Proceedings of the 9th Biennial ASME Conference on Engineering Systems Design and Analysis
ESDA2008
July 7-9, 2008, Haifa, Israel

ESDA2008-59092

APPLICATION OF THE COMPLEX ENVELOPE VECTORIZATION TO A BOUNDARY ELEMENT FORMULATION

Oliviero Giannini **Aldo Sestieri**
Dipartimento di Meccanica e Aeronautica
Università di Roma "La Sapienza", Italy

ABSTRACT

The complex envelope vectorization (CEV) is a recent method that has been successfully applied to structural and internal acoustic problems. Unlike other methods proposed in the last two decades to solve high frequency problems, CEV is not an energy method, although it shares with all the other techniques a variable transformation of the field variable. By such transformation involving a Hilbert transform, CEV allows the representation of a fast oscillating signal through a set of low oscillating signals. Thanks to such transformation it is possible to solve a high frequency dynamic problem at a computational cost that is lower than that required by finite elements. In fact, by using finite elements, a high frequency problem usually implies large matrices. On the contrary the CEV formulation is obtained by solving a set of linear problems of highly reduced dimensions. Although it was proved that CEV is in general a successful procedure, it was shown that it is particularly appropriate when the modes of the system have a negligible role on the solution. Moreover, the numerical advantage of the CEV formulation is much more pronounced when full matrices are used. Thus, for the first time it is applied to a boundary element formulation (BEM). Both external and internal acoustic fields of increasing complexity are considered: the internal and external field generated by a pulsating sphere; the external field of a forced box, where the velocity field is determined by finite elements; a set of 4 plates that form an open cavity. The results are compared with those obtained by a BEM procedure (SYSNOISE), highlighting the good quality of the proposed approach. An estimate of the computational advantage is also provided. Finally it is worthwhile to point out that the reduction of the BE matrices allows for an in-core solution even for large problems.

1. INTRODUCTION

Several techniques, alternative to SEA, have been proposed in the last decade to analyze high frequency problems, e.g. the vibration conductivity method [1,2,3] and other procedures related to it, e.g. [4], the wave intensity analysis [5], the asymptotic scaled modal analysis [6]. Most of these methods use a variable transformation, i.e. the physical variable, displacement or pressure, is replaced by some kind of energy average, that is in general more convenient from a numerical point of view and for systems subjected to uncertainties in any physical or geometrical parameter. Although energy is a very convenient variable because it well describes both the structural and acoustic fields, it introduces several problems, similar to those encountered in Statistical Energy Analysis.

The complex envelope vectorization (CEV) is another of these methods [7], but unlike SEA and other referenced procedures, the used variable in CEV is not the energy but a complex envelope variable defined through the Hilbert transform. The used transformation maps the fast oscillating response of high frequency problems into an "envelope" response characterized by a low wavenumber content, and a new formulation, computationally more efficient, is obtained. CEV presents some advantages over the energy methods, i.e. (i) the boundary conditions of the envelope problem are directly determined from the physical boundary conditions; (ii) the forcing term of the envelope equation can be directly computed from the physical load avoiding the need to estimate the input power as in the energy-based methods; (iii) unlike SEA providing a solution averaged in space, CEV gives a local information.

In [7] the theoretical formulation of CEV was presented but, for the reader convenience, it will be briefly resumed here. In that paper the successful applications of the method were deeply discussed as well as its limitations. Particularly it was pointed out that that CEV provides in general very good results, but it is particularly appropriate when the modes of the system have a negligible role on the solution, i.e. when:

- the damping of the system is relatively high;
- the direct field is preponderant with respect to the reverberant field;
- a high frequency problem (the ratio between a typical dimension of the structure with respect to the considered exciting wavelength is high) with an acceptable damping is considered;
- an external problem (no modes) is considered.

To prove the above statements, in this paper the CEV method is applied to external and internal acoustic problems, in connection to the integral boundary element formulation. In fact, since the CEV method provides the solution by solving a set of equations whose dimensions are highly reduced with respect to the physical discretized equations, it is expected that the method is computationally more efficient when full matrices are manipulated, as in the BEM approach, rather than when sparse matrices (e.g. block diagonal matrices) are obtained, as in the FEM. The CEV is applied to a set of different internal and external acoustic problems and the results are compared with those obtained by the BEM, showing the good agreement between the two methods.

