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Research Article

H_∞ Sampled-Data Control for Singular Neutral System

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This study is concerned with the H_∞ control problem for singular neutral system based on sampled-data. By input delay approach and a composite state-derivative control law, the singular system is turned into a singular neutral system with time-varying delay. Less conservative result is derived for the resultant system by incorporating the delay decomposition technique, Wirtinger-based integral inequality, and an augmented Lyapunov-Krasovskii functional. Sufficient conditions are derived to guarantee that the resulting system is regular, impulse-free, and asymptotically stable with prescribed H_∞ performance. Then, the H_∞ sampled-data controller is designed by means of linear matrix inequalities. Finally, two simulation results have shown that the proposed method is effective.

1. Introduction

In the last decades, modern control system widely used the digital computers to control continuous-time system. In the system, the continuous-time signals are transformed into discrete-time control signals by being sampled and quantized with a digital computer, and they will be transformed into continuous-time signals again by the zero-order holder. Hence, both discrete-time and continuous-time signals occur in one system with a continuous-time framework, which is called sampled-data system. Recently, three main approaches have been adopted to analyze the sampled-data systems. The first one is lifting technology [1, 2], in which the sampled-data system is transformed into a discrete-time system. However, this approach cannot deal with the uncertain sampling intervals problem. The second approach is based on the impulsive modeling of sampled-data systems in which a time-varying periodic Lyapunov function is used [3, 4]. The disadvantage of this approach is that the sampling interval of the process's output must be constant. Input delay approach, which is proposed by Fridman et al. in [5], is the third methods. It transforms the sampling period into a bounded time-varying delay, and its important advantage is that the sampling distance is not needed to be constant. Besides, the approach can deal with the problem of system with nonuniform uncertain sampling (see [6]), which is difficult for traditional lifting techniques to deal with. In the past years, the approach has considerable successful applications in neural networks system [7–9], fuzzy system [10], chaotic system [11, 12], complex dynamic system [13–15], multiagent system [16, 17], and so on.

Singular systems, also referred to as implicit systems or descriptor systems, have gained considerable attentions during the past decades. Compared to regular systems, singular systems can better describe physical systems and have wide applications in various systems such as aerospace systems, power systems, and mechanical systems. In the past years, many results have been reported to analyze the admissibility problem for singular systems [18–23]. For example, the problems of stability analysis and stabilization have been investigated in [19, 20]. The H_∞ control problem has been discussed in [21]. The passivity and dissipativity problems of singular systems with time delays have been studied in [22, 23]. In recent year, sampled-data control theory is used for singular system by proposing a control law as [24, 25]. In [24], the dissipative fault-tolerant cascade control synthesis for a class of singular networked cascade control systems (NCCS) with both differentiable and nondifferentiable time-varying delays has been studied. In [25], the problem of the event-triggered stabilization for linear singular systems based on sampled-data is considered. In [24, 25], the state feedback control laws are all considered in the system. However, to the best of our knowledge, no literatures have considered the acceleration feedback for the singular system with sampled-data. The acceleration feedback can improve the system controller's performance effectively. Moreover, it can suppress varying disturbances, so it has considerable applications in practical systems such as vibration suppression system and mechanical system. Thus, designing a composite control law which includes the state feedback and acceleration feedback for the system is the paper's aim. Then, the linear sampled-data singular system is turned into a singular neutral system with sampled-data.

