The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Cluster Space Collision Avoidance for Mobile Two-Robot Systems

Christopher A. Kitts, *Senior Member IEEE*, Kyle Stanhouse, and Piya Chindaphorn

***Abstract*—The *cluster space* state representation for multi-robot systems provides a simple means of specifying and monitoring the geometry and motion characteristics of a cluster of mobile robots. In previous work, this approach has been experimentally verified and validated for controlling the motion of mobile multi-robot systems ranging from land rovers to autonomous boats. In this paper we introduce a compact collision avoidance algorithm that operates at the level of the cluster, leading to coordinated translational and rotational motions that allow obstacles to be avoided while maintaining the relative geometry of the cluster. This paper formulates the potential-field based obstacle avoidance algorithm, describes its integration within the cluster space control architecture, and presents successful experimental results of its application to two simple, diverse multi-robot testbeds.**

I. INTRODUCTION

In the vision of multi-robot systems, application-specific performance is improved through the ability of multiple robots to provide redundancy, increase coverage or throughput, enable flexible reconfiguration, and/or provide spatially diverse functionality [1]. For mobile systems, one of the key technical considerations in enabling this vision is the technique used to coordinate the motions of the individual vehicles. Because of the physical distribution of components and the potential for limited information exchange, decentralized control approaches hold great promise [2]-[4]. Behavioral, biologically-inspired, optimization-based, and potential field techniques have been demonstrated with great success [5]-[14].

Centralized approaches exploiting global information have been successfully demonstrated [15], [16] but suffer from limited scalability and the need for global information. There are, however, a number of applications where a small number of local, cooperating robots can bring enormous value, such as material transport, regional synoptic sampling, and sensing techniques where the active stimulus and the signal reception are spatially distributed [17]-[19]. The multi-robot motion control strategy discussed in this paper, *cluster space control*, is currently being implemented via a centralized controller with global information given the applications under development, although decentralized implementations are possible.

For all of these strategies, avoiding collisions with obstacles and with other members of the formation is critical. For cases where the environment is well known and predictable, *a priori* path planning may be used. For more dynamic environments, typical strategies include both discrete and continuous approach. Discrete strategies typically involve the encoding of collision avoidance behaviors in IF-THEN production rule constructs, implemented in a broader reasoning architecture that balances this behavior with others [20]-[22]. One of the primary continuous strategies, used for both mobile robots as well as serial manipulators, is the use of potential fields for establishing avoidance forces or velocity set-points [23]-[25]; many enhancements to this approach have been proposed to address inherent drawbacks of this strategy [26]-[31]. Previous efforts at incorporating collision avoidance into multi-robot formation control systems have generally relied on superimposing a robot's formation control and/or destination-seeking commands with potential field-based avoidance functions applied to any nearby obstacles, which may include other members of the formation [5], [32]-[37]. While this approach ensures collision-free motion, it often leads to transient disturbances in controlling the shape of the formation. When maintaining formation shape is critical, the avoidance function may be applied to the virtual body, such as in [38], where a virtual perimeter is established about the formation and upon which collision avoidance forces may act; this approach uses a centralized formation controller that explicitly considers formation variables.

In this paper, we propose a novel, compact collision avoidance algorithm that operates at the level of the multi-robot cluster, and which induces both translational and rotational motions that allow obstacles to be avoided while maintaining the relative geometry of the cluster. In contrast to [38], this is done without designating a perimeter and can allow, for example, a cluster to maneuver through an obstacle field with obstacles "breaking ranks" of the formation. In general, we envision that such a technique would be combined with other approaches, such as *a priori* trajectory planning and robot-level collision avoidance, to provide a comprehensive collision avoidance strategy.

II. CLUSTER SPACE SPECIFICATION AND CONTROL

As presented in [39], the motivation of the cluster space approach is to promote the simple specification and

---

This work was supported in part by the National Science Foundation under Grants EIA0079815, E1A0082041, and CNS0619940, by NASA Ames Research Center under Grant NNA04CK64A, and by Santa Clara University under Grant TSC131; any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation, NASA Ames Research Center, or Santa Clara University.

The authors are with the Robotic Systems Laboratory, Santa Clara University, Santa Clara CA 95053 USA. Phone: 408.554.4382; fax: 408.554.5474; e-mail: {ckitts, kstanhouse, pchindaphorn}@scu.edu.

978-1-4244-3804-4/09/$25.00 ©2009 IEEE

monitoring of the motion of a mobile multi-robot system. To date, this work has focused on systems in which each robot is capable of closed-loop velocity control, a level of functionality typically built into a variety of commercially available robotic platforms.

A. Cluster Space Control

The cluster space control strategy conceptualizes the *n*-robot system as a single entity, a *cluster*, and desired motions are specified as a function of cluster attributes, such as position, orientation, and geometry. These attributes guide the selection of a set of independent system state variables suitable for specification, control, and monitoring. These state variables form the system's *cluster space*.

Cluster space state variables may be related to robot-specific state variables through a formal set of kinematic transforms. These transforms allow cluster commands to be converted to robot-specific commands, and for sensed robot-specific state data to be converted to cluster space state data.

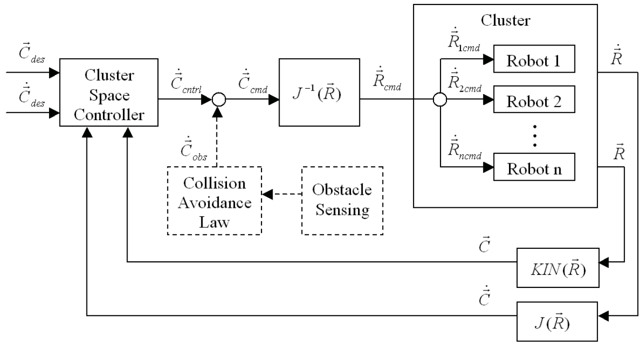


Figure 1 – Cluster Space Control Architecture for an *n*-Robot System.