Finally, an estimate of the computational advantages of CEV with respect to BEM is provided.

2. THE CEV APPROACH

The complex envelope vectorization is a numerical method that, by using a variable transformation, allows the representation of a fast oscillating signal through a set of low oscillating signals. Such transformation leads to a set of problems characterized by a low wave number excitation, even when the exciting load has a high wave number content. This means that, in practice, any high frequency problem can be solved using a coarse mesh, with a significant reduction of computation time.

The operations involved in the CEV method are the following:

• first the Fourier transform (wave number domain) of the exciting load is determined;

• the negative part of the load spectrum is canceled (Hilbert transform);

• a series of adjacent filters of appropriate width are applied to this new spectrum and each of them is shifted toward the origin of the wave number axis;

• similar operations are also performed on the structural or acoustical operator;

• by inverse Fourier transforming the whole matrices and variables, the new governing equations of the CEV problem are obtained. The unknowns are now new variables with low wave number content (complex envelope variables);

• each problem is solved separately and, at the end, the determined envelope variables are shifted toward the corresponding original wave number position and summed appropriately to obtain the physical result.

The following relationships present mathematically the operations described above.

The discrete equation of a conservative dynamic problem, forced by a harmonic force of radiant frequency ω_0 is

$$[\,-{\omega_0}^2\mathbf{M}+\mathbf{K}\,]\,\mathbf{u}=\mathbf{f} \quad \Rightarrow \quad \mathbf{A}\mathbf{u}=\mathbf{f} \tag{1}$$

First a transformation of the loading term $\mathbf{f}$ into a set of the new envelope exciting terms is performed:

$$\overleftarrow{\mathbf{f}}^{(r)}=(\mathbf{S}^{(r)}\mathbf{F}^{-1}\mathbf{W}^{(\mathbf{r})}\mathbf{F})\mathbf{f} \quad \Leftrightarrow \quad \overleftarrow{\mathbf{f}}^{(\mathbf{r})}=\mathbf{E}^{(\mathbf{r})}\mathbf{f} \tag{2}$$

where $\mathbf{F}$ is the Fourier transform operator, $\mathbf{W}$ is a bandpass filter operator, $\mathbf{S}$ is a wave number shifting operator and *(r)* is the spectral window index. Also the matrix $\mathbf{A}$ is transformed into a set of envelope operators $\mathbf{A}^{(\mathbf{r})}$ as follows:

$$\overleftarrow{\mathbf{A}}^{(r)}=(\mathbf{S}^{(r)}\mathbf{A}\mathbf{S}^{*(r)}) \tag{3}$$

being $\mathbf{S}^*$ the complex-conjugate of S. In this way, the solution of the original system of equation becomes an independent set of equations in the new unknowns $\overleftarrow{\mathbf{u}}^{(r)}$.

$$\overleftarrow{\mathbf{A}}^{(r)}\overleftarrow{\mathbf{u}}^{(r)}=\overleftarrow{\mathbf{f}}^{(r)} \tag{4}$$

Since both $\overleftarrow{\mathbf{u}}^{(r)}$ and $\overleftarrow{\mathbf{f}}^{(r)}$ are slowly oscillating signals, it is possible to reduce their dimension through an expansion operator R, that is a rectangular matrix with more rows than columns. For example, for a 6×6 matrix $\mathbf{A}$ and a reduction ratio $\tau=2$, the expansion matrix $\mathbf{R}$ can be defined as:

$$\mathbf{R}=\begin{bmatrix}1&0&0\\1&0&0\\0&1&0\\0&1&0\\0&0&1\\0&0&1\end{bmatrix}$$

whose pseudo-inverse is simply given by $\mathbf{R}^+=\mathbf{R}^T/\tau$. The operation $\mathbf{R}^+\overleftarrow{\mathbf{A}}\mathbf{R}=\overleftarrow{\mathbf{A}}_{red}$ implies a partition of the original matrix into square sub-matrices, replacing each sub-matrix with a single value obtained by averaging its elements.

