New System Design for Serial-MSK Based On Laurent Decomposition

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Abstract— **In this paper, a new approach to coding, modulation, and detection design for serial minimum-shift keying (MSK) modulation scheme is presented. The design of the new linear modulator is based on the Laurent decomposition of a well-known modified MSK scheme termed duobinary MSK. It is shown that a simple linear receiver can be designed to optimally detect the coded symbols. The detection problem for the recovery of the symbols sequence from the decision variable sequence is one corresponding to memoryless linear modulation. It is also demonstrated that the Euclidean distance between different signals is directly related to the Hamming distance between corresponding coded sequence. Therefore, optimum encoders (for a given rate and constraint length) that maximize the minimum Hamming distance can be applied.**

I. Introduction

MINIMUM-SHIFT keying is particularly important amongst constant envelope modulation schemes as it produces signals with a relatively small bandwidth and can be also expressed in the form of two binary antipodal pulse amplitude modulated (PAM) signals with non-overlapping pulses [1], [2] (a form of offset quadrature phase shift keying [OQPSK] to be precise). Thus, MSK has the performance of binary antipodal signalling in noise, and can be optimally demodulated using a simple inphase-quadrature receiver performing *symbol-by-symbol* detection. The OQPSK description of MSK is usually referred to as parallel-MSK [2]. For high data-rates applications, maintaining precise synchronization and balancing of the inphase and quadrature channel data signals on carriers is somehow difficult. In such situations, serial-MSK [3] is preferable over the parallel one.

Serial-MSK modulation and demodulation have the advantage that operations are performed serially so that the above mentioned difficulties can be totally avoided. In 1977, Amoroso *et al.*, showed how to generate serial-MSK signal using a binary phase-shift keying (BPSK) signal followed by an appropriately designed bandpass conversion filter [3]. The problem with this approach is that the phasing of the transmitter's local oscillator with respect to the data keying is critical when the ratio of carrier to data rate is not large. In such case, undesirable terms are generated which cause the serial-MSK signal envelope to fluctuate.

In 1987, Moreno and Pasupathy, constructed another approach to generate serial-MSK signals using the continuous-phase frequency-shift keying (CPFSK) description of MSK which is usually referred to as fast FSK (FFSK) [4]. In this paper, it was demonstrated that serial-MSK can be generated if the FFSK modulator is preceded by a feed-forward differential encoder. In this way, serial-MSK can be considered as a modulation with memory. When a convolutional channel encoder is combined with the modulator to improve the power efficiency, the optimum *maximum-likelihood sequence estimator* (MLSE) receiver utilizes both types of memory. A complete study of how to design convolutional codes for such modulation scheme has been exploited in [4]. These convolutional codes are chosen in such a way that when combined with serial-MSK the resulting receiver (MLSE) for the coded modulation has the smallest possible number of states. Such codes were termed matched codes. The optimization criterion in their study was to maximize the minimum Euclidean distance of the coded serial-MSK scheme for a given code rate and a fixed number of states in the optimum MLSE receiver. On the other hand, mismatched codes (convolutional coded based on conventional techniques) can achieve larger minimum Euclidean distance values at the expense of increasing the complexity of the MLSE receiver, specifically, doubling the total trellis states of the combined coded modulation system. The best mismatched encoders (with maximum free Hamming distance) for serial-MSK can be found in the literature [5], [6].

In this paper, we will present a new approach for generating serial-MSK signals which does not require the use of BPSK signal. Hence, avoiding the possibility of destroying the constant-envelope property of MSK caused by the relative phase in the BPSK signal mentioned previously. This approach is based on appropriately encoding a modified MSK scheme termed duobinary MSK and its Laurent decomposition. It is shown that a simple receiver can be constructed to optimally demodulate and detect the signal. When it comes to convolutional code design, the complexity of the receiver is independent of the structure of the channel encoder. It is demonstrated that the Euclidean distance between any two signals is linearly related to the Hamming distance of the corresponding coded symbols. Therefore, convolutional codes based on conventional techniques can be applied to achieve the *best* performance for the same MLSE decoder complexity (for a fixed code rate and constraint length) given in [4].

