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Transient Dynamic Analysis of Rotors Using SMAC Techniques: Part 1, Formulation

A new method is introduced for performing transient dynamic analysis of rotor systems using a Successive Merging and Condensation (SMAC) technique. This approach can be applied to rotor analysis problems formulated with the finite element method. Condensation is done on the partitioned equations of motions for an element, and the result is merged into the next element's equations of motion. Such manipulations result in a reduced size for the system's matrices, producing a computationally more efficient scheme. After the boundary conditions are applied, a time-marching scheme provides the transient solution at each time step.

1 Introduction

In general, the transient response of mechanical systems involving chain-like and branching systems, such as rotors and gear train systems, can be obtained by using the methodology of finite elements combined with a suitable scheme to solve the equations of motion. In rotor technology, the finite element method (FEM) is employed for spatial discretization of rotors (Lalanne and Ferraris, 1990; Nelson and McVaugh, 1976; Ruhl and Booker, 1972). This discretization approximates the infinite-dimensional spatial domain of the rotor system by a finite number of degrees of freedom (i.e., d.o.f.) coupled with suitable shape functions. The commonly used schemes to solve the discretized equations of motions are: Modal Superposition, Time-marching schemes, and Transfer Matrix Methods (TMM). These methods generally involve a large number of computations that are very time-consuming, especially when dealing with systems with a large number of d.o.f. To this end, a new method is introduced in this paper for performing transient dynamic rotor analysis that is computationally more efficient than existing methods.

In the modal superposition method, the algebraic eigenvalue problem associated with a discretized model is solved, which allows decoupling the equations of motion by way of a coordinate transformation. Each decoupled equation then can be solved separately to obtain the modal responses, which can finally be transformed into the physical response. Modal superposition methods provide for a qualitative as well as quantitative understanding of the system's behavior. For a nongyroscopic, proportional damping system model, or a conservative gyroscopic one, the associated eigenvalue problem is a real eigenvalue problem (Meirovitch, 1975 and 1980). However, for a general model it is required to deal with a complex eigenvalue problem, thereby increasing the number of computations required for solution. Also, if the number of d.o.f. of the system is large, which is usually the case, the transient analysis using a modal superposition technique involves resolving a large-scale eigenvalue problem for different speeds of the rotor. This is very costly in terms of computer resources. Yee and Tsuei (1990) have presented a Component Modal Synthesis Method for performing the transient analysis of linear systems. This method divides the system into subsystems, performs the modal analysis on the subsystems, and applies compatibility conditions. The method has been reported for systems with proportional damping.

Alternatively, one might directly use a time-marching scheme, such as Runge-Kutta, Wilson-θ, or Houbolt methods (Bathe and Wilson, 1976) to get the response in the physical coordinates. Due to the large size of the matrices, the ordinary time-marching schemes generally take large CPU times to integrate the equations of motion. A common way of reducing the size of these matrices is by using the well-known Guyan reduction scheme (Guyan, 1965). This is suitable when the inertia associated with some coordinates is much smaller than the inertia associated with other coordinates. The smaller inertia, and the forces acting on the corresponding coordinates, are then neglected. Although this results in a reduction of the computation time, the accuracy of the obtained response might be affected by the exclusion of small inertias and forces.

Transfer Matrix Methods (Rao, 1983) deal with relatively small matrices. Although this method (Degen et al., 1985; Dokainish, 1972; Mucino and Pavelic, 1981; Ohga et al., 1983) was initially used only in frequency domain, Rao et al. (1987) recently used a TMM in the time domain. In this method, the relations between state vectors of different stations are found using transfer matrices. The state vector consists of the displacement variables and their time derivatives up to the 4th order. At each time step, boundary conditions are used to solve for the displacement variables. Because of the inclusion of time derivatives of displacement variables, the size of the state vectors becomes large. Kumar et al. (1986) combined a suitable numerical scheme with TMM to perform the response analysis of a mass-spring-damper system in the time domain. At the beginning of each time step, the application of the numerical