Thus, one has:

$$\overleftarrow{\mathbf{u}}=\mathbf{R}\overleftarrow{\mathbf{u}}_{red} \tag{5}$$

(The superscript (r) used for the filters are here omitted for the sake of simplicity). The system of equations that is solved in the CEV procedure is, consequently:

$$\mathbf{R}^+\overleftarrow{\mathbf{A}}\mathbf{R}\overleftarrow{\mathbf{u}}_{red}=\mathbf{R}^+\overleftarrow{\mathbf{f}}=\overleftarrow{\mathbf{f}}_{red} \tag{6}$$

i.e.

$$\overleftarrow{\mathbf{A}}_{red}\overleftarrow{\mathbf{u}}_{red} = \overleftarrow{\mathbf{f}}_{red} \tag{7}$$

Finally the physical solution is obtained through the following inverse relationships:

$$\overleftarrow{\mathbf{u}}^{(r)}) = \mathbf{R}\overleftarrow{\mathbf{u}}^{(r)}_{red} \tag{8}$$

and

$$\mathbf{u} = Re\left[\sum_{r} \mathbf{S}^{*}\overleftarrow{\mathbf{u}}^{(r)}\right] \tag{9}$$

For the application of CEV the matrix $\mathbf{A}$, and the vectors $\mathbf{f}$ and $\mathbf{u}$, must be real. When a complex problem is considered (e.g. a non-conservative system or a boundary element formulation) it is necessary to operate differently. In this case $\mathbf{A}$ and $\mathbf{u}$ are complex, i.e. $\mathbf{A} = \mathbf{A}_R + j\mathbf{A}_I$ and $\mathbf{u} = \mathbf{u}_R + j\mathbf{u}_I$, where the subscripts R and I denote real and imaginary parts, respectively. To maintain the fundamental steps of the approach presented above, $\mathbf{u}_R$ and $\mathbf{u}_I$ must be rearranged into a new real vector $\bar{\mathbf{u}}$, and the same must be done for $\mathbf{A}_R$ and $\mathbf{A}_I$:

$$\bar{\mathbf{u}} = \begin{bmatrix} \mathbf{u_R} \\ \mathbf{u_I} \end{bmatrix}$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A_R} & -\mathbf{A_I} \\ \mathbf{A_I} & \mathbf{A_R} \end{bmatrix}$$

Finally, a new vector $\bar{\mathbf{f}}$ is defined such that

$$\bar{\mathbf{f}} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$

Thus, instead of equation (1), the alternative equation must be considered:

$$\bar{\mathbf{A}}\bar{\mathbf{u}} = \bar{\mathbf{f}} \tag{10}$$

to which the mentioned approach can be identically applied.

3. APPLICATION OF THE CEV TO A BOUNDARY ELEMENT FORMULATION

An acoustic problem under steady conditions is usually described by the Helmholtz equation with appropriate Neumann and Dirichlet boundary conditions. In the boundary element formulation such equation is transformed into an integral equation that is solved first on the boundary of the body considered and subsequently in the field of interest. Whatever the type of elements used and the number of nodes in each element, in general the discrete form of the boundary elements can be written as:

$$\mathbf{T}(\omega)\mathbf{p} = \mathbf{B}(\omega)\mathbf{v} = \mathbf{c} \tag{11}$$

where $\mathbf{T}$ and $\mathbf{B}$ depend on the Green function and are complex matrices, $\mathbf{p}$ is the complex vector of unknown pressures, and $\mathbf{v}$ the vector of known velocities on the surface of the body.