With the formal kinematics defined, we compose a system controller in which desired motions are specified and control compensations are computed in the cluster space. These compensation commands are transformed to robot space through the inverse Jacobian relationship. The resulting individual velocity commands are then provided to each of the robots, which, in turn, use their on-board closed-loop velocity control functionality to achieve this command. If robot space variables are sensed, they are transformed to the cluster space through the use of forward kinematic relationships in order to support control computations. Fig. 1 depicts a general inverse Jacobian control architecture of this type. Desired cluster space motions may be provided as regulation inputs, by a trajectory generator, by a realtime pilot, or by a higher-level application-specific controller. The dotted lines/boxes denote the addition of our new collision avoidance law, which is described in Section 3. We note that the Jacobian and inverse Jacobian matrices are functions of the cluster's pose and therefore must be updated at an appropriate rate.

We have successfully used this control approach to demonstrate cluster-space-based versions of regulated motion [40], automated trajectory control [41]-[42], human-in-the-loop piloting [43]-[44], and partitioned model-based control [45]. This work has included experiments with 2-, 3- and 4-robot planar land rover clusters [46]-[48], with 2- and 3- surface vessel systems [49], with 2- and 3-blimp aerial systems [50], [51], for robots that are both holonomic and non-holonomic, and for target applications such as escorting and patrolling [49], [52].

B. Two Robot Cluster Space Formulation

For the purposes of this paper, we consider a simple, planar, two-robot system, which we will use to demonstrate our proposed collision avoidance strategy. The conventional description of the pose of the two-robot system in Fig. 2 typically takes the form of:

$$^{G}\vec{R} = (x_1, x_2, y_1, y_2, \theta_1, \theta_2)^T \qquad (1)$$

where the variables (x_i, y_i, θ_i) specify the pose for rover *i* with respect to the global frame, {G}. We consider this a "robot-space" description of system pose.

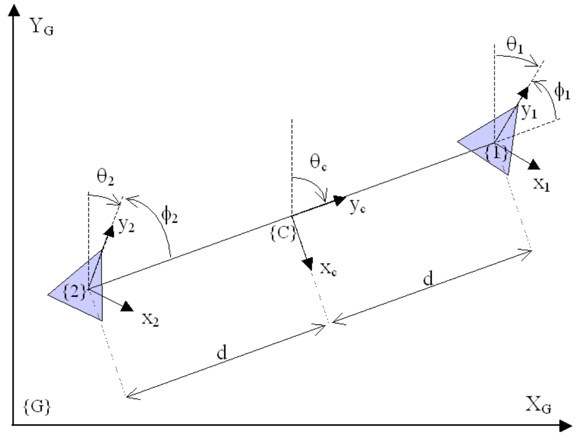


Figure 2 – Pose reference frames for the planar 2-robot system. Positive angular directions are as noted in the figure.

Alternatively, the selected cluster-space pose vector is:

$$\vec{C} = (x_c, y_c, d, \theta_c, \phi_1, \phi_2)^T \qquad (2)$$

where (x_c, y_c, θ_c) specifies the pose of frame {C} with respect to frame {G}, the ϕ_i angles specify the yaw orientations of each rover with respect to {C}, and d specifies half of the separation distance between the two rovers. In this example, we have selected to locate the {C} frame at the center of the cluster; however, we note that the cluster space method does not require this.

The mathematical relationships for the forward position kinematics and the inverse velocity kinematics, which are used in the servo loop of the inverse Jacobian controller shown in Fig. 1, are summarized in the appendix. For the experiments presented in this paper, velocity feedback was not used.

III. CLUSTER SPACE COLLISION AVOIDANCE

Our proposed cluster avoidance policy includes a repulsive translational velocity and an optional angular velocity component that adds a level of rotational maneuverability that we have found useful in relatively dense obstacle fields (e.g. when the obstacles are spaced on the order of the spacing within the robot cluster). To illustrate the formulation of our policy, consider an *n*-robot cluster in the presence of *m* arbitrarily placed obstacles.

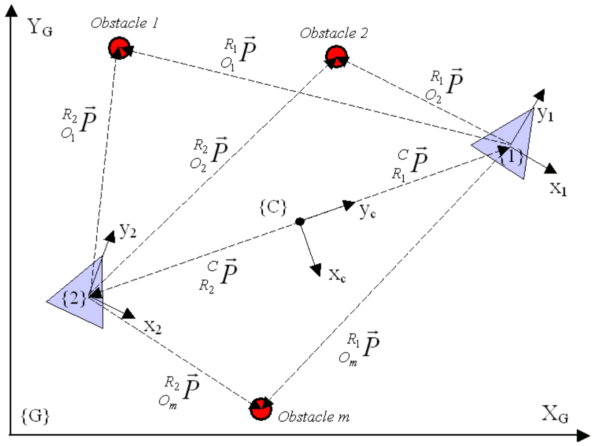


Figure 3 – An *n*-robot cluster in the presence of *m* obstacles.