Up to now, very little interest has been paid for singular neutral systems. In [26], the stability and state feedback stabilization problems of singular neutral systems are considered. Reference [27] studies the stability problems of singular neutral system with mixed delays. Reference [28] concerns the problem of the delay-dependent robust stability for neutral singular systems with time-varying delays and nonlinear perturbations. In [29], the problem of stability of singular neutral systems with multiple delays is studied. Reference [30] studies the problem of robust stability and stabilization of uncertain neutral singular systems and develops a new stability criterion of the differential operator by the final value theorem for Laplace transform. Reference [31] concerns the problem of output strictly passive control for uncertain singular neutral systems. To analyze the singular neutral systems, several methods have been proposed, among which the more popular approaches are Jensen's inequality and free weighting matrix approach. Recently Seuret has proposed a new inequality called Wirtinger-based integral inequality in [32], which can provide more accurate estimation than the Jensen inequality. So, if the inequality is employed for investigating the singular neutral systems, we can derive an improved result. Besides, delay decomposition approach, which uses the method to divide the delay interval into N equal-length subintervals, is proposed in [33]. It is worth noticing that the delay decomposition approach can reduce the conservatism. However, to the best of the authors' knowledge, little literatures have been found to study the sampled-data control problem for singular neutral system combining the delay decomposition approach with Wirtinger-based integral inequality despite its practical importance which motivates our present research work.

In the paper, the issue about H_∞ control for singular neutral system based on sampled-data is discussed. By input delay approach and a composite state-derivative control law, the linear singular system is turned into a singular neutral system with time-varying delay. By adding more information in the integral term, an augmented Lyapunov-Krasovskii functional is constructed. Less conservative results are derived for the resulting system by integrating Wirtinger-based integral inequality with delay decomposition approach. Sufficient conditions are derived to guarantee that the resulting system is regular, impulse-free, and asymptotically stable with prescribed H_∞ performance. Then the H_∞ sampled-data controller is obtained by means of linear matrix inequalities. Finally, we give two illustrate examples to show that the proposed method is effective.

2. Problem Formulation

The linear singular system is considered as follows:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + B_1 u(t) + Bw(t), \quad t > 0, \\ y(t) &= Cx(t) + Du(t), \quad t \in [-d, 0], \\ x(t) &= \phi(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^n$ is the control input and the initial condition, $w(t)$ is the disturbance input vector, and $\phi(t)$ is the compatible initial function. A, B, and B_1 are known constant matrices; matrix $E \in R^{n\times n}$ is assumed to be singular and the rank $E = r \le n$.

In this paper, the state variable of the system is assumed to be measured at the sampling instant $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$; that is, in the interval $t_k \le t \le t_{k+1}$, only $x(t_k)$ is available for control purposes. The sampling period follows the assumption that it is bounded by a constant d; that is,

$$t_{k+1} - t_k \le d, \quad \forall k \ge 0,\ d > 0. \tag{2}$$

Then, for system (1), we choose the composite state feedback control law as

$$u(t) = u(t_k) = K_1 x(t_k) + K_2 \dot{x}(t_k), \quad t_k \le t < t_{k+1}, \tag{3}$$

where t_k represents the sampling instant and $u(t_k)$ represents the discrete-time control signal. K_1 and K_2 are the controller gain matrix that will be designed.

By substituting (3) into (1), we obtain

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_1 x(t_k) + G_1 \dot{x}(t_k) + Bw(t), \\ &\qquad t > 0, \\ y(t) &= Cx(t) + D_1 x(t_k) + D_2 \dot{x}(t_k), \\ x(t) &= \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{4}$$

where

$$A_1 = B_1K_1,$$
$$G_1 = B_1K_2,$$
$$D_1 = DK_1, \tag{5}$$
$$D_2 = DK_2.$$

Remark 1. By the composite state-derivative feedback control law (3), system (1) is converted to the singular system in neutral type. The state feedback as well as acceleration feedback is of great significance in actual sampled-data control system. For example, in the dynamic positioning (DP) ship system, by measuring the acceleration and velocity of the DP ship, the acceleration and velocity feedback loop are established, which can improve the DP controller's performance and suppress varying disturbances like wind, waves, and ocean currents greatly.

Remark 2. Note that (4) involves both discrete and continuous signals, which is more different and practical than continuous-time control approach for singular neutral system. Besides, because the parameter uncertainties exist in the sampled-data control system, the traditional lifting technique is difficult to deal with the problem.

Throughout the paper, we introduce the definitions as follows.