This problem is analogous to the one discussed in the previous section, so that the CEV can be applied to it as explained above. In this case, however, there are different possibilities. Assume either an internal or external acoustic problem, i.e. consider a general body, some surfaces of which vibrate under an external load. In any standard BEM procedure, first a finite element code is used to determine the response of the vibrating surfaces. Then the boundary elements are used to determine the pressure on the body surfaces and finally an algebraic equation provides the field pressure. With the CEV, it is possible to solve the field velocity by applying directly the CEV, but this procedure is not convenient because the CEV solution of the structural problem introduces some errors that propagate into the acoustic solution. Moreover the computation time is not too heavy for these type of problems. Therefore, as with the standard BEM, the field velocity is solved by FEM. With respect to the matrix $\mathbf{T}$ there are two possibilities:

• one can first determine the matrix $\mathbf{T}$ by the BEM, then compute the envelope matrix $\overleftarrow{\mathbf{T}}$ and finally introduce the reduction operation to get $\overleftarrow{\mathbf{T}}_{red}$;

• one can first determine the matrix $\mathbf{T}$ by the BEM, then introduce the reduction operator to get $\mathbf{T_{red}}$ and finally compute the envelope matrix $\overleftarrow{\mathbf{T}}_{red}$.

Both procedures produce good results, but the second one is certainly much more convenient computationally.

With respect to the term $\mathbf{c}$, here the chance is unique, i.e. it is necessary to use the BEM to compute $\mathbf{c}$, then pass to the envelope $\overleftarrow{\mathbf{c}}$ and finally determine $\overleftarrow{\mathbf{c}}_{red}$

4. TEST CASES

Three different cases of increasing complexity are considered to show the features of the solutions that can be obtained by the CEV and the quality of results:

• a pulsating sphere (both internal and external acoustic problems)

• the external field generated by a vibrating rectangular box

• the external acoustic field radiated by a benchmark structure made of a set of coupled plates.

The results presented throughout this section are compared with results obtained by the BEM code SYSNOISE. The following comparisons are presented.

• Point spectra of the external pressure field. Such graphs allow to check how the CEV solution captures the physics of the wave propagation in the medium.

• Vectorized surface pressure fields. For a given frequency the CEV solution is compared with the reference solution (SYSNOISE). Such graphs show the errors induced by the CEV solution and permits to evaluate the quality of the outputs. The x axis of these graphs represents the node number.

• Vectorized external/internal pressure fields. Such graphs allow to check whether and how the errors induced on the surface propagate on the internal/external field. Moreover, for internal problems, these graphs permit to show how the CEV solution captures the cavity resonances.

• Colormap of pressure fields either on the surface or in the field of interest.

4.1 Pulsating sphere in air

The pulsating sphere has radius $r = 0.1m$, and the boundary velocity condition, normal to the surface, is $v = 0.01m/s$. The problem is solved by SYSNOISE with 1016 DOFS, and the reduction ratio used in CEV is 8.

4.1.1 External field

The external field is computed at radial points, from the surface of the sphere up to 15 m (figure 1).

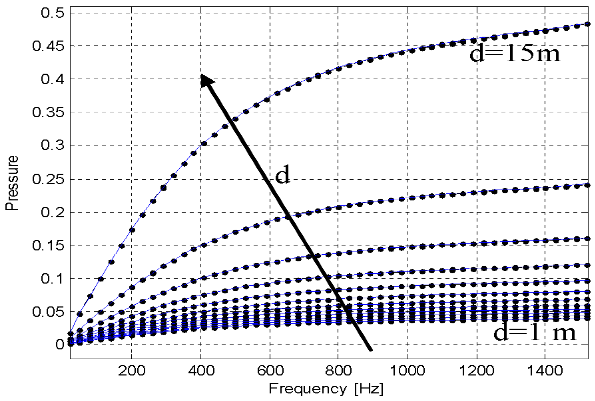

Figure 1: Pressure spectrum at a distance d ranging from 1m to 15m from the center of the sphere. Comparison between the CEV solution (dots) and the reference solution (blue)

This first trivial example is presented to show a reference behavior of the CEV method on a simple case. It is worthwhile to point out (figure 2) that the external pressure field is in perfect agreement with the reference solution even if the surface pressure computed by CEV has an oscillating behavior around the exact field computed by SYSNOISE.