Referring to Fig. 3, we develop the collision avoidance policy by first considering a conventionally-defined repulsive velocity contribution for robot *i* due to obstacle *j* of the form [24]-[25]:

$$ {}_{R_i}\vec{V}_{Oj} = -K_{O_j}\left(\frac{ {}^{R_i}_{O_j}\vec{P} }{ \left| {}^{R_i}_{O_j}\vec{P} \right|^{r} } \right) \tag{3} $$

where ${}^{R_i}_{O_j}\vec{P}$ is the position vector from robot *i* to obstacle *j*, K_{O_j} is a gain associated with obstacle *j*, and *r* is greater than or equal to one to ensure a greater repulsive effect for closer objects and with its exact value selected to provide the desired amount of field decay.

The total repulsive velocity for robot *i* due to all obstacles is:

$$ {}_{R_i}\vec{V}_{O} = \sum_{j=1,m} {}_{R_i}\vec{V}_{O_j} \tag{4} $$

As our cluster-level collision avoidance policy, we define the translational repulsive velocity for the cluster to be the sum of all of the individual robot repulsive velocities. The cluster's angular repulsive velocity, which in practice we have used optionally, is specified as the sum of the first moments of each robot's net repulsive velocity about the cluster's centroid:

$$ \begin{pmatrix} {}_{C}\vec{V}_{O} \\ {}_{C}\vec{w}_{O} \end{pmatrix} = \begin{pmatrix} \sum\limits_{i=1,n} {}_{R_i}\vec{V}_{O} \\ \sum\limits_{i=1,n} {}^{C}_{R_i}\vec{P} \times {}_{R_i}\vec{V}_{O} \end{pmatrix} \tag{5} $$

where ${}^{C}_{R_i}\vec{P}$ is the position vector from the cluster's centroid to robot *i*.

The dotted elements in Fig. 1 show how this cluster space collision avoidance policy is integrated with the cluster space controller. Collision avoidance velocity requirements are summed with velocity commands from the cluster space controller, and the net cluster space velocity vector is transformed to robot space via the inverse Jacobian.

As this work represents preliminary exploration of the incorporation of collision avoidance policies within the cluster space control framework, several simplifications have been made. First, the obstacle's repulsive field extends to infinity; practical application would use a modified function that is zero outside of a predetermined detection region [53]-[54]. Second, local minima, singular configurations, and deadlocks, which are common drawbacks of potential field-based approaches [26], are certainly possible with the current policy; such issues could ultimately be addressed through techniques such as varying the shape of the avoidance region or adding dithering or circulatory fields around the obstacles [27]-[31], [35]. Third, the current approach explores cluster level collision avoidance without robot level avoidance capabilities; such robot level behavior is ultimately desired in order to ensure vehicle-level safety [35]. Fourth, computational complexity in the current implementation limits scalability of the technique as implemented in the work described in this paper; ongoing work in this area includes decentralized implementations of the cluster space technique, exploration of multi-rate controllers, and computing avoidance functions only for the "exterior" robots in a cluster. Fifth, kinematic redundancy within the mobile robot cluster has yet to be explored as a means of achieving optimal motion; as an example of this, note that a two robot cluster could possibly avoid a collision be either translating or rotating in such a manner as to avoid an obstacle. Finally, the work presented here is offered without proof of stability; such a proof is being prepared for publication in a separate venue.

IV. EXPERIMENTAL RESULTS

To explore and demonstrate the proposed cluster space collision avoidance technique, we have experimentally implemented it in two simple, diverse two-robot systems. This section summarizes the characteristics of each system and presents selected experimental results.

A. Differential Drive Two-Robot Results

Our first set of experiments was performed using two differential drive robots that operated outdoors with cluster separations on the order of 5-30 meters and within a sparse obstacle field. The optional angular component of the collision avoidance policy was not used for these experiments. Due to the non-holonomic constraints of this system, only four cluster degrees of freedom are independently specified by the operator; robot heading for both vehicles are determined automatically.

1) *Testbed Description*

This testbed consisted of two commercially-available differential drive chasses with custom sensing, communication, and control equipment as well as an off-board control computer for human interfacing and control computation, as depicted in Fig. 4a [42], [44], [55], [56]. Each chassis is an ActivMedia Robotics Amigobot with eight body-mounted sonar around its circumference. The robots are capable of closed loop velocity control through the use of wheel encoders and a Renesas SH7144-based microcontroller.

On top of each robot is mounted a student-integrated equipment suite that includes a WAAS-enabled Garmin 18 GPS receiver, a Devantech CMPS03 digital compass, and a Maxstream 9Xstream™ 900 MHz serial radiomodem used for wireless communication with an off-board controller. A modular network of BasicX-24 microcontrollers is used for component interfacing and data handling. These microcontrollers collect, filter, and process sensor data, send sensor data to an off-board control computer, and route robot velocity commands supplied by the off-board control computer from the radiomodem to the robot chassis microcontroller.

