Mixed CO2-Water Injection Into Geothermal Reservoirs: A Numerical Study

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ABSTRACT

Heat recovery from deep/ultra-deep geothermal reservoirs and CO_2 storage in geologic formations are promising techniques for reducing CO_2 emissions. Both techniques involve injection of fluid into deep saline aquifers, oil/gas reservoirs, or stimulated fractured crystalline formations. A potential alternative to storing CO_2 in aquifers involves dissolution of CO_2 in the return (cold) water stream of geothermal doublets. The efficiency of injection and sweep, as well as the safety of operations highly depends on reservoir physical, thermal, and compositional properties, which may change following CO_2 and/or cooled water injection and potential reaction. However, although there is convincing evidence that thermal, flow, and chemical processes are strongly coupled, the nature of the coupling is not yet fully understood. In order to understand the spatial and temporal hydrothermal and chemical effects on targeted geologic media of CO_2 injection, a geometrically flexible approach for solving reactive non-isothermal density–viscosity-dependent flow and reactive transport in low-enthalpy geothermal systems is utilized. The method is applied to simulate the non-isothermal reactive flow transport in a doublet geothermal system. Several injection scenarios are analyzed. The overall heat recovery and CO_2 -storage capacity are calculated for different formations. Furthermore, the influence of both the induced viscosity and porosity changes on flow and heat transfer in such systems are investigated.

1. INTRODUCTION

There is an increased interest in recovering geothermal energy from deep geological resources as an approach for reducing CO_2 emissions. Additionally, injecting CO_2 into geothermal geological formation is considered as one of the mitigation methods to reduce CO_2 emissions further (Pruess 2006, Salimi and Wolf 2012). Dissolved CO_2 injection into geothermal reservoirs has several advantages compared to the supercritical CO_2 -sequestration. These include: less risk of leakage and unwanted release from storage because of reduced buoyancy effects and not over-pressurizing the aquifers, and no salt precipitation due to formation dry-out.

Many factors control the dynamics of the cold water and CO_2 injection into geothermal aquifers, including: density and viscosity variations induced by cold water penetrating into the hot reservoir (Saeid et al., 2014); porous media heterogeneity (Ferguson 2008, Diersch and Kolditz, 2002; Salimi and Wolf 2012); seasonal fluctuations in discharge and injection temperature (Saeid et al., 2014); phase changes (Salimi and Wolf 2012); CO_2 trapping and immobilization (Kuhn et al., 2009, Tao and Bryant, 2012); and the chemistry of the formation fluid and rock present in the system (Xu et al., 2006).

One of the primary concerns is that chemical reactions induced by CO_2 combined with cooled water injections may affect the overall heat recovery and CO_2 -storage capacity. A possible consequence of the interaction between CO_2 and the rock is the formation of a low pH solution that can alter rock mineralogy. This interaction, however, takes place at high temperatures and associated pressure and might thus follow different rules than observed at normal conditions. The subsequent porosity evolutions will cause problems or ease for injectivity (Andreani et. al., 2008).

We aim to advance our understanding of thermal and reactive processes in low-enthalpy geothermal aquifers where dissolved CO_2 in re-injected cold-water intrudes into the hot geothermal formations. Specific questions to be addressed are: What are the chemical effects of CO_2 flow on alteration of the porosity and permeability of the porous formations under reservoir conditions? How these influence the overall heat recovery? What are the roles of heterogeneity in such systems?

A flexible and accurate numerical model is utilized to study the combined effect of different chemical, heat and flow processes on performance of geothermal doublets. In this work we demonstrate how porosity evolution and viscosity variations caused by thermal and chemical stresses can alter the heat transfer in heterogeneous media.

2. METHODOLOGY

This section presents an overview of the numerical models employed for this study as well as the methods for energy analyses.

