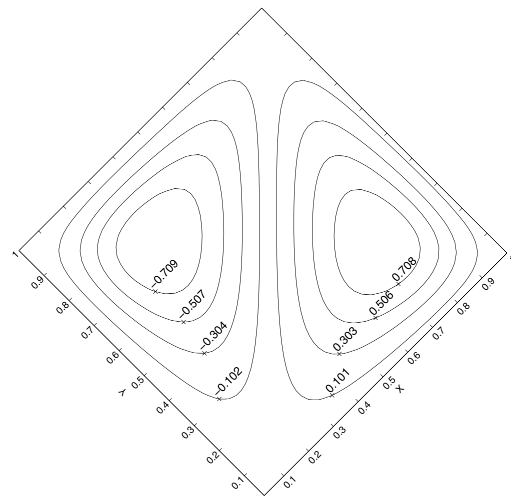

Figure 2: Streamline plot for $\gamma = 45^0$ and $Ra = 10^4$

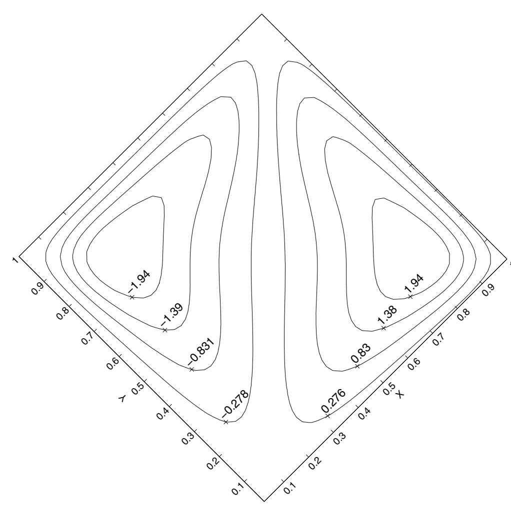

Figure 3: Streamlines for $\gamma = 45^0$ and $Ra = 1.5\times10^5$

The local Nusselt number can be defined based on the maximum temperature difference in pure conduction in one-

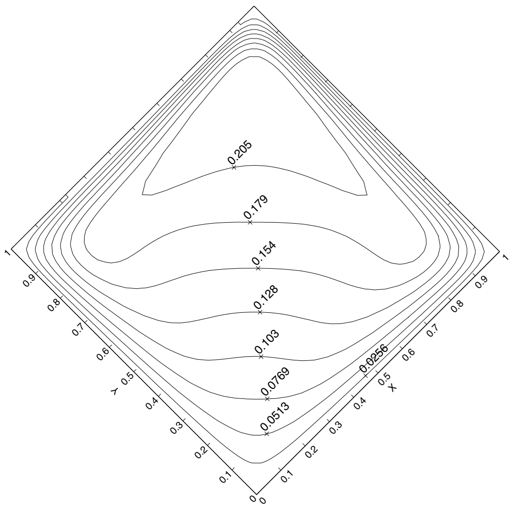

Figure 4: Isotherms for $\gamma = 45^0$ and $Ra = 1.5 \times 10^5$

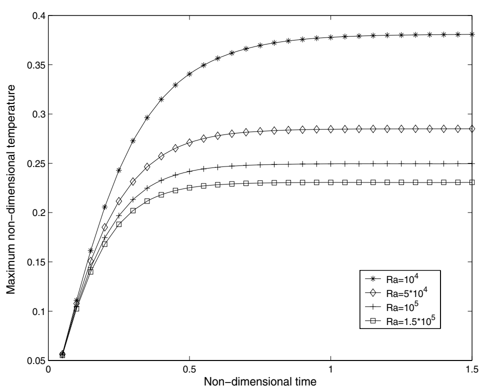

Figure 5: Maximum non-dimensional temperature in the cavity for $\gamma = 45^0$

dimensional layer containing uniform internal heat source and the corresponding characteristic length $\frac{L}{2}$ [5]. In terms of dimensionless variables, the local Nusselt number is expressed as,

$$Nu = -0.5\frac{\partial\theta}{\partial N}_{wall} \tag{9}$$

where, N is the dimensionless wall coordinates and the average Nusselt number is given by,