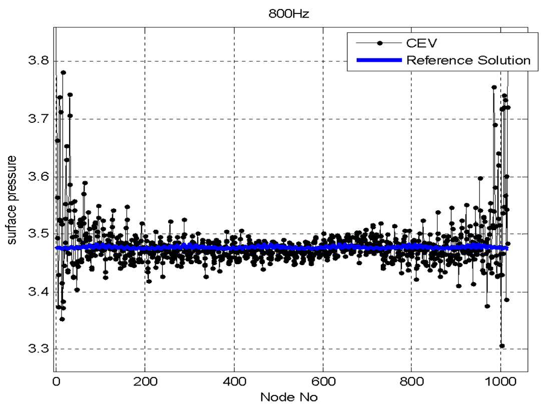

Figure 2: Pressure field on the surface of the sphere for a frequency f =800Hz. Comparison between the CEV solution (dots) and the reference solution (blue)

4.1.2 Internal field

The internal pressure field is computed in internal points of the sphere. This case is presented to check whether the CEV formulation is able to reproduce the cavity resonances of a system. Figure 3 shows the point pressure spectrum at an internal point of the spherical cavity. A perfect agreement between the CEV and the reference solution can be observed.

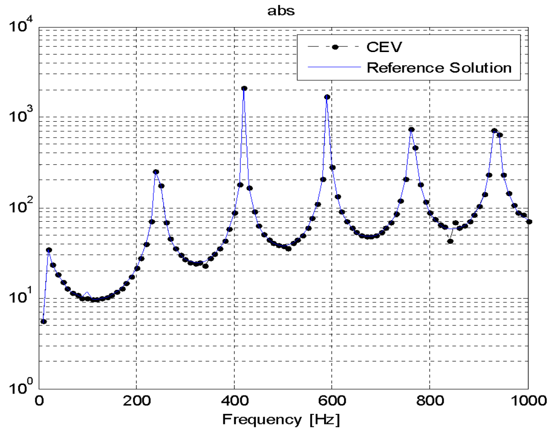

Figure 3: Point pressure spectrum inside the cavity. Comparison between CEV solution (dots) and the reference solution (blue)

4.2 External field generated by a loaded box

The rectangular box dimensions are $20 \times 22 \times 24\times$ cm, and the top face is loaded by two point forces of amplitude 1000 N, with a flat spectrum between 700 and 2500 Hz, figure 4. The thickness of the plates is 2mm, and the material is steel. The boundary velocity conditions are computed by a finite element analysis. The degrees of freedom used by the BEM is 1176 and the CEV uses a reduction ratio 21. The external pressure field is computed at points along a line normal to the top face of the box. can be observed.

In this more complex external problem the normal surface velocity changes on the surface and the CEV solution differs from the reference solution. Nevertheless, the two solutions are close enough so that the CEV pressure field represents a good approximation of the actual pressure field, as can be stated by observing the results presented in figure 5 where the spectra of the CEV and reference solutions are compared in the range 700-2500 Hz. Moreover, the directionality of the pressure field is well captured by the CEV solution (not shown in figure). The effect of the reduction ratio was also considered on this test. Figure 6 shows the average spectrum determined on a sphere surrounding the box with a radius of 15 m from the center of the box.

The frequency range analyzed ranges from 1800 to 1900 Hz. Is is possible to note two important aspects:

- increasing the reduction factor from 21 up to 168 the

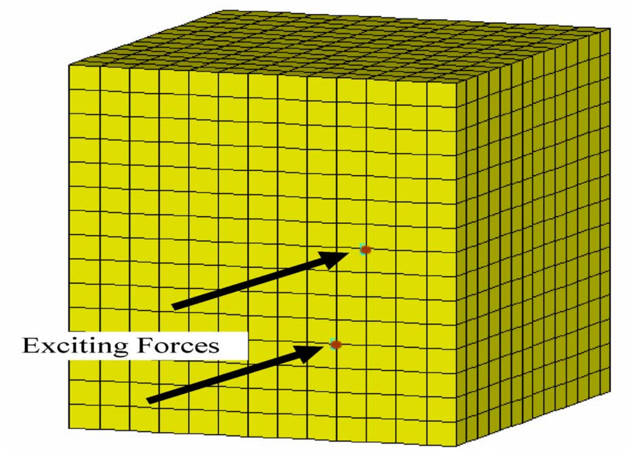

Figure 4: Geometry and position of the exciting forces

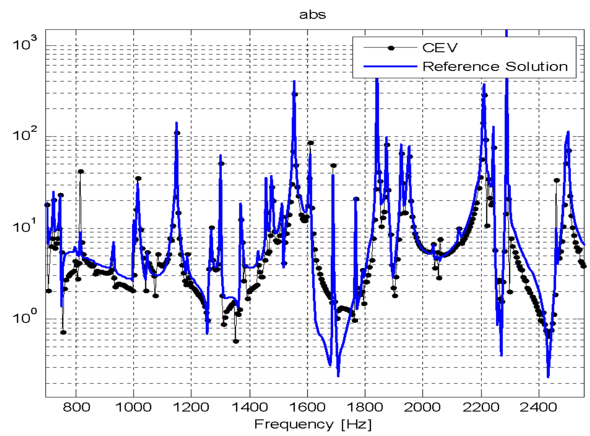

Figure 5: Averaged pressure spectrum at a distance d=15m from the box. Comparison between CEV solution (dots) and the reference solution (blue)

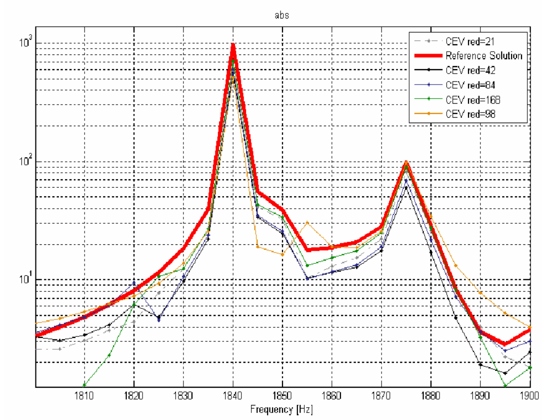

Figure 6: Point spectrum at a distance d=15 m from the box. Comparison between reference solution (red) and the CEV solution for different values of the reduction factor (dotted lines)

qualitative behavior of the solution is almost unchanged, even if there is obviously a slight degrade;

• the quality of the CEV solution does not have a trend that follows strictly the reduction factor, even if the CEV solution becomes less accurate when the reduction factor increases.

However it is worthwhile to point out that, for a reduction factor of 168, the dimensions of the CEV problem has only 14 DOFs.

4.3 External field radiated by a benchmark

The system considered is made of four steel coupled plates. Three of them are rigid and the fourth one is flexible (thickness 20 mm) and excited by a set of flat spectra forces applied to three different nodes (1000 N each) (figure 8). The boundary velocity conditions are determined by a FEA. The number of DOFS used is 855 and the reduction factor used in CEV is 19. The external pressure field is computed over a set of points laying on a plane internal to the open cavity between the plates. The chosen geometry of the benchmark permits to have an acoustic field characterized by stationary waves (the top and bottom plates are parallel) and traveling waves. Moreover, because of the combination of rigid and flexible plates, the normal velocity field presents discontinuities at the boundary between the plates. The results presented concern the pressure spectrum in a reference point of the field (figure 9) and the pressure field computed on the internal plane for two different