2.1 Non-isothermal multi-component flow and transport

A modeling platform called CSMP++ (Matthäi et al., 2007) is employed to solve for non-isothermal multi-component flow and transport in low enthalpy geothermal aquifers. This platform features an object-oriented application programmer interface (API), designed for the simulation of complex geological processes and their interactions (Matthäi et al., 2009; Nick et al., 2011). This geometrically flexible and stable transport algorithm can resolve complex geological structures and many orders of magnitude of permeability variations which are ubiquitous in geological formations (Milliotte and Matthäi, 2014). CSMP++ relies on a non-oscillatory higher-order accurate finite-element node-centered finite-volume (FEFV) scheme for solving time-dependent advection-dispersion problems (Matthäi et al., 2009). An algebraic multigrid method for system of equations, SAMG, (Stüben, 1999) is employed as the solver for solving flow and transport equations.

2.2 Reactive solver

Biogeochemical Reaction Network Simulator (BRNS) (Regnier et al., 2002), for solving a set of coupled non-linear equations modeling reactive solutes, is used as a flexible numerical engine applicable for a variety of reactive transport problems in subsurface environments such as, groundwater contamination, interaction between groundwater and sea water (Thullner et al., 2009; Nick et al., 2013). One of the main features of BRNS is an automated procedure for code generation using a MAPLE interface. This provides high flexibility to include and combine alternative biogeochemical process descriptions in the model. Using this interface, kinetically controlled and equilibrium reactions, and combinations of both can be specified with arbitrary form of equations describing the transformations of the chemical species. A description on how symbolic programming can be used to create the model specific source code necessary to the numerical solution of the governing equations is explained in Regnier et al. (2002).

2.3 Coupled THC modeling

It is essential to utilize computational tools with predictive capabilities. CSMP++ and BRNS are coupled to simulate nonisothermal reactive multi-component transport in heterogeneous porous media that are discretized with spatially variably refined unstructured grids to allow a realistic representation of the flow geometry. The transient flow, advection–dispersion and reaction are computed in a sequential manner (e.g. Steefel and MacQuarrie, 1996).

Flow, heat transfer and solute transport are nonlinear processes as the fluid viscosity and density affecting the velocity field are a function of the temperature and concentration. In deep geothermal systems heat and chemical stresses can also cause physical alterations, which may have a significant effect on flow and reaction rates. As a consequence such alterations will lead to changes in permeability and porosity of the formations due to mineral precipitation and dissolution. These changes imply that the flow field needs to be recalculated if the porous media properties, such as porosity and permeability, and fluid properties, such as fluid density and viscosity, change due to transport or reactions (e.g. Nick et al., 2013; Raoof et al., 2013). To address these problems, we apply sequential non-iterative operator splitting: in each time step first the transport solution is obtained and then the reaction part is solved using the reactive solver of BRNS (Nick et al., 2013).

2.4 Reactions

In this work, it is assumed that calcium carbonate is the only reactive mineral present in the formation. The equilibrium parameters are defined as a function of temperature. The dissolution of calcite has been proposed to occur via three parallel reactions (Plummer et al., 1978):

$$CaCO_{3}+H^{+} \longleftrightarrow Ca^{2+}+HCO_{3}^{-}$$

$$CaCO_{3}+H_{2}CO_{3} \longleftrightarrow Ca^{2+}+2HCO_{3}^{-}$$

$$CaCO_{3} \longleftrightarrow Ca^{2+}+CO_{3}^{2-}$$
(1)

Several studies have addressed the dissolution–precipitation behavior of calcite. Following the approach of Li et al. (2008), we describe the rate of calcite dissolution and precipitation by a Transition State Theory (TST) rate law based on the three parallel reactions as identified by Plummer et al. (1978) and Chou et al. (1989):

$$Rate = A(k_1 a_{H^+} k_2 a_{H_2CO_3} + k_3)(1 - a_{CO_3^{2-}} a_{Ca^{2+}} / K_{eq})$$
(2)

where A is the reactive surface area of calcite, k_1 , k_2 and k_3 are experimentally determined rate constants and are a function of temperature (Plummer et al., 1982), a_{H^+} , $a_{H_2 CO^*}$, $a_{Ca^{2+}}$ and $a_{CO^{2-}}$ are the activities of the subscripted species and K_{eq} is the solubility product for calcite.