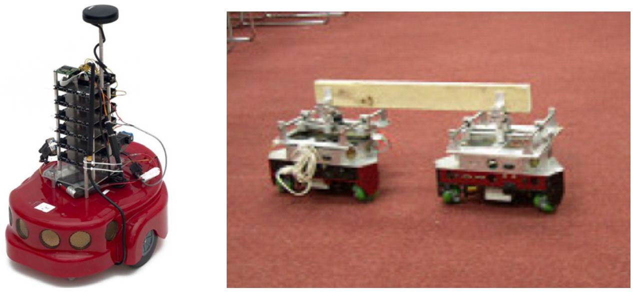
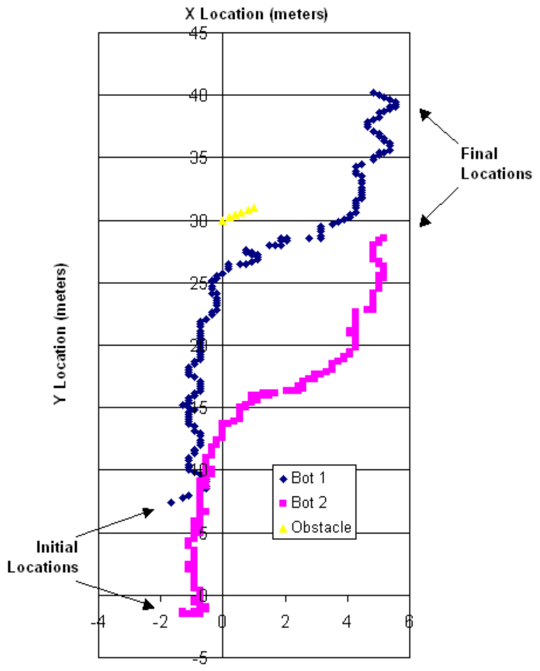
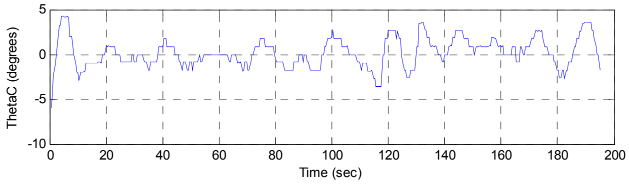

(a) (b)

Figure 4 – Experimental Testbed. (a) Two of these differential drive vehicles were used for the experiments described in Section IV.A; (b) These omnidrive vehicles were used for the experiments described in Section IV.B.

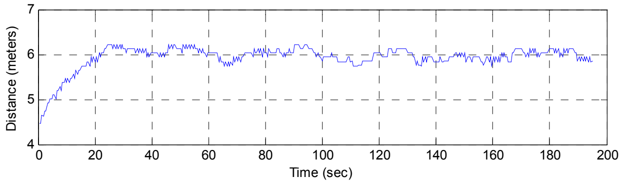
The off-board control computer is a Pentium IV-class personal computer that runs a graphical user interface (GUI) allowing a human operator to specify robot or cluster-level directives and to monitor activity. The computer executes a compiled Matlab-generated control program that ingests sensor data from the GUI, performs the desired control computation, and returns new velocity set-points to be sent to each robot. The controller executes at approximately 3 Hz and uses a value of r = 1 for the obstacle avoidance policy.



(a) Overhead view of cluster motion



(b) Cluster heading (θ_c) time history



(c) Cluster size (d) time history

Figure 5 – Experimental Results. (a) The two robots move in a coordinated fashion in order to avoid the obstacle (shown in yellow). (b) They maintain cluster heading to within +/- 4 degrees. (c) They maintain separation distance to within +/- 0.5 meters.

2) *Experimental Results*

A variety of collision avoidance experiments have been conducted using this two-robot testbed, to include integration with velocity regulating, trajectory following, and pilot-in-the-loop controllers. As a simple example, we show here results when the two robots have been commanded to drive North at 10 m/min with a separation of 12 m and a cluster heading of 0 degrees. A single obstacle (a large piece of cardboard about 2 m in length) was placed in the path of the cluster. Obstacle sensing was performed by the sonar on board the robots.

Fig. 5 shows the results of this test [55]. In (a) an overhead view of the resulting cluster motion is shown, and in (b) the time histories of several key cluster variables are provided. As is seen, the cluster successfully avoids the obstacle while maintaining control of its cluster parameters. Performance was limited by the accuracies of the GPS and compass components (specified to +/- 3 m and +/- 5 deg, respectively) and by the use of manually tuned proportional-only controllers.

B. *Omniwheel Two-Robot Results*

Our second set of experiments was performed using two holonomic, omniwheel drive robots that operated indoors with cluster separations on the order of 0.5-2 meters and multiple, tightly-space obstacles. The optional angular component of the collision avoidance policy was used for these experiments.

1) *Testbed Description*

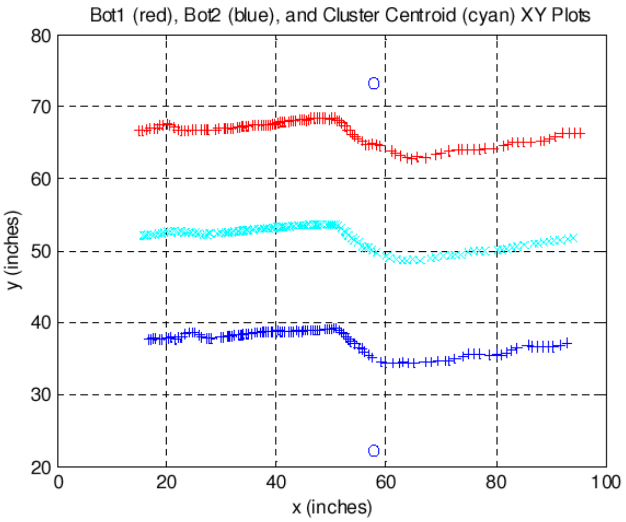
This testbed consisted of two custom, student-developed omniwheel chasses with custom sensing, communication, and control equipment as well as an off-board control computer for human interfacing and control computation, as depicted in Fig. 4b [57], [58]. The robots are capable of closed loop velocity control through the use of wheel encoders and industrial PID controllers. An Atmel ATMEGA128-based microcontroller serves as the on-board computer with duties that include computing wheel velocity set-points via the robot's inverse kinematic transforms, receiving robot-level velocity commands from an off-board computer via a Maxstream 9Xstream™ 900 MHz serial radiomodem.