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scheme establishes relationships between the displacement variables and their derivatives. For this reason, the state vectors do not contain any time derivatives of the displacement variables. This makes the size of the state vector smaller than the one required in the method proposed by Rao et al. (1987). Another recently developed TMM algorithm (Subbiah, 1988), referred to as Transient Properties Transfer Approach (TPTA), marches out the solution of rotor problems in time domain. In TPTA, the dynamical properties (forces and displacements) of one node are transferred to the next node, with the maximum size of the matrices being twice the number of d.o.f. at each node (usually two displacements and two rotations). The advantage in using this approach is that storage space and computation time requirements are much less than those required by time-marching schemes and by the modal superposition method. Furthermore, the TMM as well as direct time-marching schemes, in general, can handle nonconservative gyroscopic and nongyroscopic systems and are capable of analyzing nonlinear systems. As efficient as this recently developed approach is, a complete transient rotor analysis might still require a large CPU time.

The method introduced in this paper, the *Successive Merging and Condensation* (SMAC) method, is suitable for transient dynamic rotor analysis and is capable of handling linear and nonlinear, as well as conservative and nonconservative, rotor systems. Condensation and merging manipulations are performed with the purpose of reducing the size of matrices being handled. The resulting matrices are of order equal to the number of degrees of freedom modeled at each node of the rotor's FE model. The proposed SMAC method involves the computation and storage of the coefficient matrices. These, for each time step, are then multiplied by the modified column matrices obtained from the merging stage. The mathematical formulation, and the algorithm for proper computer implementation, of the SMAC method are presented in this paper.

2 Equations of Motion

2.1 Finite Element Formulation. A rotor consists of a shaft, disks, and bearings which can be modeled using an FEM formulation as n discrete elements or subsystems (Fig. 1). The end points of an element are called nodes (or stations), and they are located such that each disk or bearing of the rotor coincides with a node. Each element is assumed to be continuous in diameter, with discontinuity of diameter allowable only

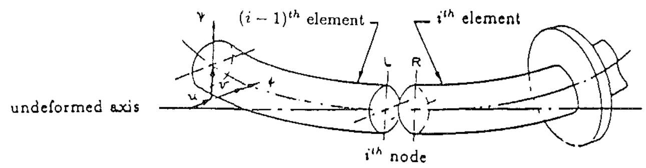

Fig. 1 Model of a typical rotor system

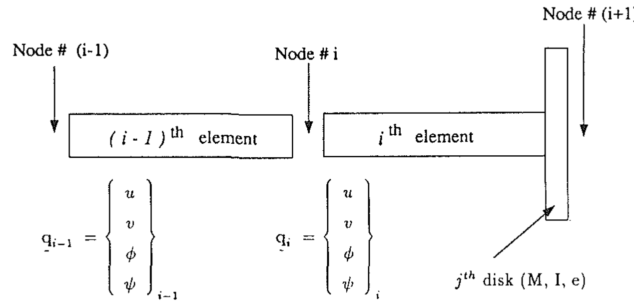

Fig. 2 Degrees of freedom for discretized model

between elements. The degrees of freedom for a typical finite element formulation (Nelson and McVaugh, 1976) are shown in Fig. 2.

The equations of motion for an element take the form

$$\mathbf{M}^i\ddot{\mathbf{x}}^i(t)+\mathbf{C}^i\dot{\mathbf{x}}^i(t)+\mathbf{K}^i\mathbf{x}^i(t)=\mathbf{F}^i(t) \tag{1}$$

where $\mathbf{M}^i$ and $\mathbf{K}^i$ denote the symmetric mass and stiffness matrices, respectively, for the ith element. For a physical system, $\mathbf{M}^i$ is a positive definite matrix, and $\mathbf{K}^i$ is a semipositive definite matrix. For a conservative model, $\mathbf{C}^i$ represents the skew symmetric gyroscopic matrix, and for a nonconservative model which includes damping, $\mathbf{C}^i$ represents a positive definite, or positive semi-definite, matrix. The vector $\mathbf{F}^i(t)$ represents the generalized force vectors for the ith element, which is partially due to the eccentricities of the shafts and the disks. The vector $\mathbf{x}^i(t)$ consists of displacement variables for the ith element.