2.5 Energy production

The thermal capacity $(P_{doublet})$ of a geothermal doublet can be expressed as:

$$P_{doublet} = Q \rho_{f} c_{f} (T_{production} - T_{injection})$$
⁽³⁾

with $P_{doublet}$ (in W), Q the flow rate of the doublet (m³/s), ρ_f the water density (kg/m³), c_f the specific heat of the formation water (J/kg K) and T the fluid temperature (in degrees C or K).

2.6 Required pump energy and Coefficient of performance (COP)

The power, P_{pump} , [W] needed for a pump in a doublet system can be calculated as,

$$P_{pump} = \frac{Q\Delta P}{\eta} \tag{4}$$

where η is the pump efficiency [-] and where pressure, P, is expressed in Pa. Coefficient of performance (COP) is defined as the ratio of thermal capacity of geothermal energy, $P_{doublet}$, to required pump energy, P_{pump} .

2.7 Model set-up

A finite-element node-centered finite-volume model is used to examine mechanisms controlling the fate of CO_2 injected in geothermal aquifers (Figure 1). In this study the geothermal reservoirs are modeled as a two-dimensional (2D) horizontal porous medium initially filled with hot formation water. We consider a geothermal reservoir with a length of 1500 m, a width of 1000m, and a height of 100m. In this model, we define fluid and rock properties as functions of temperature. We take also into account porosity evolution induced by reactions within the porous rock. It is assumed that the reservoir is at the depth of 3000 m and its temperature is estimated to be equal to 100 °C. The produced hot water is cooled down to 40 °C. We assume initially that all the concentrations in the reservoir are in an equilibrium state with CaCO₃. Initially, the reservoir is saturated with hot water. Then, a cold mixture of CO_2 -water is injected through the injection well and, subsequently, water and CO_2 are produced through the production well. Table 1 shows the reservoir properties utilized for the numerical simulations. All the simulations are conducted for a period of 50 years under a constant pressure difference of 50 bars applied between the production and injection wells. A CO_2 concentration of 1.0 mol/L is considered in the injection well. The initial discharge is equal to 217 m³/hr.



Figure 1: Numerical experiment set-up and permeability field with $\lambda_x = \lambda_y = 100 \text{ m}$, $\sigma^2 = 0.1$ and $\mu = 1 \times 10^{-13} \text{ m}^2$

In order to consider heterogeneity in our simulations a random field generator, FGEN, developed by Robin et al. (1993) is used to generate the permeability fields assuming the permeability k is log-normally distributed (e.g. Nick 2008). An isotropic Gaussian correlation function for the log-permeability can be used given by:

$$R(h) = e^{-(h/\lambda)^2}$$
⁽⁵⁾

where λ denotes correlation length and *h* denotes Euclidean distance. Therefore, a normal distribution with sample size given by the number of elements in the mesh can be formed with a given mean permeability, μ , and a given variance, σ^2 .

Table 1:	List	of rock	and	fluid	properties
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Parameter	Value	Unit
Mean Permeability	1×10 ⁻¹³	m^2
Porosity	20	0⁄0
Rock density	2650	Kg.m ⁻³
Fluid thermal conductivity	0.67	$W.m^{-1}.K^{-1}$
Rock thermal conductivity	3	$W.m^{-1}.K^{-1}$
Fluid specific heat capacity	4190	J.kg ⁻¹ .K ⁻¹
Rock specific heat capacity	980	J.kg ⁻¹ .K ⁻¹

3. RESULTS

Considering non-isothermal horizontal flow in a heterogeneous porous medium, a series of simulations on several heterogeneous aquifers is carried out to examine the effect of heterogeneity together with porosity and viscosity alteration on heat transfer in geothermal aquifers.