As with the previous testbed, an off-board Pentium IV-class control computer runs a Matlab-based control program, allowing a human operator to specify robot or cluster-level directives and to monitor activity. The control program ingests data from a student-developed overhead vision system that tracks the motion of the robots over a workspace of approximately 15' x 15'; the control program performs the desired control computation and returns new velocity set-points to be sent to each robot. The controller executes at approximately 2 Hz and uses a value of r = 2 for the obstacle avoidance policy.

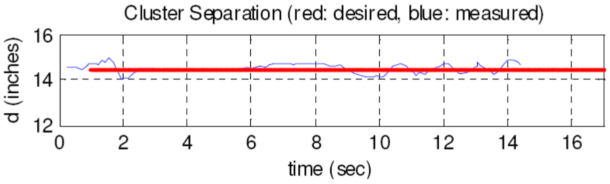
2) *Experimental Results*

A variety of collision avoidance experiments have been conducted using this two-robot testbed, to include integration with velocity regulating, trajectory following, and pilot-in-the-loop controllers. As a simple example, we show here results when the two robots have been commanded to drive in the positive X direction at 1 inch/sec with a separation of 14.5 inches and with an initial cluster heading of 0 degrees [59]. Two obstacles were placed in the path of the cluster and were sensed by the same overhead vision system that was used to track robot position.

Fig. 6 shows the results of using only the translational component of the collision avoidance law. As can be seen, the cluster moves as one in order to drive evenly through the two obstacles. Fig. 7 shows the results of using the full collision avoidance law, to include the rotational component.

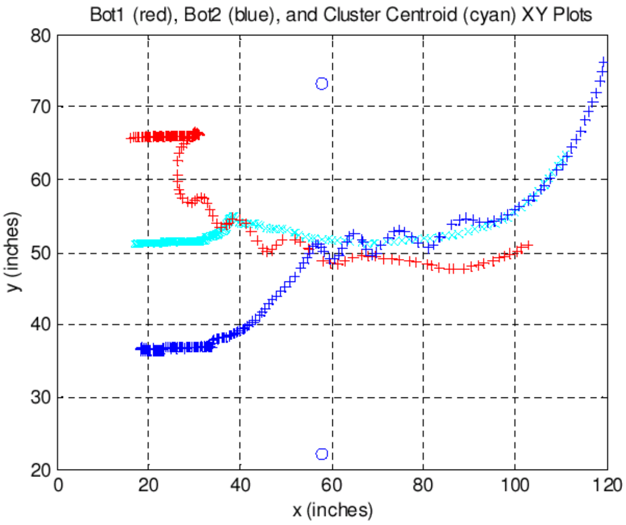


(a) Overhead view of cluster moving through 2 obstacles.

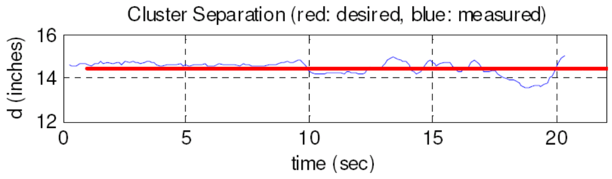


(b) Time history of cluster size.

Figure 6 – Experimental Results. Cluster maneuvering through two obstacles using only the translational control law component.



(a) Overhead view of cluster moving through 2 obstacles



(b) Time history of cluster size.

Figure 7 – Experimental Results. Cluster maneuvering through two obstacles using both the translational and rotational components of the law.

As is seen, the cluster reorients itself in order to maneuver through the center obstacle field. Performance was limited by the accuracy of the vision system [59], which was evaluated to have an accuracy of better than 0.5 inch across the workspace (95% confidence level) and by the use of manually tuned proportional-only controllers.

C. *Comparison of Results*

The results from these experiments demonstrate that our collision avoidance algorithm ensures that obstacles are avoided while also maintaining the integrity of the cluster geometry during the avoidance maneuver. The experiments also highlight the optional use of the law's rotational component, which is particularly useful for more demanding maneuvering scenarios. This produces a distinctly different behavior than robot-level approaches in which either (a) only robots near obstacles move out of the way, such as in [5], or (b) robots out of range of an obstacle maintain formation in a reactive manner, based on varying inter-robot positions that propagate through the formation from the robots that are directly avoiding collisions, such as in [37].

V. ONGOING AND FUTURE WORK

As reviewed in Section II.A, we have successfully applied the cluster space specification and control approach to a wide variety of mobile robot systems varying in number, mobility characteristics, and sensing schemes; this work has been applied to land rovers, aerial vehicles, and surface ships, and it has ranged from fully automated control to pilot-in-the-loop applications. Current initiatives include scaling the approach to larger clusters with higher-DOF vehicles (which lead to much larger and more cumbersome Jacobian transforms, although this can be alleviated somewhat through the judicious choice of state variables), developing a stability proof for the non-holonomic controller, integrating a spoken dialog interface to allow verbal command of the system, and integrating higher-level application-specific controllers suitable for field applications.

With respect to our collision avoidance policy, we are currently exploring a number of extensions. First, we have begun a more rigorous evaluation of the policy when it is implemented with robot-level collision avoidance laws and other features of our recent cluster space work, such as the use of model-based nonlinear controllers. Second, we plan to apply the law to clusters with a greater number of robots moving through fields with a greater number of obstacles; due to computational constraints, we anticipate the need to develop extensions to our policy such as the use of limited regions of interaction, dual rate controllers, and minimal representations of the cluster and the obstacle environment. Third, we are extending our current stability proof to include use of this policy. Fourth, we will incorporate this policy into our diverse array of robotic field systems that operate in land, sea, air and space and which are routinely used to perform world-class scientific and advanced technology demonstration missions [60].