For convenience, the following vector and matrix partitions are introduced:

$$\mathbf{M}^i=\begin{bmatrix}\mathbf{m}_{11} & \mathbf{m}_{12}\\ \mathbf{m}_{21} & \mathbf{m}_{22}\end{bmatrix}^i,\ \mathbf{C}^i=\begin{bmatrix}\mathbf{c}_{11} & \mathbf{c}_{12}\\ \mathbf{c}_{21} & \mathbf{c}_{22}\end{bmatrix}^i,\ \mathbf{K}^i=\begin{bmatrix}\mathbf{k}_{11} & \mathbf{k}_{12}\\ \mathbf{k}_{21} & \mathbf{k}_{22}\end{bmatrix}^i \tag{2}$$

Nomenclature

- $\mathbf{C}$ = damping matrix
- $\mathbf{D}, \mathbf{E}, \mathbf{Y}$ = matrices of size 4×4
- $\mathbf{F}(t)$ = generalized force vector
- $\mathbf{G}$ = gyroscopic matrix
- $\mathbf{K}$ = stiffness matrix
- ${}^i\mathbf{K}'_{jk}, {}^i\mathbf{K}_{jk}$ = submatrices of size 4×4
- $\mathbf{M}$ = mass matrix
- $\mathbf{a}, \mathbf{b}$ = vectors, dependent upon $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$ and numerical scheme
- $\mathbf{c}_{jk}$ = submatrix of damping matrix $\mathbf{C}^i$
- $\mathbf{f}_i$ = generalized force vector for ith node
- $\mathbf{f}_i^{ext}$ = external force at ith node, due to unbalance
- $\mathbf{g}$ = vector of dimension 4
- $\mathbf{k}_{jk}$ = submatrix of stiffness matrix $\mathbf{K}^i$
- $\mathbf{m}_{jk}$ = submatrix of mass matrix $\mathbf{M}^i$
- n = number of elements in rotor system
- $\mathbf{q}_i$ = state vector of displacement for ith node
- $\dot{\mathbf{q}}$ = time derivative of $\mathbf{q}$
- $\ddot{\mathbf{q}}$ = second time derivative of $\mathbf{q}$
- $\mathbf{s}_1, \mathbf{s}_2$ = vectors of dimension 4
- t_o = initial time of integration
- $\mathbf{v}$ = vector, dependent upon $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{a}$, $\mathbf{b}$
- $\mathbf{x}(t)$ = state vector of displacement at time t
- $\dot{\mathbf{x}}$ = time derivative of $\mathbf{x}$
- $\ddot{\mathbf{x}}$ = second time derivative of $\mathbf{x}$
- $\mathbf{z}$ = a vector of dimension 4
- Δt = time step for the numerical integration
- θ = constant in the Wilson-θ method
- α, β = constants, dependent upon Δt and the numerical scheme

Subscript

- i = used for showing association with the ith node

Superscript

- i = used for showing association with the ith element

Presuperscript

- R or L = shows the association of the vector with the right or left side elements of the node described by the subscript

$$\mathbf{x}^i(t)=\begin{Bmatrix}\mathbf{q}_i(t)\\ \mathbf{q}_{i+1}(t)\end{Bmatrix}^i,\ \mathbf{F}^i(t)=\begin{Bmatrix}\mathbf{f}_i(t)\\ \mathbf{f}_{i+1}(t)\end{Bmatrix}^i \tag{3}$$

where $\mathbf{q}_i(t)$ and $\mathbf{f}_i(t)$ refer to the ith nodal displacements and generalized forces, respectively. In FEM, valid matrices are formulated for all the elements and then assembled into global matrices to get the discretized equations of motion for the rotor system. These equations still remain to be solved. The statement of the problem can be posed as follows:

Given: $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$

Find: $\mathbf{x}(t+\Delta t)$, $\dot{\mathbf{x}}(t+\Delta t)$ and $\ddot{\mathbf{x}}(t+\Delta t)$

A solution can then be obtained by applying a suitable scheme, such as modal superposition, step-by-step time-marching procedures, TMM, or the proposed SMAC method, which is presented next.