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3.1 Homogeneous versus heterogeneous porous media

Considering all detail heterogeneity in field scale models is computationally costive and using simplified models is desirable. It is known that the dispersive behavior of solute transport is a function of scale, correlation length and heterogeneity (Berkowitz et. al., 2006). Moreover, if the viscosities or densities of injection fluid and the formation fluid are different, both porous media and fluid properties control the dispersion (e.g. Nick et. al., 2009). Therefore, variable dispersivity, early breakthrough times, and long tails of breakthrough curves are characteristic of such solute transport.

Here, the focus is on presenting the effect of formation heterogeneity on non-isothermal and non-reactive flow and solute transport in geothermal reservoirs. Figure 2 shows the cold plume after 10 years of operation in a homogeneous aquifer and a heterogeneous aquifer. The mean value of the permeability of the heterogeneous field is equal to the permeability of the homogeneous medium. Note that pressure at the injection and production wells are kept constant for the entire 50 years of simulations. With this condition, the total discharge alters in time due to the viscosity effect induced by injection of cold water into a warm reservoir. This effect is more pronounced near the injection wellbore where the hydraulic conductivity decreases noticeably. For example, after 2 years of production the initial discharges (217 m³/hr) for both heterogeneous and homogeneous aquifers decline to 188 and 158 m³/hr, respectively. It can be seen in Figure 3 that this unfavorable effect on discharge is less pronounced in the heterogeneous aquifer when compared to the results of the homogeneous formations. This favorable effect of heterogeneity on flow is mainly due to a process known as viscous crossflow (Zapata and Lake, 1980). The viscous crossflow process occurs when the injected cold fluid from high permeable regions is diverted to low permeable regions due to the change of formation fluid viscosity induced by temperature changes. The effect of the viscous crossflow process on total discharge is maximized during the first 15 years of operation and diminishes as the whole reservoir becomes cold.



Figure 2: Scaled temperature fields after 10 years of operation for a homogeneous geothermal aquifer ($k=1 \times 10^{-13}$) and a heterogeneous formation ($\lambda_x = \lambda_y = 100 \text{ m}, \sigma^2 = 2$ and $\mu = 1 \times 10^{-13} \text{ m}^2$)



Figure 3: Scaled temperature and CO₂ concentration measured at the production well together with produced energy and scaled discharge for both homogeneous aquifer ($k=1 \times 10^{-13}$) and heterogeneous aquifer ($\lambda_x = \lambda_y = 100$ m, $\sigma^2 = 2$ and $\mu = 1 \times 10^{-13}$ m²)

The results show that heterogeneous media causes early breakthrough times for both injected cold water and CO_2 fronts. This is due to two reasons: a) presence of high permeability regions providing fast flow pathways, and b) the viscose crossflow effect on total discharge for the heterogeneous formation minimizing the unfavorable effect of viscosity changes on the total flow. It is clear that the favorable effect of heterogeneity on increasing discharge plays against the unfavorable effect of heterogeneity on the lifetime.

These two effects, therefore, influence the produced energy. In this example, the total produced energy during the first ~ 20 years of production is greater for heterogeneous formation than that of homogeneous formation.

A series of simulations are conducted on different formations with different values of log permeability variance. Figure 4 illustrates the effect the degree of heterogeneity has on the calculated lifetime, produced energy, pump required energy and COP. These are the ensemble values calculated for several randomly generated permeability fields. Note that the lifetime of this project is defined as the time when the temperature at the production drops by 20%. Net energy is calculated as the difference between the total energy produced during the life time and the total energy used for pumping. It is evident, for these sets of heterogeneous fields ($\lambda_x = \lambda_y = 100 \text{ m}$ and $\mu = 1 \times 10^{-13} \text{ m}^2$), that heterogeneity has an overall unfavorable effect on the performance of low-enthalpy geothermal systems.