$$\overline{Nu} = \int_0^1 Nu dN \tag{10}$$

There are four types of local Nusselt numbers: Nu_L, Nu_R, Nu_B and Nu_T corresponding to left, right, bottom and top walls of the cavity, respectively. Figure 6 shows the local Nusselt distribution on four walls at $Ra = 10^5$ and $\gamma = 15^0$. Along the top wall, the local Nusselt number Nu_T is greater than that by conduction only. On the other hand, local Nusselt number along the bottom wall Nu_B is lower than that by conduction only. The values of local Nusselt number along the left and right walls are smaller in the lower portion and greater in the upper portion of the enclosure than those for conduction only. A comparison of the local

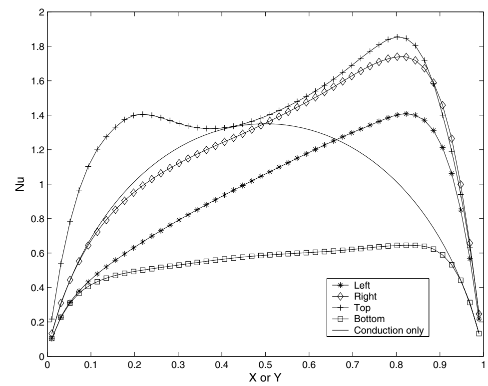

Figure 6: Local Nusselt number distributions on each wall for $\gamma = 15^0$ and $Ra = 10^5$

heat transfer rate with convection, q_{conv}, to that which would occur with two-dimensional pure conduction, $q_{2D,cond}$, is used for investigating the influence of the natural convection. To describe the ratio of these two heat transfer rates, a modified local Nusselt number is defined as [5],

$$Nu^+ = \frac{q_{conv}}{q_{2D,cond}} \tag{11}$$

Hence, the modified local Nusselt number can be expressed as,

$$Nu^+ = \frac{\pi^2 Nu}{16\sum_{n=0}^{\infty}\frac{sin[(2n+1)\pi x/L].\tanh[(2n+1)\pi/2]}{(2n+1)^2}} \tag{12}$$

It is to be noted that the local Nusselt number Nu is related to the absolute magnitude of the heat transfer rate, whereas the modified local Nusselt number Nu^+ is related to the heat transfer relative to that of a two-dimensional conduction case.

As can be seen from Fig. 7, ${Nu_L}^+$ decreases as the inclination angle γ increases. Figure 8 shows that ${Nu_R}^+$ increases in the

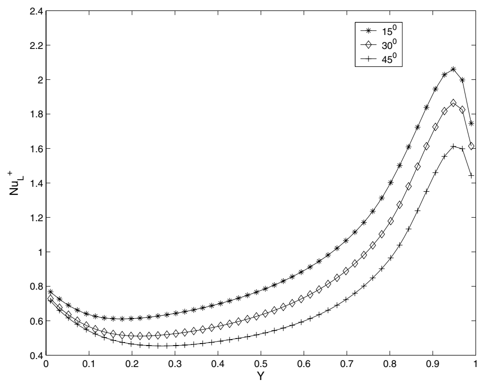

Figure 7: $Nu_L{}^+$ distribution for different values of γ at $Ra = 10^5$

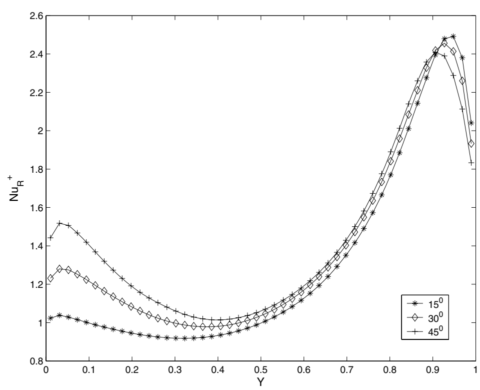

Figure 8: $Nu_R{}^+$ distributions for different values of γ at $Ra = 10^5$

lower portion but remains almost constant in the upper portion of the enclosure as the inclination angle increases.