3 Proposed SMAC Method

The proposed SMAC algorithm is an alternate procedure for performing the integration process in rotor dynamics. This method makes use of the fact that the displacements of one node can be expressed in terms of the displacements of the next node, thereby allowing the elimination of the unknown forces from the original equations for the element. Consequently, the size of the matrices being dealt with becomes smaller, half the size required by TMM, thus significantly reducing the computational requirements.

3.1 Formulation of the Method. The formulation begins by expressing the velocity and the acceleration coordinates in terms of the associated displacement coordinates. There are many existing numerical schemes (e.g., Wilson-θ method, Houbolt methods) which allow writing of $\ddot{\mathbf{x}}(t+\Delta t)$ and $\dot{\mathbf{x}}(t+\Delta t)$ in the following form:

$$\ddot{\mathbf{x}}(t+\Delta t)=\alpha\mathbf{x}(t+\Delta t)+\mathbf{a}(t) \tag{4}$$

$$\dot{\mathbf{x}}(t+\Delta t)=\beta\mathbf{x}(t+\Delta t)+\mathbf{b}(t) \tag{5}$$

where α and β are constants dependent upon the time-step size (Δt) chosen, and where $\mathbf{a}(t)$ and $\mathbf{b}(t)$ are vectors dependent upon $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, and $\ddot{\mathbf{x}}(t)$.

Substitution of Eq. (4) and Eq. (5) into Eq. (1) results in the following:

$$(\alpha\mathbf{M}^i+\beta\mathbf{C}^i+\mathbf{K}^i)\mathbf{x}^i(t+\Delta t)+\mathbf{M}^i\mathbf{a}^i(t)+\mathbf{C}^i\mathbf{b}^i(t)=\mathbf{F}^i(t+\Delta t) \tag{6}$$

This can further be written as

$$\begin{bmatrix}{}^i\mathbf{K}'_{11} & {}^i\mathbf{K}'_{12}\\ {}^i\mathbf{K}'_{21} & {}^i\mathbf{K}'_{22}\end{bmatrix}\begin{Bmatrix}\mathbf{q}_i(t+\Delta t)\\ \mathbf{q}_{i+1}(t+\Delta t)\end{Bmatrix}=\begin{Bmatrix}{}^R\mathbf{f}_i(t+\Delta t)-{}^R\mathbf{v}_i(t)\\ {}^L\mathbf{f}_{i+1}(t+\Delta t)-{}^L\mathbf{v}_{i+1}(t)\end{Bmatrix} \tag{7}$$

where

$$\begin{bmatrix}{}^i\mathbf{K}'_{11} & {}^i\mathbf{K}'_{12}\\ {}^i\mathbf{K}'_{21} & {}^i\mathbf{K}'_{22}\end{bmatrix}=\alpha\mathbf{M}^i+\beta\mathbf{C}^i+\mathbf{K}^i$$

$$\begin{Bmatrix}{}^R\mathbf{v}_i(t)\\ {}^L\mathbf{v}_{i+1}(t)\end{Bmatrix}=\mathbf{M}^i\mathbf{a}^i(t)+\mathbf{C}^i\mathbf{b}^i(t)$$

$$\mathbf{a}^i(t)=\begin{Bmatrix}\mathbf{a}_i(t)\\ \mathbf{a}_{i+1}(t)\end{Bmatrix}^i,\ \mathbf{b}^i(t)=\begin{Bmatrix}\mathbf{b}_i(t)\\ \mathbf{b}_{i+1}(t)\end{Bmatrix}^i \tag{8}$$

with $\mathbf{a}^i(t)$ and $\mathbf{b}^i(t)$ partitioned consistent with $\mathbf{x}^i(t)$. Superscripts R and L represent right- and left-side elements, respectively, of the ith node.