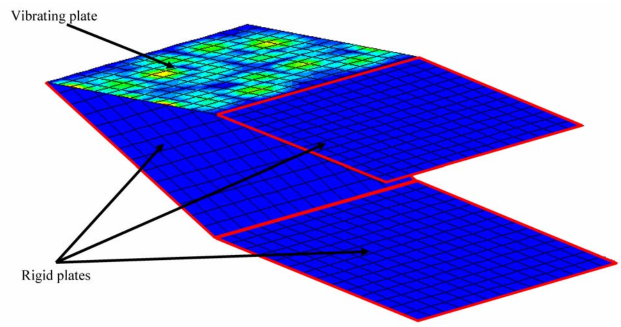

Figure 7: Characteristics of the benchmark case

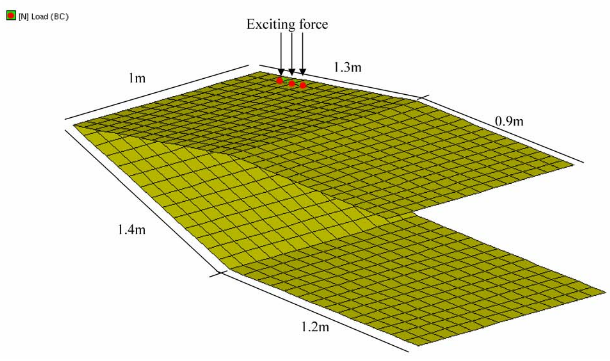

Figure 8: Geometry of the benchmark case

frequencies (130 and 1125 Hz), figures 10 and 11. The CEV results are in very good agreement with the reference solution for rather low frequencies and maintain such good agreement even for high frequencies of the exciting force, up to the limit of the considered mesh.

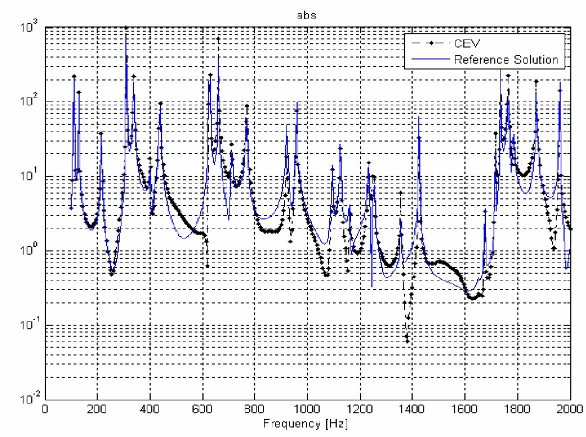

Figure 9: Pressure point spectrum at a point of the field. Comparison between CEV solution (dots) and the reference solution (blue)

5. CONCLUDING REMARKS AND CONSIDERATION ABOUT THE COMPUTATIONAL ADVANTAGES

The application of the Complex Envelope vectorization to a boundary element formulation, and particularly to external acoustic problems, is presented. This application was expected to be quite efficient because the CEV seems to be more appropriate for systems where the effect modes is negligible. It is important to note that the solution determined by the CEV both on the surface and in the external field is usually bounded between 3dB from the reference solution. However it can be observed that also the internal field is computed efficiently. As well, it is worthwhile to point out that the CEV solution is generally able to maintain the spatial phase pattern on both the surface and the internal field, providing significant results with a computational time that is in general much shorter than the standard solution obtained by the BEM. Finally, the CEV solution seems not to be affected by an increase of the complexity of the system or the adopted reduction factor.

About the computational advantages, for a systematic comparison of CEV with the standard BEM technique, the following aspects may be considered. The time for the solution of the pressure field on the surface can be reduced "at will", (see the example of the radiating box) maintaining a reasonable agreement between the reference solution and the CEV solution, by increasing the reduction factor. However, by increasing the reduction factor, it is necessary to increase the number of windows. For each of them a set of operations must be carried out, so that it is probably possible to determine an

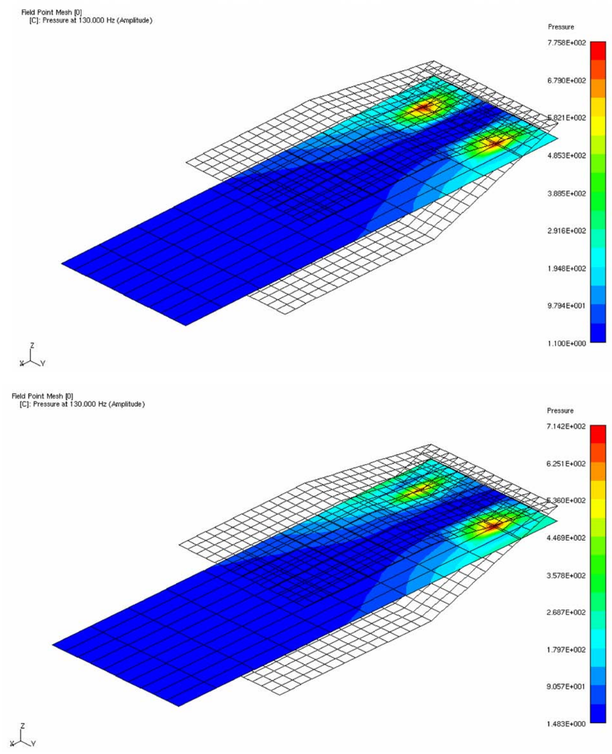

Figure 10: Internal pressure field at 130Hz. Up: reference solution, down: CEV solution