II. Serial-MSK based on FFSK

Fig. 1 shows a block diagram of a coded serial-MSK system generated using an FFSK modulator. At time t a binary k-tuple

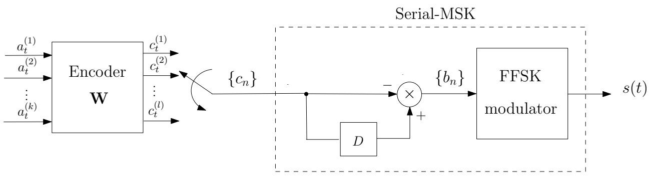

Fig. 1. Coded serial-MSK system generated using FFSK modulator and a feed-forward differential encoder. The D block represents a one symbol delay.

$(a_t^{(1)}, a_t^{(2)}, ..., a_t^{(k)})$, $a_n^{(m)} \in \{-1, 1\}$ produces at the output of the channel encoder $\mathbf{W}$ a binary l-tuple $(c_t^{(1)}, c_t^{(2)}, ..., c_t^{(l)})$, $c_n^{(m)} \in \{-1, 1\}$, which then pass through the feed-forward differential encoder to produce the $b_n \in \{-1, 1\}$ symbols using the mapping rule $b_n = -c_n c_{n-1}$.

The FFSK modulator generates signals with frequencies separated by half the symbol rate. If we define the frequencies $f_- = f_c - 1/4T$ and $f_+ = f_c + 1/4T$, where f_c is the carrier frequency and T is the coded symbol period, then the transmitted serial-MSK signal $s(t)$, according to [4], can be mathematically expressed as

$$s(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_- t + \theta(t)), \tag{1}$$

where E is the transmitted energy per symbol, and $\theta(t)$ is the "tilted" excess-phase that is given by

$$\theta(t) = \frac{\pi}{2T} \int_0^t \sum_m b_m h(\tau - mT)\, d\tau + \frac{\pi t}{2T}, \tag{2}$$

where $\{..., b_{-1}, b_0, b_1, ...\}$ is a binary symbol stream input to the FFSK modulator. At the symbol transitions, $\theta(t)$ is given by

$$\theta_n = \theta(nT) = \frac{\pi}{2} \sum_{m=0}^{n-1} (1 + b_m), \tag{3}$$

which can only be exactly one of two values (modulo π): 0, π. Equivalently, θ_n can be expressed in terms of the channel coded sequence $\{c_n\}$ as

$$\theta_n = \frac{\pi}{2} \sum_{m=0}^{n-1} (1 - c_m c_{m-1}), \tag{4}$$

with $c_{-1} = -1$, assuming $\theta_0 = 0$ at $t = 0$. The excess-phase can be described by the trellis diagram shown in Fig. 2.

To improve the performance of serial-MSK, a natural thought would be to employ error-control coding. The design of convolutional codes for such modulation scheme has been considered in [4]. It has been shown that the performance and complexity of the optimum (MLSE) receiver depend on the structure of the code. These codes cannot always provide the best performance while achieving the lowest complexity possible of the MLSE receiver. Such codes were termed matched codes and the search for good codes applied to serial-MSK was conducted using computer-aided search. Next, we will show a new approach for generating serial-MSK signal that yields a simplification in the modulation, detection and coding design for serial MSK.

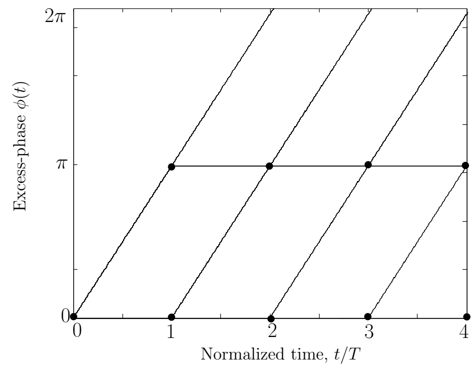

Fig. 2. The excess-phase trellis of Serial-MSK for $t \geq 0$.

III. Precoded Duobinary MSK and Its Laurent Decomposition

We would like to consider in this section a new approach for generating serial-MSK signals using duobainry MSK. Duobinary MSK is an instance of a modified MSK that may be described as employing correlative coding [10] using coding polynomial $(1 + D)/2$. Fig. 3 shows a block diagram for

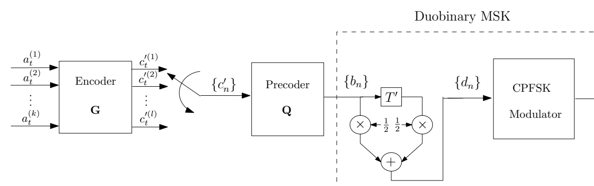

Fig. 3. Coded duobinary MSK system employing precoding.

a coded duobinary MSK transmitter employing precoding. The channel encoder $\mathbf{G}$ generates the symbols $c'_n \in \{-1, 1\}$ which then pass through the precoder $\mathbf{Q}$ to produce the $b_n \in \{-1, 1\}$ symbols using the following mapping rule $b_n = (-1)^{n+1} c'_n c'_{n-1}$.