Definition 3 (see [36]). (1) The sampled-data control of singular neutral system (4) with $w(t) = 0$

$$E\dot{x}(t) = Ax(t) + A_1x(t_k) + G_1\dot{x}(t_k) \tag{6}$$

is said to be regular and impulse-free if the pair (E, A) is regular and impulse-free.

(2) System (6) is said to be asymptotically admissible, if it is regular, impulse-free, and asymptotically stable.

Definition 4. System (4) is said to have H_∞ performance γ if the following inequality is satisfied:

$$\|y(t)\|_2 \le \gamma \|w(t)\|_2 \tag{7}$$

for all nonzero $w(t) \in L_2[0, \infty)$ under zero initial condition, where $\gamma > 0$.

Using the input delay approach, the state feedback controller $u(t)$ is rewritten as

$$\begin{aligned} u(t) &= u(t_k) = u(t - (t - t_k)) = u(t - \tau(t)), \\ &\qquad t_k \le t < t_{k+1}, \ \tau(t) = t - t_k, \end{aligned} \tag{8}$$

where the time-varying delay $\tau(t)$ is piecewise-linear satisfying

$$\begin{aligned} &0 \le \tau(t) \le d, \\ &\dot{\tau}(t) = 1, \quad t \ne t_k. \end{aligned} \tag{9}$$

Thus, the singular neutral system based on sampled-data in (4) can be converted to the singular neutral system with time-varying delay $\tau(t)$ as follows:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_1x(t - \tau(t)) + G_1\dot{x}(t - \tau(t)) \\ &\quad + Bw(t), \quad t > 0, \\ y(t) &= Cx(t) + D_1x(t - \tau(t)) + D_2\dot{x}(t - \tau(t)), \\ x(t) &= \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{10}$$

where $A_1 = B_1K_1$, $G_1 = B_1K_2$, $D_1 = DK_1$, and $D_2 = DK_2$.

For obtaining the main results, we state the lemma as follows.

Lemma 5 (see [37]). *For a given matrix $R > 0$ and scalars a and b satisfying $a < b$, the following inequality holds for all continuously differentiable function ω in $[a, b] \to R^n$*

$$\begin{aligned} &\int_a^b \dot{w}^T(s) R\dot{w}(s)\, ds \\ &\quad \ge \frac{1}{b-a}(w(b) - w(a))^T R(w(b) - w(a)) \\ &\quad + \frac{3}{b-a}\Omega^T R\Omega, \end{aligned} \tag{11}$$

where

$$\Omega = w(b) + w(a) - \frac{2}{b-a}\int_a^b w(s)\, ds. \tag{12}$$

3. Main Results

The H_∞ sampled-data control problem for singular neutral system is studied in this section. By constructing an augmented Lyapunov-Krasovskii functional, combining the delay decomposition technique with the Wirtinger-based integral inequality, less conservative results can be derived.

Theorem 6. *For given scalar $d > 0$, the closed-loop system (6) is asymptotically admissible, if there exist symmetric positive-definite matrices P, S, Q_i, Z_i, $i = 1, \ldots, N$, such that*

$$E^T P = P^T E \geq 0, \tag{13}$$

$$\begin{bmatrix}
\Xi_1^1 & \Xi_2^1 & 0 & \cdots & 0 & \Xi_{N+2}^1 & 0 & \cdots & 0 & 0 & \Xi_{2N+3}^1 & \Xi_{2N+4}^1 & A^T \\
* & \Xi_2^2 & \ddots & \ddots & \cdots & \Xi_{N+2}^2 & \ddots & \ddots & \vdots & \vdots & 0 & 0 & 0 \\
* & * & \ddots & \Xi_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \Xi_N^N & \Xi_{N+1}^N & \vdots & \ddots & \ddots & \Xi_{2N+1}^N & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & \Xi_{N+1}^{N+1} & 0 & \cdots & 0 & \Xi_{2N+1}^{N+1} & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & -12E^T Z_i E & 0 & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & -12E^T Z_N E & 0 & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & \Xi_{2N+2}^{2N+2} & \vdots & \vdots & 0 \\
* & * & * & * & * & * & * & * & * & * & 0 & \vdots & A_1^T \\
* & * & * & * & * & * & * & * & * & * & * & 0 & G_1^T \\
* & * & * & * & * & * & * & * & * & * & * & * & -\delta
\end{bmatrix} < 0, \tag{14}$$