Figure 4: Dependence of lifetime, net energy, pump required energy, and COP on the degree of heterogeneity for heterogeneous geothermal aquifers ($\lambda_x = \lambda_y = 100$ m and $\mu = 1 \times 10^{-13}$ m²).

3.2 COMBINED CO2 INJECTION WITH COLD WATER

Here, we consider cold mixed CO_2 -water injection into a geothermal reservoir in order to understand the effect of CO_2 sequestration on performance of low-enthalpy geothermal systems. Injection of low pH solution induced by CO_2 dissolution may alter the reservoir characteristics through change in porosity and subsequently the permeability of the reservoir. Three aquifers are considered for simulations: an aquifer with no reactive mineral, an aquifer with 2% reactive mineral and an aquifer with 10% reactive mineral (i.e. calcite). In the two systems with reactive mineral, porosity of the formation near the injection wellbore increases enhancing the permeability. This favorable effect increases the total discharge and, consequently, the recoverable heat in place is swept faster under a constant pressure difference between the wells. These behaviors are illustrated by Figure 5, where the results of simulations for three different scenarios are shown. The results obtained from these simulations highlight the combined adverse effect of viscosity and the positive effect of increased permeability on flow.



Figure 5: The effect of initial reactive mineral in the aquifers on temperature of produced fluid and discharge under constant pressure gradient between the injection and production wells.

Figure 6 demonstrates the CO_2 breakthrough curves showing that the CO_2 front is far ahead of the temperature front as the speed of the thermal front is retarded by a factor of ~3 in this example. Enhanced flow causing earlier CO_2 and cold-water breakthrough is one of the main effects of dissolution in the reservoir. Because of the significant increase in the discharge for the scenario with CO_2 injection into a reactive system, the recovered energy is improved, particularly, during the first 30 years of operation. It is however evident that the lifetime of the geothermal project is reduced considerably due to a large increase in discharge.

Several simulations are conducted on different heterogeneous aquifers with different degrees of heterogeneity. In Figure 7, we illustrate the results of two injection scenarios: cold-water injection with and without CO_2 injection. The analysis of these results of the simulations for various heterogeneous aquifers highlights a drop in total lifetime and a minimized effect on recovered energy due to CO_2 injection.

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Figure 6: Temperature and CO₂ concentration measured at the production well together with produced energy and discharge for a heterogeneous aquifer ($\lambda_x = \lambda_y = 100 \text{ m}, \sigma^2 = 2 \text{ and } \mu = 1 \times 10^{-13} \text{ m}^2$) with and without reactive minerals. The thicker lines represent the results of the system without reaction.



Figure 7: Dependence of lifetime and produced net energy on the degree of heterogeneity for heterogeneous geothermal aquifers ($\lambda_x = \lambda_y = 100 \text{ m}$ and $\mu = 1 \times 10^{-13} \text{ m}^2$) for cold water injection wit and without dissolved CO₂.

4. CONCLUSIONS

This paper reports numerical results of the non-isothermal multicomponent reactive flow and transport in heterogeneous geothermal aquifers. We investigate the role of heterogeneity, viscosity alteration and porosity evolution on heat transfer in geothermal aquifers. The following conclusion arising from this analysis can be drawn:

- 1. Permeability heterogeneities in a geothermal aquifer considerably impact both heat recovery and lifetime. Suggesting that characterization of geothermal reservoirs is essential for evaluating energy recovery in such reservoirs.
- 2. Studying the effect of viscosity changes induced by formation fluid temperature alteration reveals that ignoring these effects leads to significant errors for calculating the lifetime and produced energy for geothermal systems.
- 3. The temperature induced viscosity effect has a less adverse effect on overall heat production in the formation with large variation of permeability field mainly due to viscous crossflow process.
- 4. Cold mixed CO₂-water injection into a geothermal reservoir results in CO₂ storage in the subsurface with eventual decreases in the lifetime of the project due to increases of the flow rate. The increase in flow rate, however, enhances the total recovered energy.

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