The transmitted duobinary MSK signal can be expressed as

$$s(t) = \sqrt{\frac{2E'}{T'}} \cos(2\pi f_c t + \phi(t)), \tag{5}$$

where E' is the transmitted energy per c'_n symbol, T' is the c'_n symbol period, and the excess-phase $\phi(t)$ in this case is given by

$$\phi(t) = \frac{\pi}{2T'} \int_0^t \sum_m d_m h(\tau - mT') d\tau, \tag{6}$$

where $d_m = (b_m + b_{m-1})/2 \in \{-1, 0, 1\}$ is the input to the CPFSK modulator, and $h(t)$ is a NRZ rectangular pulse shape of duration T'. Fig. 4 shows the excess-phase trellis diagram for duobinary MSK signal.

Following the development given by Laurent [8], the low-pass equivalent signal $s_\ell(t)$ of duobinary MSK may be represented by a linear superposition of two amplitude-modulated

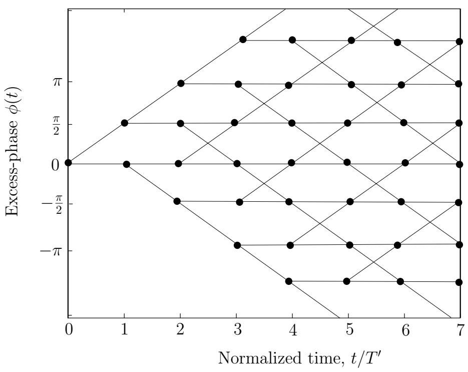

Fig. 4. The excess-phase trellis of duobinary MSK for $t \geq 0$.

pulse trains (PAM). The PAM representation of $s_\ell(t)$, with respect to f_c, from [8] is

$$s_\ell(t) = \sqrt{\frac{2E'}{T'}} \sum_{k=0}^{1} \sum_{n=-\infty}^{\infty} \beta_{k,n} c_k(t - nT'), \qquad (7)$$

where the Laurent coefficients $\beta_{0,n}$ and $\beta_{1,n}$ can be expressed as

$$\beta_{0,n} = \begin{cases} c'_n, & \text{for } n \text{ odd;} \\ -jc'_n, & \text{for } n \text{ even,} \end{cases} \qquad (8)$$

while,

$$\beta_{1,n} = \begin{cases} -b_{n-1}c'_n, & n \text{ even;} \\ -jb_{n-1}c'_n, & n \text{ odd,} \end{cases} \qquad (9)$$

and the Laurent pulses, $c_0(t)$ and $c_1(t)$, are given respectively by

$$c_0(t) = \begin{cases} S(t)S(t+T'), & 0 \leq t \leq 3T'; \\ 0, & \text{otherwise,} \end{cases} \qquad (10)$$

$$c_1(t) = \begin{cases} S(t)S(t+3T'), & 0 \leq t \leq T'; \\ 0, & \text{otherwise,} \end{cases} \qquad (11)$$

where

$$S(t) = \begin{cases} \sin\left(\frac{\pi t}{4T'}\right), & t \in [0, 4T']; \\ 0, & \text{otherwise.} \end{cases} \qquad (12)$$

It has been shown in [7] that applying two stages of coding, the last being a double repetition code[1], the lowpass equivalent signal of duobinary MSK can be expressed as

$$s_\ell(t) = \sqrt{\frac{2E}{T}} \sum_{n=-\infty}^{\infty} c_n g(t - nT), \qquad (13)$$

where $T = 2T'$ is the c_n symbol period. This situation is depicted in Fig. 5, where the channel encoder $\mathbf{G}$ is decomposed into channel encoder $\mathbf{W}$ followed by a double repetition code.

The pulse $g(t)$ provided in (13) is given by

$$g(t) = p(t - T/2) + j\tilde{p}(t), \qquad (14)$$

for which

$$p(t) = c_0(t) - c_1(t + T/2), \qquad (15)$$

[1]A technique used to simplify the coding design for generalized MSK with two symbol period pulse shape duration.