It is interesting to note that, in case of horizontal cavity, the steady-state is not approached for Rayleigh number equal to or greater than 5×10^4. Figures 9 to 12 show streamlines during twelve time instants during one period of oscillation for $Ra = 10^5$. At $\tau = 0.64$ a pair of secondary counter-rotating rolls is situated near center of the top wall of the cavity in addition to main rolls. Then at $\tau = 0.675$, left secondary roll joins with right main roll and the right secondary roll is destroyed gradually ($\tau = 0.675, \tau = 0.71, \tau = 0.74$). At $\tau = 0.835$ two new secondary rolls appear and gradually enlarge. From $\tau = 0.87$, the whole process repeats itself being reflected at the vertical symmetry line of the cavity until same flow pattern as at $\tau = 0.64$ is reached again. At $\tau = 0.87$, the right secondary roll gets attached to the left main roll and left secondary roll gets destroyed gradually($\tau = 0.900, \tau = 0.925, \tau = 0.950$). At $\tau = 1.045$ new secondary rolls appear near the top wall of cavity.

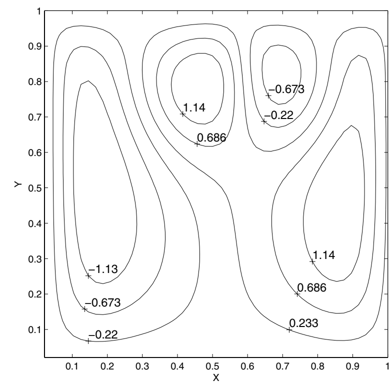

Figure 9: Streamline plot for $\gamma = 0^0$,$Ra = 10^5$, and $\tau = 0.640$

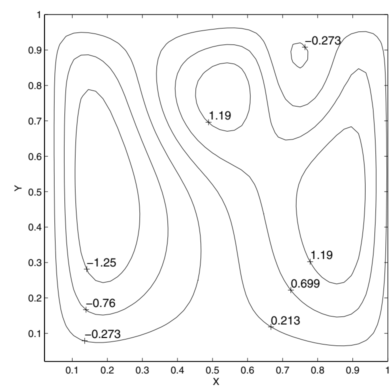

Figure 10: Streamline plot for $\gamma = 0^0$,$Ra = 10^5$, and $\tau = 0.675$

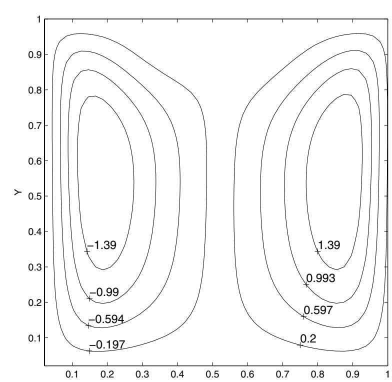

Figure 11: Streamline plot for $\gamma = 0^0$,$Ra = 10^5$, and $\tau = 0.800$

Conclusions

Two-dimensional laminar natural convection in an inclined square enclosure containing uniform internal heat generation is

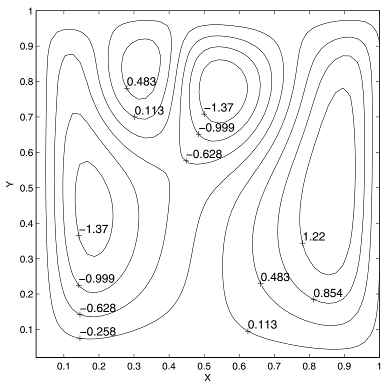

Figure 12: Streamline plot for $\gamma = 0^0$,$Ra = 10^5$, and $\tau = 0.870$

studied here. Two-dimensional unsteady governing equations are solved by implicit scheme and ADI method. SIMPLE algorithm with staggered grid is used for calculation. For inclination angles of 45^0, 30^0 and 15^0 and Rayleigh numbers up to 1.5×10^5, steady-state solutions are approached after initial transient period. For these cases two counter-rotating rolls of the fluid are present in the cavity. In case of horizontal cavity, for Rayleigh number greater than or equal to 5×10^4, *periodic* solutions are obtained. For such cases, two *unstable* secondary rolls are present near the center of top wall, in addition to main rolls. The secondary rolls are dissipated and recreated during one period of oscillation.

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