where

$$\Xi_1^1 = P^T A + A^T P + S_1 + Q_1 - 4E^T Z_1 E,$$

$$\Xi_{N+1+i}^i = \frac{d}{N E^T R_i E} + 6E^T Z_i E,$$

$$\Xi_{N+1+i}^{i+1} = -\frac{d}{N E^T R_i E} + 6E^T Z_i E, \quad i = 1, \ldots, N$$

$$\Xi_{2N+3}^1 = PA_1,$$

$$\Xi_{2N+4}^1 = PG_1,$$

$$\Xi_i^i = -Q_{i-1} + Q_i - 4E^T Z_{i-1} E - 4E^T Z_i E, \quad i = 2, \ldots, N \tag{15}$$

$$\Xi_{i+1}^i = -2E^T Z_i E, \quad i = 1, \ldots, N,$$

$$\Xi_{N+1}^{N+1} = -Q_N - 4E^T Z_N E - S_1,$$

$$\Xi_{2N+2}^{2N+2} = -E^T S_2 E,$$

$$\delta = \left(\frac{d}{N}\right)^2 \sum_{i=1}^{N} E^T Z_i E + E^T S_2 E.$$

Proof. First of all, we prove system (6) is regular and impulse-free. Since rank $E = r \leq n$, there exist nonsingular matrices $\widehat{M}$ and $\widehat{H}$ such that

$$\overline{E} = \widehat{M} E \widehat{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{16}$$

Similar to (16), we define

$$\overline{A} = \widehat{M} A \widehat{H} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$
$$\overline{P} = \widehat{M}^{-T} P \widehat{H} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \tag{17}$$

From (13) and the expressions (16) and (17), we can obtain that $P_{12} = 0$. Then we premultiply and postmultiply $\Xi_1^1 < 0$ by $\widehat{H}^T$ and $\widehat{H}$, respectively; the inequality $A_{22}^T P_{22} + P_{22}^T A_{22} < 0$ can be obtained, which implies A_{22} is nonsingular and the pair (E, A) is regular and impulse-free. Then, by Definition 3, system (6) is regular and impulse-free.

Next, the asymptotically stability of system (6) will be proved. An augmented Lyapunov- Krasovskii functional is chosen as follows:

$$V(t)=\sum_{i=1}^{6}V_i(t),\quad t\in[t_k,t_{k+1}),\tag{18}$$

$$\begin{aligned}
&V_1(t)=x(t)^T E^T P x(t),\\
&V_2(t)=\int_{t-d}^{t} x(s)^T S_1 x(s)\,ds,\\
&V_3(t)=\int_{t-d}^{t} \dot{x}(s)^T E^T S_2 E\dot{x}(s)\,ds,\\
&V_4(t)=\sum_{i=1}^{N}\int_{t-id/N}^{t-(i-1)d/N} x^T(s)\,Q_i x(s)\,ds,\\
&V_5(t)\\
&\quad=\sum_{i=1}^{N}\int_{t-id/N}^{t-(i-1)d/N} x^T(s)\,ds\,E^T R_i E\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds,\\
&V_6(t)=\frac{h}{N}\sum_{i=1}^{N}\int_{-id/N}^{-(i-1)d/N}\int_{t+\theta}^{t}\dot{x}^T(s)\,E^T Z_i E\dot{x}(s)\,ds\,d\theta.
\end{aligned}\tag{19}$$