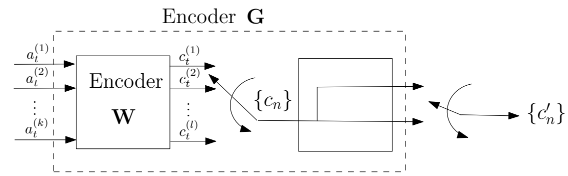

Fig. 5. The decomposition of channel encoder $\mathbf{G}$ in Fig. 3 into channel encoder $\mathbf{W}$ followed by a double repetition code.

and

$$\tilde{p}(t) = c_1(t - 3T/2) - c_0(t). \qquad (16)$$

A plot of the pulses $p(t)$ and $\tilde{p}(t)$ is shown in Fig. 6.

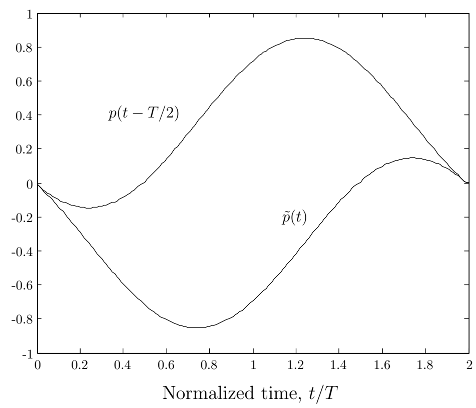

Fig. 6. The impulse response of the real and imaginary parts of $g(t)$, i.e., $p(t - T/2)$ and $\tilde{p}(t)$.

It can be shown that the excess phase of the special coded duobinary MSK signal at $t = 2nT' = nT$ is related to the encoder $\mathbf{W}$ output sequence $\{c_n\}$ by

$$\phi_n = \frac{\pi}{2} \sum_{m=0}^{n-1} (1 - c_m c_{m-1}), \qquad (17)$$

which is identical to the excess-phase values at the symbol transition ($t = nT$) for serial-MSK signal generated using FFSK modulator (i.e., θ_n) given in (4). Fig. 7 shows the

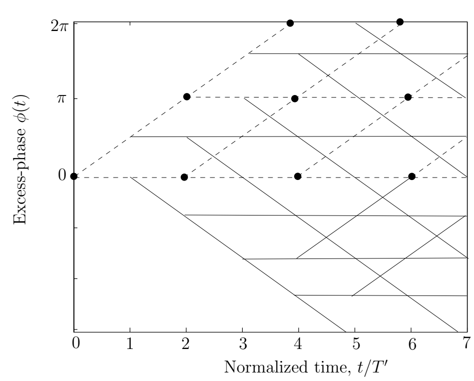

Fig. 7. Possible phase trajectories for coded duobinary MSK signal (shown in dashed lines) generated using the system shown in Fig. 3, when the channel encoder $\mathbf{G}$ is decomposed in two stages as depicted in Fig. 5.

possible trajectories of $\phi(t)$ for such coded duobinary MSK which are shown in dashed lines.

Now, consider the case where encoder $\mathbf{G}$ is just a double repetition code (i.e., $\mathbf{W}$ is the identity mapping $c_n = a_n$). Assuming a_n are independent identically distributed random variables which take on values $+1$ or -1 with equal probability, the spectral density function of the lowpass equivalent signal $s_\ell(t)$ given in (13) can be expressed as [9]

$$S_{s_\ell}(f) = \frac{1}{T_b}|G(f)|^2, \quad (18)$$

where T_b is the information bit period, and $G(f)$ is the Fourier transform of $g(t)$. By evaluating the Fourier transform, it can be easily shown that $G(f)$ can be expressed as

$$G(f) = \frac{T_b}{2\pi} e^{-j(2\pi f T_b - \pi/4)} \frac{\sin(2\pi f T_b)}{f T_b(1 - 2 f T_b)}. \quad (19)$$

Using (18), we find that

$$S_{s_\ell}(f) = \frac{T_b}{8\pi^2}\left(\frac{\sin(2\pi f T_b)}{f T_b(1 - 2 f T_b)}\right)^2. \quad (20)$$

It is interesting to note that the power spectrum of precoded duobinary MSK employing double repetition code is identical to the one given for (precoded) MSK with no coding shifted in frequency by $1/4T_b$, i.e.,

$$S_{s_\ell}(f + 1/4T_b) = \frac{16 T_b}{\pi^2}\left(\frac{\cos(2\pi f T_b)}{1 - 16 f^2 T_b^2}\right)^2. \quad (21)$$

From (17) and (21) we can conclude that the output signal of the system depicted in Fig. 3 is identical to a serial-MSK signal, provided that the channel encoder $\mathbf{G}$ is decomposed into two stages as shown in Fig. 5.