In order to proceed, the following claim is made and then proved:

Claim: If the following relationship exists for the ith node

$$\mathbf{Y}_i\mathbf{q}_i(t+\Delta t)=\{{}^L\mathbf{f}_i(t+\Delta t)-{}^L\mathbf{v}_i(t)\}-\mathbf{z}_i(t+\Delta t) \tag{9}$$

where $\mathbf{Y}_i$ and $\mathbf{z}_i(t+\Delta t)$ are known, then the same type of relationship will exist for the $(i+1)$th node.

Proof: Merging Eq. (9) into Eq. (7) yields the following expression:

$$\begin{bmatrix}{}^i\mathbf{K}'_{11}+\mathbf{Y}_i & {}^i\mathbf{K}'_{12}\\ {}^i\mathbf{K}'_{21} & {}^i\mathbf{K}'_{22}\end{bmatrix}\begin{Bmatrix}\mathbf{q}_i(t+\Delta t)\\ \mathbf{q}_{i+1}(t+\Delta t)\end{Bmatrix}=\begin{Bmatrix}{}^R\mathbf{f}_i(t+\Delta t)+{}^L\mathbf{f}_i(t+\Delta t)-({}^R\mathbf{v}_i(t)+{}^L\mathbf{v}_i(t))-\mathbf{z}_i(t+\Delta t)\\ {}^L\mathbf{f}_{i+1}(t+\Delta t)-{}^L\mathbf{v}_{i+1}(t)\end{Bmatrix} \tag{10}$$

where the unknown forces ${}^R\mathbf{f}_i(t)$ and ${}^L\mathbf{f}_i(t)$ at the ith node are related to the known external nodal force $\mathbf{f}_i^{ext}(t)$ by

$${}^R\mathbf{f}_i(t)+{}^L\mathbf{f}_i(t)=\mathbf{f}_i^{ext}(t) \tag{11}$$

Similarly, the sum of ${}^R\mathbf{v}_i(t)$ and ${}^L\mathbf{v}_i(t)$ can be expressed as $\mathbf{v}_i(t)$,

$${}^R\mathbf{v}_i(t)+{}^L\mathbf{v}_i(t)=\mathbf{v}_i(t)=[\mathbf{m}_{21}^{i-1}\quad \mathbf{m}_{22}^{i-1}+\mathbf{m}_{11}^i\quad \mathbf{m}_{12}^i]\begin{Bmatrix}\mathbf{a}_{i-1}(t)\\ \mathbf{a}_i(t)\\ \mathbf{a}_{i+1}(t)\end{Bmatrix}+[\mathbf{c}_{21}^{i-1}\quad \mathbf{c}_{22}^{i-1}+\mathbf{c}_{11}^i\quad \mathbf{c}_{12}^i]\begin{Bmatrix}\mathbf{b}_{i-1}(t)\\ \mathbf{b}_i(t)\\ \mathbf{b}_{i+1}(t)\end{Bmatrix} \tag{12}$$

thereby reducing the number of computations required by the analysis. Combining Eqs. (11) and (12) with Eq. (10) yields the expression:

$$\begin{bmatrix}{}^i\mathbf{K}_{11} & {}^i\mathbf{K}_{12}\\ {}^i\mathbf{K}_{21} & {}^i\mathbf{K}_{22}\end{bmatrix}\begin{Bmatrix}\mathbf{q}_i(t+\Delta t)\\ \mathbf{q}_{i+1}(t+\Delta t)\end{Bmatrix}=\begin{Bmatrix}\mathbf{f}_i^{ext}(t+\Delta t)-\mathbf{v}_i(t)-\mathbf{z}_i(t+\Delta t)\\ {}^L\mathbf{f}_{i+1}(t+\Delta t)-{}^L\mathbf{v}_{i+1}(t)\end{Bmatrix} \tag{13}$$

where the following notation is used:

$$\begin{aligned}{}^i\mathbf{K}'_{11}+\mathbf{Y}_i&={}^i\mathbf{K}_{11}\\ {}^i\mathbf{K}'_{12}&={}^i\mathbf{K}_{12}\\ {}^i\mathbf{K}'_{21}&={}^i\mathbf{K}_{21}\\ {}^i\mathbf{K}'_{22}&={}^i\mathbf{K}_{22}\end{aligned} \tag{14}$$