Calculating the derivative of $V(t)$, we can get that

$$\begin{aligned}
&\dot{V}_1(t)=2x(t)^T PE\dot{x}(t),\\
&\dot{V}_2(t)=x(t)^T S_1 x(t)-x(t-d)^T S_1 x(t-d),\\
&\dot{V}_3(t)=\dot{x}(t)^T E^T S_2 E x(t)-\dot{x}(t-d)^T E^T S_2 E\dot{x}(t\\
&\quad -d),\\
&\dot{V}_4(t)=\sum_{i=1}^{N}\left[x^T\left(t-\frac{(i-1)d}{N}\right)Q_i x\left(t-\frac{(i-1)d}{N}\right)\right.\\
&\quad\left.-x^T\left(t-\frac{id}{N}\right)Q_i x\left(t-\frac{id}{N}\right)\right],\\
&\dot{V}_5(t)=\sum_{i=1}^{N}\left[\left(x^T\left(t-\frac{(i-1)d}{N}\right)-x^T\left(t-\frac{id}{N}\right)\right)\right.\\
&\quad\left.\cdot E^T R_i E\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds\right],\\
&\dot{V}_6(t)=\sum_{i=1}^{N}\left[\left(\frac{d}{N}\right)^2\dot{x}^T(t)\,E^T Z_i E\dot{x}(t)-\frac{d}{N}\right.\\
&\quad\left.\cdot\int_{t-id/N}^{t-(i-1)d/N}\dot{x}^T(t)\,E^T Z_i E\dot{x}(t)\,ds\right].
\end{aligned}\tag{20}$$

Employing Lemma 5, we have

$$\begin{aligned}
-\frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N}\dot{x}^T(t)\,E^T Z_i E\dot{x}(t)\,ds \le{}& \begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right)\\ x\left(t-\frac{id}{N}\right)\\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N}x(s)\,ds\end{bmatrix}^T\\
&\cdot\begin{bmatrix}-4E^TZ_iE & -2E^TZ_iE & 6E^TZ_iE\\ * & -4E^TZ_iE & 6E^TZ_iE\\ * & * & -12E^TZ_iE\end{bmatrix}\begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right)\\ x\left(t-\frac{id}{N}\right)\\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N}x(s)\,ds\end{bmatrix}.
\end{aligned}\tag{21}$$

Then

$$\dot{V}_6(t)\le\sum_{i=1}^{N}\left[\left(\frac{d}{N}\right)^2\dot{x}^T(t)\,E^T Z_i E\dot{x}(t)\right.$$

$$+\begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right) \\ x\left(t-\frac{id}{N}\right) \\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds \end{bmatrix}^T \begin{bmatrix} -4E^TZ_iE & -2E^TZ_iE & 6E^TZ_iE \\ * & -4E^TZ_iE & 6E^TZ_iE \\ * & * & -12E^TZ_iE \end{bmatrix} \begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right) \\ x\left(t-\frac{id}{N}\right) \\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds \end{bmatrix}\Bigg]. \tag{22}$$

Substituting inequalities (22) into $\dot{V}(t)$ it can be concluded that

$$\begin{aligned}\dot{V}(t) &\leq 2x(t)^T PE\dot{x}(t) + x(t)^T S_1x(t) - x(t-d)^T S_1x(t-d) + \dot{x}(t)^T E^TS_2E\dot{x}(t) - \dot{x}(t-d)^T E^TS_2E\dot{x}(t-d) \\ &+ \sum_{i=1}^{N}\left[x^T\left(t-\frac{(i-1)d}{N}\right)Q_ix\left(t-\frac{(i-1)d}{N}\right) - x^T\left(t-\frac{id}{N}\right)Q_ix\left(t-\frac{id}{N}\right)\right] \\ &+ \sum_{i=1}^{N}\left[\left(x^T\left(t-\frac{(i-1)d}{N}\right) - x^T\left(t-\frac{id}{N}\right)\right)E^TR_iE\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds\right] + \sum_{i=1}^{N}\Bigg[\left(\frac{d}{N}\right)^2 \dot{x}^T(t)E^TZ_iE\dot{x}(t) \\ &+ \begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right) \\ x\left(t-\frac{id}{N}\right) \\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds \end{bmatrix}^T \begin{bmatrix} -4E^TZ_iE & -2E^TZ_iE & 6E^TZ_iE \\ * & -4E^TZ_iE & 6E^TZ_iE \\ * & * & -12E^TZ_iE \end{bmatrix} \begin{bmatrix} x\left(t-\frac{(i-1)d}{N}\right) \\ x\left(t-\frac{id}{N}\right) \\ \frac{d}{N}\int_{t-id/N}^{t-(i-1)d/N} x(s)\,ds \end{bmatrix}\Bigg] \leq \varsigma^T(t)\left(\Theta + \psi^T\delta\psi\right) \\ &\cdot \varsigma(t),\end{aligned} \tag{23}$$