IV. New Serial-MSK Modulator and Receiver Design

A. The Modulator

Equations (13)–(16) suggest another strategy to modulate the serial-MSK signal other than the non-linear FFSK modulator (depicted in Fig. 1) using a simple linear modulator. Fig. 8 shows a block diagram for a baseband model of a modulator

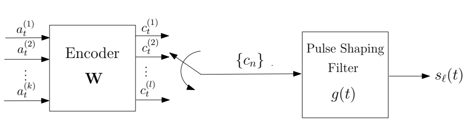

Fig. 8. Equivalent complex baseband model of the new serial-MSK modulator. The impulse response of the pulse shaping filter $g(t)$ is as given in (14).

based on these results. The output waveform $s_\ell(t)$ is a PAM signal with overlapping pulses of duration $2T$ that carries the coded symbols $\{c_n\}$ *serially* at a rate $1/T$. Therefore, inter-symbol interference exists in this signal, where it affects a finite number of symbols. Inter-symbol interference (ISI) introduces *memory* in the signal, which is the case for serial-MSK. The optimum detection of signals with ISI is based on *maximum-likelihood sequence estimation* (MLSE) which is typically implemented using the Viterbi algorithm.

B. Simplified Demodulation and Detection Design

Based on (13), i.e., expressing serial-MSK as a PAM signal, we can design a simple linear receiver (assuming $f_c T \gg 1$) for recovering the coded symbols c_n. Fig. 9 shows a linear

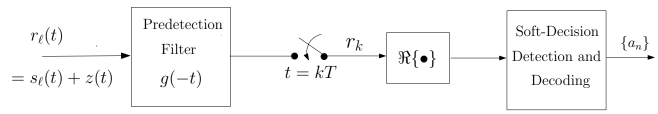

Fig. 9. Simple linear receiver for serial-MSK. The decoder used here is the inverse of the channel encoder $\mathbf{W}$.

coherent receiver used to (soft) detect coded serial-MSK generated using the new modulator. This receiver is similar to that of coded BPSK, except that a different predetection filter is used.

Let us assume the lowpass equivalent of the received signal $r_\ell(t)$ is simply the transmitted signal $s_\ell(t)$ corrupted by an additive white Gaussian noise process $z(t)$ with power spectrum $\mathcal{N}_0$. Thus the received signal can be expressed as

$$r_\ell(t) = s_\ell(t) + z(t). \quad (22)$$

It is well-known [9, p. 600] that filtering the signal with filter matched to the pulse shape and sampling the filtered signal at the end of the symbol periods produces a set of *sufficient statistics* for the symbol stream, from which the symbols can then be recovered. Fig. 9 shows a receiver that implements this approach to arrive at an optimal receiver.

It can be easily verified that the real part of the matched filter's sampled output at $t = kT$ (i.e, r_k), is given by

$$\Re\{r_k\} = \sqrt{2ET} c_k + n_k, \quad (23)$$

where $\{n_k\}$ are *uncorrelated* zero-mean Gaussian random variables (hence independent) with variance equal to $\mathcal{N}_o T$. Lets consider the situation where no coding is applied ($\mathbf{W}$ in Fig. 8 is the identity mapping). In this case, *symbol-by-symbol* detection (hard decision) is optimal in the sense of minimizing the probability of error of detecting the information symbol a_n.