VI. SUMMARY AND CONCLUSIONS

We have proposed a multi-robot collision avoidance policy appropriate for use with our previously developed cluster space technique for specifying and controlling the motion and geometry of a multi-robot system. This collision avoidance policy consists of a repulsive velocity command that is superimposed with other navigation commands in the cluster space and then converted to robot-specific velocity commands through the use of the system's inverse Jacobian. The collision avoidance command has a translational component and an optional angular component that can be applied depending on the nature of the environment and the maneuvering requirements.

We have demonstrated this policy and its use with our cluster space controller through successful experiments with two diverse two-robot systems, both of which were able to navigate as specified while avoiding obstacles. The use of the angular component of the collision avoidance law proved to be particularly effective for our experiments with a relatively tight maneuvering requirement. Overall, our positive results show that this reactive technique effectively simplifies the specification and control of a multi-robot system given that a) real-time cluster pilots receive automated aid in avoiding collisions, and b) navigation trajectories can be specified without considering dynamic obstacles given that collision avoidance corrections will be computed and applied in realtime. Our ongoing and future work in this field is focused on extending this capability to larger clusters and to more cluttered environments in order to improve the controllability and applicability of multi-robot systems.

APPENDIX

Given the selection of cluster space state variables presented in Section II.A, the forward position kinematic relationships for the two-robot system are [39], [40]:

$$x_c = \frac{x_1 + x_2}{2} \qquad y_c = \frac{y_1 + y_2}{2}$$

$$d = \frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad \phi_1 = \theta_1 - ATAN2(x_1 - x_2, y_1 - y_2)$$

$$\theta_c = ATAN2(x_1 - x_2, y_1 - y_2) \qquad \phi_2 = \theta_2 - ATAN2(x_1 - x_2, y_1 - y_2)$$

where ATAN2 is a computational function that calculates a four-quadrant arc tangent with a range of [π, -π] [61].

By differentiating the position kinematic expressions, the velocity kinematics may be expressed as in the form of a Jacobian matrix. The inverse Jacobian relates robot and cluster velocities in the following manner:

$$\dot{R} = \begin{bmatrix} 1 & 0 & \sin(\theta_c) & d\cos(\theta_c) & 0 & 0 \\ 1 & 0 & -\sin(\theta_c) & -d\cos(\theta_c) & 0 & 0 \\ 0 & 1 & \cos(\theta_c) & -d\sin(\theta_c) & 0 & 0 \\ 0 & 1 & -\cos(\theta_c) & d\sin(\theta_c) & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \dot{C}$$

ACKNOWLEDGMENT

The authors thank student researchers who assisted with performing experiments, especially Brian Tully and Tran To, who have contributed to testbed development, and who have contributed to the development and evolution of the cluster space control approach for multi-robot systems.

REFERENCES

[1] C. Kitts and M. Egerstedt, “Design, Control and Applications of Real-World Multirobot Systems.” IEEE Robotics and Automation Magazine, v 15, n 1, March 2008, p. 8.

[2] D. Siljak, *Decentralized Control of Complex Systems*, New York: Academic, 1991.

[3] M. Ikeda, “Decentralized control of large-scale systems,” in *Three Decades of Mathematical Systems Theory*, H. Nijmeijer and J. Schumacher, Eds. Berlin: Springer-Verlag, pp. 219-242, 1989.

[4] T. Yang, H. Yu, M. Fei, and L. Li, “Networked control systems: a historical review and current research topics,” *Measurement and Control*, v 38, n 1, February 2005, pp. 12-16.

[5] T. Balch and R. Arkin, “Behavior-based formation control for multirobot teams,” *IEEE Transactions on Robotics and Automation*, v 14, n 6, December 1998, pp. 926-939.

[6] E. Flinn, “Testing for the ‘boids’,” *Aerospace America*, v 43, n 6, June, 2005, pp. 28-29.

[7] O. Khatib, “The Potential Field Approach and Operational Space Formulation in Robot Control,” *Proc. Fourth Yale Workshop on Applications of Adaptive Systems Theory*, Yale University, New Haven, Connecticut, May 1985, pp. 208-214.

[8] N. E. Leonard and E. Fiorelli, “Virtual leaders, artificial potentials and coordinated control of groups,” *Proc. 2001 IEEE Conf. Decision and Contro*l, 2001, pp. 2968–2973.

[9] R. M. Murray, Recent research in cooperative control of multi-vehicle systems,” *Journal of Dynamics Systems, Measurement and Control*, v 129, n 5, 2007, pp. 571-583.

[10] W. B. Dunbar and R. M. Murray, “Distributed receding horizon control for multi-vehicle formation stabilization,” *Automatica*, v 42, n 4, 2006, pp. 549-558.

[11] P. Ögren, E. Fiorelli, and N. E. Leonard, “Cooperative control of mobile sensor networks: adaptive gradient climbing in a distributed environment,” *IEEE Transactions on Automatic Control*, v 49, n 8, 2004, pp. 1292-1302.

[12] E. W. Justh and P. S. Krishnaprasad, “Equilibria and steering laws for planar formations,” *Systems and Control Letters*, v 52, 2004, pp. 25-38.

[13] R. Sepulchre, D. Paley, and N. E. Leonard, “Collective motion and oscillator synchronization,” Proceedings of the 16th Symposium on Mathematical Theory of Networks and Systems, Leuven, Belgium, 2004, p. 308.