Using Eq. (13), $\mathbf{q}_i(t+\Delta t)$ can be expressed in terms of $\mathbf{q}_{i+1}(t+\Delta t)$

$$\mathbf{q}_i(t+\Delta t)=-{}^i\mathbf{K}_{11}^{-1}{}^i\mathbf{K}_{12}\mathbf{q}_{i+1}(t+\Delta t)+{}^i\mathbf{K}_{11}^{-1}(\mathbf{f}_i^{ext}(t+\Delta t)-\mathbf{v}_i(t)-\mathbf{z}_i(t+\Delta t)) \tag{15}$$

Equation (13) can now be used in conjunction with Eq. (15) to yield the following expression for $\mathbf{q}_{i+1}(t+\Delta t)$:

$$({}^i\mathbf{K}_{22}-{}^i\mathbf{K}_{21}{}^i\mathbf{K}_{11}^{-1}{}^i\mathbf{K}_{12})\mathbf{q}_{i+1}(t+\Delta t)=({}^L\mathbf{f}_{i+1}(t+\Delta t)-{}^L\mathbf{v}_{i+1}(t))-{}^i\mathbf{K}_{21}{}^i\mathbf{K}_{11}^{-1}(\mathbf{f}_i^{ext}(t+\Delta t)-\mathbf{v}_i(t)-\mathbf{z}_i(t+\Delta t)) \tag{16}$$

It can be seen that Eq. (16) is of the same form as Eq. (9), where

$$z_{i+1}(t+\Delta t)={}^i\mathbf{K}_{21}{}^i\mathbf{K}_{11}^{-1}(\mathbf{f}_i^{ext}(t+\Delta t)-\mathbf{v}_i(t)-\mathbf{z}_i(t+\Delta t)) \tag{17}$$

and

$$\mathbf{Y}_{i+1}=({}^i\mathbf{K}_{22}-{}^i\mathbf{K}_{21}{}^i\mathbf{K}_{11}^{-1}{}^i\mathbf{K}_{12}) \tag{18}$$

thus validating the claim previously made.

It should be noted that Eq. (9) is also valid for $i=1$, where the following conditions hold:

$$\mathbf{Y}_1 = \mathbf{0}$$
$$^{L}\mathbf{f}_1(t+\Delta t) = \mathbf{0}$$
$$^{L}\mathbf{v}_1(t) = \mathbf{0}$$
$$\mathbf{z}_1(t+\Delta t) = \mathbf{0} \tag{19}$$

allowing for the determination of all $\mathbf{z}_i(t+\Delta t)$ and $\mathbf{Y}_i$ ($i=1, 2, \ldots, n+1$). For $i=n+1$, Eq. (9) can also be written as

$$\mathbf{Y}_{n+1}\mathbf{q}_{n+1}(t+\Delta t) = \{^{L}\mathbf{f}_{n+1}(t+\Delta t) - {}^{L}\mathbf{v}_{n+1}(t)\} - \mathbf{z}_{n+1}(t+\Delta t) \tag{20}$$

with the following boundary conditions for the rotor:

$$^{L}\mathbf{f}_{n+1}(t+\Delta t) = \mathbf{f}^{ext}_{n+1}(t+\Delta t) \tag{21}$$

$$^{L}\mathbf{v}_{n+1}(t) = [\mathbf{m}^{n}_{12} \quad \mathbf{m}^{n}_{22}]\begin{Bmatrix} \mathbf{a}_n(t) \\ \mathbf{a}_{n+1}(t) \end{Bmatrix} + [\mathbf{c}^{n}_{12} \quad \mathbf{c}^{n}_{22}]\begin{Bmatrix} \mathbf{b}_n(t) \\ \mathbf{b}_{n+1}(t) \end{Bmatrix} \tag{22}$$

hence, $\mathbf{q}_{n+1}(t+\Delta t)$ can be calculated. Additionally, all $\mathbf{q}_i(t+\Delta t)$ ($i=n, n-1, \ldots, 1$) can be calculated as well using Eq. (15). It is important to mention that all matrices $\mathbf{Y}_i$ and $\mathbf{K}_i$ ($i=1, 2, \ldots, n+1$) are calculated only once for a constant rotational speed of the system.