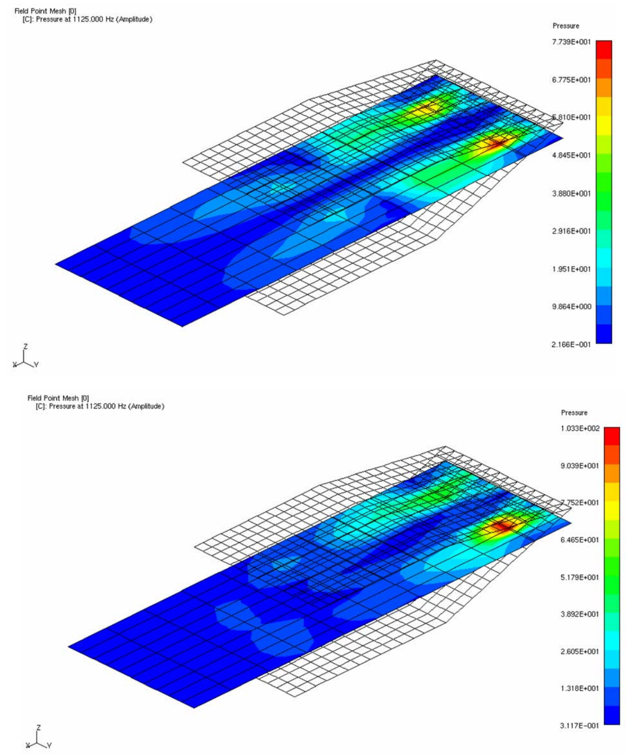

Figure 11: Internal pressure field at 1125Hz. Up: reference solution, down: CEV solution

optimum reduction value. For the cases presented, that are characterized by about one thousand degrees of freedom, the best compromise is a reduction factor of about 20. For this value the solution of the CEV equations requires a computational time 6 times lower than the standard solution. However, it is expected that, by a more efficient implementation of the code, the computational time using CEV should be reduced to an order of magnitude of the reduction factor τ.

6. ACKNOWLEDGEMENT

This paper have been granted by MUR. The authors acknowledge the collaboration of LMS that provided a temporary license of SYSNOISE to work on this subject.

7. REFERENCES

[1] Le Bot A., 1998, “Geometric diffusion of vibrational energy and comparisons with the vibrational conductivity approach”, *J. Sound and Vibration*, vol. 212, pp. 637-647.

[2] Ichchou M.N., Jezequel J., 1996, , “Comments on models of the energy flow in vibrating membranes and on simple models of the energetics of transversely vibrating plates”, *J. Sound and Vibration* vol. 195(4), pp. 679-685.

[3] Wohlever J.C., Bernhard R.J., 1992, , “Mechanical energy flow models of rods and beams”, *J. Sound and Vibration*, vol. 153, pp. 1-19.

[4] Le Bot A., 1998, “A vibroacoustic model for high frequency analysis”, *J. Sound and Vibration*, vol. 211(4), pp. 537-554.

[5] Langley R.S., 1995, , “On the vibrational conductivity approach to high frequency dynamics for two-dimensional structural components”, *J. Sound and Vibration*, vol. 182(4), pp. 637-657.

[6] De Rosa S., Franco F., Ricci F., Marulo F., 1997, “Fisrt assessment of the energy based similitude for the evaluation of the damped structural response”, *J. Sound and Vibration*, vol. 204(3), pp. 540-548.

[7] Giannini O., Carcaterra A., Sestieri A., 2007, “High frequency vibration analysis by the complex envelope vectorization”, *Int. J. of the Acoust. Soc. of Am.*, vol. 121(6), pp. 3472-3483.