[14] D. M. Stipanovic, G. Inalhan, R. Teo, and C. J. Tomlin, “Decentralized overlapping control of a formation of unmanned aerial vehicles,” *Automatica*, v 40, 2004, pp. 1285-1296.

[15] H. Yamaguchi and T. Arai, “Distributed and autonomous control method for generating shape of multiple mobile robot group,” *Proceedings 1994 EEE/RSJ International Conference on Intelligent Robots and Systems*, 1994, pp. 800-807.

[16] K. Tan, and M. Lewis, “Virtual Structures for High-Precision Cooperative Mobile Robotic Control,” *Proceedings of the 1996 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1996, pp. 132-139.

[17] M. Hashimoto, F. Oba and T. Eguchi, “Dynamic control approach for motion coordination of multiple wheeled mobile robots transporting a single object,” *Proceedings 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1995, pp. 1944-1951.

[18] D. Rus, B. Donald and J. Jennings, “Moving furniture with teams of autonomous robots,” *Proceedings 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1995, pp. 235-248.

[19] C.P. Tang, R.M. Bhatt, M.Abou-Samah, and V. Krovi, “Screw-theoretic analysis framework for cooperative payload transport by mobile manipulator collectives,” *IEEE/ASME Transactions on Mechatronics*, v 11, n 2, April 2006, pp. 169-178.

[20] R. Brooks, “A Robust Layered Control System for a Mobile Robot,” *IEEE Journal of Robotics and Automation*, v. RA-2, n. 1, 1986, pp. 14-23.

[21] J. Connell and P. Viola, “Cooperative Control of a Semi-Autonomous Mobile Robot,” *Proceedings of the IEEE International Conference on Robotics and Automation*, 1990, pp. 1118-1121.

[22] N. Nilsson, “Teleo-Reactive Programs for Agent Control,” *Journal of Artificial Intelligence Research*, v. 1, 1994, pp. 139-158.

[23] B. Krogh, “A Generalized Potential Field Approach to Obstacle Avoidance Control,” SME-RI Technical Paper MS84-184, Society of Manufacturing Engineers, Dearborn MI, 1984.

[24] O. Khatib, “Real-Time Obstacle Avoidance for Manipulators and Mobile Robots,” *Proceedings of the IEEE International Conference on Robotics and Automation*, St. Louis MO, 1985, pp. 500-505.

[25] R. Arkin, “Motor Schema-Based Mobile Robot Navigation,” *International Journal of Robotics Research*, v. 8, n. 4, 1989, pp. 92-112.

[26] Y. Koren and J. Borenstein, “Potential Field Methods and Their Inherent Limitations for Mobile Robot Navigation,” *Proceedings of the IEEE International Conference on Robotics and Automation*, Sacramento, CA, 1991, pp. 1398-1404.

[27] J. Kim and P. Khosla, "Real-Time Obstacle Avoidance Using Harmonic Potential Functions," *IEEE Transactions on Robotics and Automation*, v. 8, n. 3, 1992, pp. 338-349.

[28] C. Connolly and R. Grupen, "On the Applications of Harmonic Functions to Robotics," *Journal of Robotic Systems*, v. 10, n. 7, 1993, pp. 931-946.

[29] W. Tianmiao and Z. Bo, "Time-Varying Potential Field-Based on Perception-Action Behaviors of Mobile Robots," *Proceedings of the IEEE International Conference on Robotics and Automation*, Nice, France, 1992, pp. 2549-2554.

[30] R. Arkin, “Motor Schema Based Navigation for Mobile Robots: An Approach to Programming by Behavior,” *Proceedings of the IEEE Conference on Robotics and Automation*, Raleigh, NC, 1987, pp. 264-271.

[31] R. Clark, R. Arkin, and A. Ram, “Learning Momentum: On-Line Performance Enhancement for Reactive Systems,” *Proceedings of the IEEE International Conference on Robotics and Automation*, Nice, France, May 1992, pp. 111-116.

[32] Q. Jia and G. Li, “Formation control and obstacle avoidance algorithm of multiple AUV’s based on potential functions and behavior rules,” *Proceeedings 2007 IEEE International Conference on Automation and Logistics*, Aug. 18-21, 2007, pp. 569-573.

[33] F. Schneider and D. Wildermuth, “A potential field based approach to multi robot formation navigation,” *Proceedings 2003 IEEE International Conference on Robotics, Intelligent Systems, and Signal Processing*, Oct., 2003, pp. 680-685.

[34] Y. Liang and H. Lee, “Decentralized formation control and obstacle avoidance for multiple robots with nonholonomic constraints,” *Proceedings 2006 American Control Conference*, June 14-16, pp. 5596-5601.

[35] S. Mastellone, D. Stipanovic, C. Graunke, and M. Spong, “Formation Control and Collision Avoidance for Multi-Agent Non-holonomic systems: Theory and Experiments,” *The International Journal of Robotics Research,* v. 27, n. 1, Jan. 2008, pp.107-126.

[36] A. Fujimori, T. Saito, and G. Bohacs, “Formated navigation of mobile robots with obstacle avoidance,” *Proceedings 2006 IEEE Conference on Robotics, Automation and Mechatronics,* 2006, p 6.

[37] R. Dougherty, V. Ochoa, Z. Randles, and C. Kitts, “A behavioral control approach to formation-keeping through an obstacle field,” *Proceedings IEEE Aerospace Conference*, v 1, 2004, pp. 168-175.