3.2 Computer Implementation. The algorithm for the introduced SMAC method followed in the computer implementation is given below:

(1) Input the data for the rotor system.
(2) Obtain the equations of motion for each finite element formulations.
(3) Get the initial conditions (i.e., $\mathbf{q}_i(t_0)$ and $\dot{\mathbf{q}}_i(t_0)$), and calculate $\ddot{\mathbf{q}}_i(t_0)$, for $i=1, 2, \ldots, n+1$.
(4) Set the value of Δt, and calculate the values of α and β.
(5) Form all $\mathbf{K}'$ matrices, (Eq. (7)) for $i=1, 2, \ldots n$.
(6) Set $^{1}\mathbf{K}_{11} = {}^{1}\mathbf{K}'_{11}$
Form $^{i-1}\mathbf{D}$, $^{i}\mathbf{K}_{11}$, $^{i-1}\mathbf{E}$, for $i=2, 3, \ldots n+1$ as follows

$$^{i-1}\mathbf{D} = {}^{i-1}\mathbf{K}'_{21}\,{}^{i-1}\mathbf{K}^{-1}_{11}$$
$$^{i}\mathbf{K}_{11} = {}^{i}\mathbf{K}'_{11} + {}^{i-1}\mathbf{K}'_{22} - {}^{i-1}\mathbf{D}\,{}^{i-1}\mathbf{K}'_{12}$$
$$^{i-1}\mathbf{E} = {}^{i-1}\mathbf{K}^{-1}_{11}\,{}^{i-1}\mathbf{K}'_{12}$$

(7) For each time step:
  (*a*) Depending upon the numerical method utilized and the values of $\mathbf{q}_i(t)$, $\dot{\mathbf{q}}_i(t)$ and $\ddot{\mathbf{q}}_i(t)$, calculate $\mathbf{a}_i(t)$ and $\mathbf{b}_i(t)$ for $i=1, 2, \ldots n+1$.
  (*b*) Calculate $\mathbf{v}_i(t)$ as given in Eq. (12), for $i=1, 2, \ldots n+1$ (for $i=1$, all the quantities $\mathbf{m}'_s$, $\mathbf{c}'_s$ with superscript $(i-1)$ are equal to zero).
  (*c*) Calculate all external forces $\mathbf{f}^{ext}_i(t+\Delta t)$
  (*d*) Set $\mathbf{g}_1 = \mathbf{f}^{ext}_1(t+\Delta t) + \mathbf{v}_1(t)$.
  Get $\mathbf{g}_i$, for $i=2, 3, \ldots n+1$ as follows:
  $$\mathbf{g}_i = {}^{i}\mathbf{K}^{-1}_{11}[\mathbf{f}^{ext}_i(t+\Delta t) + \mathbf{v}_i(t) - {}^{i-1}\mathbf{D}\,\mathbf{g}_{i-1}]$$
  (*e*) Set $\mathbf{q}_{n+1}(t+\Delta t) = \mathbf{g}_{n+1}$.
  Get $\mathbf{g}_i(t+\Delta t)$, for $i=n, n-1, \ldots 2, 1$ as follows:
  $$\mathbf{q}_i(t+\Delta t) = \mathbf{g}_i - {}^{i}\mathbf{E}\,\mathbf{q}_{i+1}(t+\Delta t)$$
  (*f*) Calculate $\ddot{\mathbf{q}}_i(t+\Delta t)$ and $\dot{\mathbf{q}}_i(t+\Delta t)$, for $i=1, 2, \ldots n+1$, Eq. (4) and Eq. (5).
  (*g*) Go to step (7*a*) after setting the value of time t to $t+\Delta t$.