V. Coded Serial-MSK

A. The Euclidean Distance

It is well-known [9] that for any coded-modulation system, the asymptotic performance of the optimum receiver in additive white Gaussian noise channel (AWGN) in terms of the asymptotic average probability of bit error $\mathcal{P}_e$ may be defined as

$$\mathcal{P}_e \triangleq Q\left(\sqrt{\frac{E_b}{\mathcal{N}_o}\, d^2_{\min}}\right), \quad (24)$$

where E_b is the average energy transmitted per information bit, $\mathcal{N}_o$ is the one-sided mean power spectral density of the noise process, $Q(x)$ is the familiar "Q-function" given by $Q(x) = \int_x^\infty 1/\sqrt{2\pi} e^{-t^2/2}\, dx$, and $d^2_{\min}$ is the normalized free squared Euclidean distance defined as

$$d^2_{\min} = \min_{i \neq j} \frac{1}{2E_b} \int_{-\infty}^{\infty} |s_i(t) - s_j(t)|^2\, dt. \quad (25)$$

It can be easily shown that the minimum Euclidean and Hamming distances are linearly related as

$$d^2_{\min} = 2R_c H^W_{\min}, \tag{26}$$

where R_c is the channel encoder $\mathbf{W}$ rate and $H^W_{\min}$ is the minimum Hamming distance between all coded sequences generated by encoder $\mathbf{W}$. The immediate consequence of (26) is that, optimum convolutional encoders $\mathbf{W}$ (with maximum free Hamming distances) can be applied to achieve the *best* performance. In addition, a simplification on the search for best codes applied to such modulation is achieved.

B. Receiver Complexity

The penalty for using error-control coding, besides the decreased bandwidth efficiency, is receiver complexity. Since in serial-MSK the phase is continuous, the signal contains memory. As such, to best recover the symbols requires this memory to be taken into account which requires we use a maximum-likelihood sequence estimator (MLSE) receiver. When convolutional code is combined with such modulation scheme, both the encoder and modulation memory contribute to the total memory of the coded-modulation scheme. In other words, the complexity of the MLSE receiver in terms of the number of trellis states S_V is determined by the number of states in the combined coded modulation trellis, in contrast to memoryless modulation where the combined trellis has the same number of states as that of the convolutional code.

Several attempts have been performed to reduce the complexity of the optimum receiver while achieving near optimum performance. In [4], Moreno ans Pasupathy, had constructed special convolutional codes called "matched codes" [9] which, when combined with serial-MSK modulator, reduce the complexity of the MLSE receiver (in terms of the number of states in the combined coded modulation trellis S_V) while achieving good coding gains. However, it has been shown in the previous section that generating serial-MSK using the new modulator, the coded symbols can be optimally recovered using a simple linear receiver. The complexity of such receiver is determined by the number of trellis states of the "decoder" that corresponds to the channel encoder $\mathbf{W}$ with constraint length v and rate R_c. This means that the memory of the modulator has no effect on the complexity of the receiver.

VI. Numerical Results

In this section, we present numerical results of the search for the best convolutional codes applied to serial-MSK generated using the system depicted in Fig. 8. The results are presented in Table I. Note that these codes have been previously reported in literature (for memoryless modulation) [5], [6] where only encoders $\mathbf{W}$ of rate $1/2$, $2/3$ are reported of constraint length up to 5. For every best code found, the (normalized) minimum square Euclidean distance is given All codes that are reported here for the new serial-MSK achieve the same free Euclidean distance for the same codes applied to linear modulation schemes such as BPSK and OQPSK. Moreover, in most cases these codes achieve better performance than the codes reported in [4] (applied to the system shown in Fig. 1) for the same decoder complexity.

TABLE I

A comparison of the $d^2_{\min}$ achieved in convolutionally coded serial-MSK signal generated using the FFSK modulator and the new modulator for the same decoder complexity.

States S_V	Code Rate	$d^2_{\min}$ Reported in [4]	$d^2_{\min}$ New Serial-MSK
2	1/2	3.00	3.00
	2/3	2.67	2.67
4	1/2	4.00	5.00
	2/3	4.00	4.00
8	1/2	6.00	6.00
	2/3	5.33	5.33
16	1/2	6.00	7.00
	2/3	6.67	6.67
32	1/2	8.00	8.00
	2/3	8.00	8.00

VII. Conclusion

A new linear modulator has been designed to generate serial-MSK. It is shown that a simple linear receiver can be constructed to optimally demodulate and detect coded serial-MSK signal. It is demonstrated that the detection problem for the recovery of the symbol sequences is one corresponding to memoryless linear modulation. It is also shown that the receiver complexity is not affected by the memory of the modulator. As such, the optimum convolutional codes designed using conventional techniques are the best candidates for serial-MSK generated using the new modulator. In most cases, these codes, for a given code rate and constraint length outperform convolutional codes found in [4] for the same decoder complexity.

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