[38] C. De La Cruz and R. Carelli, “Dynamic model based formation control and obstacle avoidance of multi-robot systems,” *Robotica*, v. 26, n. 3, May, 2008, pp. 345-356.

[39] C. Kitts and I. Mas, “Cluster space specification and control of mobile multi-robot systems,” *IEEE/ASME Transactions on Mechatronics*, v 14, n 2, 2009, pp. 207-218.

[40] R. Ishizu, “The Design, Simulation and Implementation of Multi-Robot Collaborative Control from the Cluster Perspective,” C. Kitts, Adv., M.S. Thesis, Dept. Elec Eng, Santa Clara University, Santa Clara, CA, December 2005. Available at http://rsl.engr.scu.edu.

[41] P. Connolley, “Design and Implementation of a Cluster Space Trajectory Controller for Multiple Holonomic Robots,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2006. Available at http://rsl.engr.scu.edu.

[42] T. To, “Automated Cluster Space Trajectory Control of Two Non-Holonomic Robots,” C. Kitts, Adv., M.S. Thesis, Dept. Comp Eng, Santa Clara University, Santa Clara, CA, June 2006. Available at http://rsl.engr.scu.edu.

[43] M. Kalkbrenner, “Design and Implementation of a Cluster Space Human Interface Controller, including Applications to Multi-Robot Piloting and Multi-Robot Object Manipulation,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2006. Available at http://rsl.engr.scu.edu.

[44] B. Tully, “Cluster Space Piloting of a Nonholonomic Multi-Robot System,” C. Kitts, Adv., M.S. Thesis, Dept. Comp Eng, Santa Clara University, Santa Clara, CA, June 2006. Available at http://rsl.engr.scu.edu.

[45] R. Lee, “Model-Based Cluster-Space Control of Multiple Robot Systems,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2007. Available at http://rsl.engr.scu.edu.

[46] I. Mas, O. Petrovic, and C. Kitts, “Cluster space specification and control of a 3-robot mobile system,” *Proceedings of the IEEE International Conference on Robotics and Automation*, Pasadena, CA, 2008, pp. 3763-3768.

[47] I. Mas, J. Acain, O. Petrovic, and C. Kitts, “Error characterization in the vicinity of singularities in multi-robot cluster space control,” *Proceedings of the IEEE International Conference on Robotics and Biomimetics*, Bangkok, Thailand, December 2008, pp. 1911-1917.

[48] E. Girod, “Design and Implementation of Four Robot Cluster Space Control,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2008. Available at http://rsl.engr.scu.edu.

[49] P. Mahacek, I. Mas, O. Petrovic, J. Acain, and C. Kitts, “Cluster space control of autonomous surface vessels,” *Marine Technology Society Journal,* v 43, n 1, 2009, pp. 13-20.

[50] M. S. Agnew, “Cluster Space Control of a 2-Robot Aerial System,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2009. Available at http://rsl.engr.scu.edu.

[51] P. M. Dal Canto, “Three Blimp Cluster Space Control: Derivation and Implementation,” C. Kitts, Adv., M.S. Thesis, Dept. Mech Eng, Santa Clara University, Santa Clara, CA, June 2009. Available at http://rsl.engr.scu.edu.

[52] I. Mas, S. Li, J. Acain, and C. Kitts, “Entrapment/Escorting and Patrolling Missions in Multi-Robot Cluster Space Control,” accepted *IEEE/RSJ International Conference on Intelligent Robots and Systems*, St. Louis MO, October 2009.

[53] D. M. Stipanovic, P.F. Hokayem, M. W. Spong, and D. D. Siljak, “Cooperative Avoidance Control for Multiagent Systems,” *Journal of Dynamic Systems, Measurement and Control*, v 129, n 5, 2007, pp. 699-707.

[54] G. Leitmann and J. Skowronski, “Avoidance control,” *Journal of Optimization Theory and Applications*, v 23, 1977, pp. 581-591.

[55] P. Chindaphorn, “Cluster Space Obstacle Avoidance for Two Non-Holonomic Robots,” C. Kitts, Adv., M.S. Thesis, Dept. Comp Eng, Santa Clara University, Santa Clara, CA, June 2006. Available at http://rsl.engr.scu.edu.

[56] P. Chindaphorn, S. En Huang, T. To, B. Tully, and S. Young, “Decabot: An Experimental Testbed for Collaborative Multi-Robot Navigation,” C. Kitts, Adv., Undergraduate Thesis, Santa Clara University, Santa Clara, CA, June 2005.

[57] M. Barycza, P. Connolly, R. Kottenstette, and E. Redwine, “OmniFleet: The Design and Implementation of Multiple Holonomic Robots,” C. Kitts and N. Quinn, Advs., Undergraduate Thesis, Santa Clara University, Santa Clara, CA, June 2004.

[58] A. Moreland, A. Cuaresma, J. de la Fuente, K. Kerns, and V. Reddy, “Omnibot II: The Design and Implementation of Dual Robot Collaborative Object Transportation,” C. Kitts, Adv., Undergraduate Thesis, Santa Clara University, Santa Clara, CA, June 2005.

[59] K. Stanhouse, “Cluster Space Obstacle Avoidance for Mobile Multi-Robot Systems,” C. Kitts, Adv., M.S. Thesis, Dept. Elec Eng, Santa Clara University, Santa Clara, CA, December 2006. Available at http://rsl.engr.scu.edu.

[60] C. Kitts, “Surf, turf, and above the Earth,” *IEEE Robotics and Automation Magazine*, v 10, n 3, September 2003, pp. 30-36.

[61] MATLAB Function Reference, The Mathworks, Natick, MA, 2008.