A flowchart for the proposed SMAC method is presented in Fig. 3. In this particular implementation the equations of motion were obtained using a FEM formulation corresponding to a conservative system (Lalanne and Ferraris, 1990; Nelson and McVaugh, 1976). The Wilson-θ numerical scheme was applied in the implementation of Step 7 [Eqs. (4) and (5)] of the algorithm. The algorithm developed is applicable to single-shaft rotor systems, and development of the appropriate subroutines for extension to multishaft rotor system is currently under way.

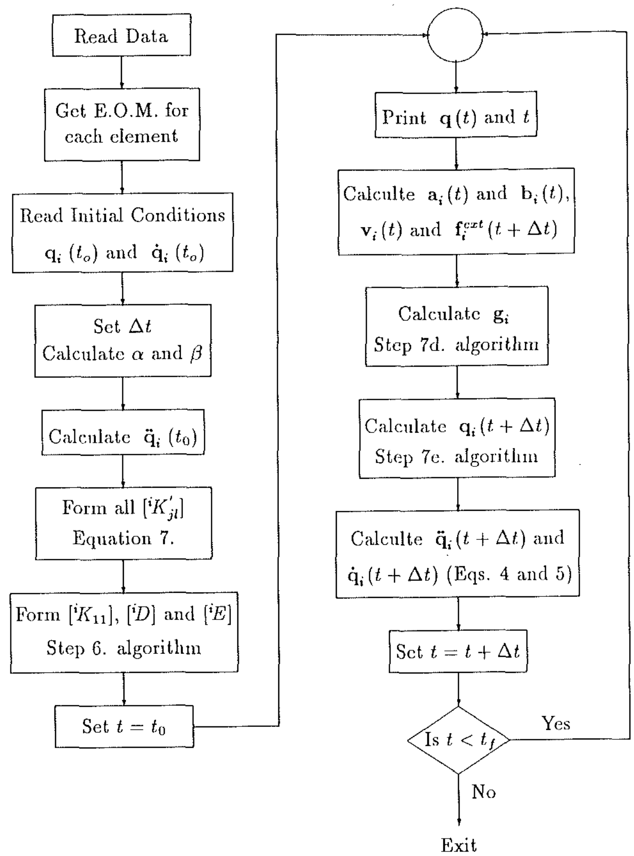

Fig. 3 Flow-chart for the proposed SMAC algorithm

4 Implementation Issues

Two issues of importance in the evaluation of a numerical method for time-dependent phenomena are: stability and time-step size. The numerical stability of the proposed SMAC method is solely dependent upon the numerical scheme used in solving Eq. (4) and Eq. (5). Since the Wilson-θ method was utilized, in order to ensure accuracy and unconditional stability an appropriate value of the parameter θ should be selected (Bathe and Wilson, 1973).

Selection of the time-step size is important since accuracy and CPU time increase by decreasing it. In transient response, the selection of the time step basically depends on the rotor speed and the frequencies of the lower modes of the system. The use of FEM formulations results in inaccurate magnitudes for the higher modes, indicating that smaller time steps may not always produce more accurate results. Numerical examples and performance evaluations are presented in Part 2 of this paper (Ratan and Rodriguez, 1992).

5 Summary

A new method for the transient dynamic analysis of rotor systems has been presented. The proposed method, referred to as the Successive Merging and Condensation (SMAC) method, employs condensation and merging techniques on the equations of motion of a rotor system. These manipulations of the FEM-based matrices make the introduced scheme computationally more efficient than existing algorithms. A Claim, and its Proof, have been presented as the basis for the formulation of the proposed algorithm. The SMAC method is capable of handling linear as well as nonlinear, and conservative as well as nonconservative gyroscopic systems. The formulation presented is appropriate for single-shaft systems, and extensions to multishaft systems are currently under study.

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