Basin-scale internal-wave tomography using a large-aperture vertical array

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abstract: Basin-scale internal wave tomography has the potential to be a very important oceanographic tool to discover the sources and sinks of internal wave energy. Basin-scale acoustic transmissions are ideal for internal wave tomography since the long-range allows for the build-up of the large internal wave acoustic effects. Use of a vertical receiving array simplifies the task of acoustic normal mode detection, wavefront identification in terms of specific geometrical optics ray paths and their use enhances the number of acoustic observables that can be used in an internal wave inversion. Using wavefront fluctuations inference of the range-average internal wave energy as a function of depth forms the basis of a linear inverse problem, yet inversions for other internal wave parameters like power law spectral slope and vertical mode number bandwidth are more complex nonlinear inverses. Inversions using acoustic normal mode observables can only be done using Monte Carlo methods.

INTRODUCTION

Basin-scale acoustic internal wave tomography has the potential to be a very important oceanographic tool. The idea, which was first suggested by Flatté in 1983(1), is very simple in principle. The acoustic travel time difference between a fluctuating ocean and the average ocean can be expressed as,

$$\delta T = \int_{\Gamma} ds \frac{\delta c(\vec{r})}{\bar{c}^2(\vec{r})} \tag{1}$$

where Γ is a geometrical optics ray path, $\delta c(\vec{r})$ is an internal wave induced sound speed fluctuation and $\bar{c}(\vec{r})$ is the average ocean sound speed. In general $\delta c/\bar{c}$ is at most of order 10^{-3}, but for basin scale ranges the acoustic effect accumulates significantly. Squaring Eq. (1) and taking the expectation value the variance of travel time is seen to be a double integral over the sound speed fluctuation correlation function ρ,

$$\tau^2 = \langle \delta T^2 \rangle = \int_{\Gamma_1} \frac{ds_1}{\bar{c}^2(s_1)} \int_{\Gamma_1} \frac{ds_2}{\bar{c}^2(s_2)} \rho(s_1, s_2). \tag{2}$$

For basin-scale acoustic transmissions τ^2 is of the order 10^{-4} s to 10^{-3} s which is easily observable. Time series observations of τ^2 over an ocean basin have important information of the possible sources and sinks of internal wave energy. For example, variability of τ^2 on the fortnightly time scale would indicate a tidal source of internal wave energy, while variability on the seasonal timescale would indicate wind generation sources due to the development of the winter or summer storm track. If acoustic paths which sample high mesoscale energy regions have larger τ^2 this indicates possible mesoscale sources.

Observations of τ^2 can be obtained from a single hydrophone, but the use of a vertical array allows the use of other acoustic fluctuation quantities like vertical coherence. Denoting $\chi(1)$ as the log amplitude, and $\phi(1)$ as the phase at some point (1) and if it is assumed that $\langle \chi(1)\chi(2) \rangle = \langle \chi(1)\phi(2) \rangle = \langle \chi(2)\phi(1) \rangle = 0$ and χ and ϕ are Gaussian random variables then the coherence between two points (1) and (2) is closely given by

$$\langle \psi(1)\psi^*(2) \rangle \simeq \exp(-D(1,2)/2) \tag{3}$$

and D, the phase structure function, is expressed as,

$$D(1,2) = \sigma^2 \langle (\int_{\Gamma_1} ds_1 \frac{\delta c}{c^2} - \int_{\Gamma_2} ds_2 \frac{\delta c}{c^2})^2 \rangle \simeq 2\Big(\tau_1^2 - \int_{\Gamma_1} \frac{ds_1}{\bar{c}^2(s_1)} \int_{\Gamma_2} \frac{ds_2}{\bar{c}^2(s_2)} \rho(s_1, s_2)\Big) \tag{4}$$

where Γ_1 and Γ_2 are ray paths terminating at points (1) and (2). For data collected by the Acoustic Thermometry of Ocean Climate (ATOC) program and future data from the North Pacific Acoustic Observatory

(NPAL) we intend to use travel time variance τ^2, and vertical and temporal coherence as acoustic observables for internal wave tomography.

INTERNAL-WAVE TOMOGRAPHY

The primary quantity of interest for internal wave tomography is the phase structure function D, since its evaluation is directly linked to the calculation of both τ^2 and time and depth coherence. For acoustic energy which does not sample the upper few hundred meters of the ocean, the Garrett-Munk (GM) internal wave displacement spectrum (2), $F_j(k_x, k_y)$, in terms of the internal wave mode number j, and horizontal wavenumbers k_x and k_y, serves as a good starting point for the calculation of the sound speed correlation function ρ

$$\begin{aligned} \rho(\Delta x, \Delta z, \Delta t; \bar{z}) &= (\delta c_0)^2 \left((N(\bar{z})/N_0)^3 + \sum_{n=1}^{nmax} a_n A_n(\bar{z}) \right) \times \\ &\mathrm{Re} \left(\sum_j \int dk_x \int dk_y \int F_j(k_x, k_y) \exp(i(k_x \Delta x + k_z(\bar{z}, j)\Delta z - \omega(k_x, k_y, j)\Delta t)) \right) \end{aligned} \tag{5}$$

where δc_0 is the reference rms sound speed variation (typically 1.0 m/s), N is the buoyancy frequency profile, N_0 is a reference buoyancy frequency (typically 3 cph), $k_z(\bar{z}, j) = \pi j N(\bar{z})/n0B$ is the WKB vertical wavenumber, $\omega^2 = f^2 + (k_x^2 + k_y^2)(n0B/(\pi j))$ is the WKB dispersion relation, and $n0B = \int_0^{z_b} N(z)dz$. The GM internal wave spectrum with a variable power law exponent on the vertical mode number spectrum is,

$$F_j(k_x, k_y) = M_j \frac{1}{(j^2 + j_*^2)^p} M_k \frac{k_j (k_x^2 + k_y^2)^{1/2}}{(k_x^2 + k_y^2 + k_j^2)^2} \tag{6}$$

where $k_j = \pi j f / n0B$, and M_j and M_k are normalizations so the spectrum integrates to unity. The parameters to be established by the inverse are the $\{a_n\}$, j_* and p. The $\{a_n\}$ coefficients are corrections to the WKB depth scaling for internal wave energy, j_* is the modal bandwidth, and p is the modal power law (nominally $\{a_n\} = 0$, $p = 1$, and $j_* = 3$ for GM). Because ρ depends linearly on the coefficients $\{a_n\}$ these parameters can be inferred using linear inverse theory. The inverse for the parameters p, and j_* however will require non-linear methods. Procedures to deal with this mixed inverse will need to be developed.

For the case of acoustic energy with significant interaction with the upper few hundred meters of the ocean and/or the air/sea interface, there is no good "first guess" internal wave model to work from. A model proposed by T. Duda (3) based upon doppler sonar observations off the Southern California coast(4) could be a starting point.

Finally the outlined internal wave tomography method depends on resolving acoustic wavefronts. We have found that for basin scale transmissions only a portion of the arrival can be resolved in terms of wavefronts; these are the geometrical optics ray paths which cycle steeply through the ocean sound channel(5). For acoustic energy which travels close to the sound channel axis internal wave induced fluctuations are very large, obliterating the wavefront pattern. With a vertical array this energy can be treated in terms of the time spreading of acoustic normal mode arrivals(6). At this point there is no way to do the internal wave inverse problem using normal mode observables without resorting to time consuming Monte Carlo numerical simulations.

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