where

$$\varsigma(t) = \left[x^T(t) \;\; x^T\left(t-\frac{d}{N}\right) \;\; \cdots \;\; x^T(t-d) \;\; \frac{1}{N}\int_{t-d/N}^{t} x^T(s)\,ds \;\; \cdots \;\; \frac{1}{N}\int_{t-d}^{t-(N-1)d/N} x^T(s)\,ds \;\; \dot{x}^T(t-d) \;\; x^T(t-\tau(t)) \;\; \dot{x}^T(t-\tau(t))\right],$$

$$\psi = \begin{bmatrix} A & 0 & \cdots & 0 & BK_1 & 0 & \cdots & 0 & BK_2 \end{bmatrix},$$

$$\Theta = \left[\begin{array}{cccccccccccc}
\Xi_1^1 & \Xi_2^1 & 0 & \cdots & 0 & \Xi_{N+2}^1 & 0 & \cdots & 0 & 0 & \Xi_{2N+3}^1 & \Xi_{2N+4}^1 \\
* & \Xi_2^2 & \ddots & \ddots & \cdots & \Xi_{N+2}^2 & \ddots & \ddots & \vdots & \vdots & 0 & 0 \\
* & * & \ddots & \Xi_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots \\
* & * & * & \Xi_N^N & \Xi_{N+1}^N & \vdots & \ddots & \ddots & \Xi_{2N+1}^N & \vdots & \vdots & \vdots \\
* & * & * & * & \Xi_{N+1}^{N+1} & 0 & \cdots & 0 & \Xi_{2N+1}^{N+1} & \vdots & \vdots & \vdots \\
* & * & * & * & * & -12E^T Z_i E & 0 & \cdots & \cdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & -12E^T Z_N E & 0 & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & \Xi_{2N+2}^{2N+2} & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & * & 0 & \vdots \\
* & * & * & * & * & * & * & * & * & * & * & 0
\end{array}\right]. \tag{24}$$

By Schur complement, inequality (14) implies

$$\Theta + \psi^T \delta \psi < 0, \tag{25}$$

where Θ is defined in (24); it is clear from (25) that $\dot{V}(t) < 0$; hence, system (6) is asymptotically stable. This completes the proof. □

Remark 7. In [38, 39], integral of delay term is always adopted to construct an augmented Lyapunov-Krasovskii functional. Therefore, in order to consider the information of delay term sufficiently in our constructed functional (18), an integral term of

$$\int_{t-id/N}^{t-(i-1)d/N} x^T(s)\, ds\, E^T R_i E \int_{t-id/N}^{t-(i-1)d/N} x(s)\, ds \tag{26}$$

is added in $V_4(x_t)$.

Remark 8. In the Theorem 6, delay decomposition approach, which is established by decomposing the delay intervals $\tau(t)$ into N equidistant subintervals, is employed for constructing the Lyapunov-Krasovskii functional (18). It is noted that the delay decomposition approach can derive less conservative results. Besides, the Wirtinger-based integral inequality is used for estimating the derivative $\dot{V}_6(t)$. It is pointed out that, compared to Jensen inequality in [40–42], the Wirtinger-based integral inequality has tighter upper bounds and less conservatism. Thus, incorporating delay decomposition approach with Wirtinger-based integral inequality, we can derive less conservative result for the system.

Next, the H_∞ performance will be considered for system (10).

Theorem 9. *For given scalar $d > 0$ and a prescribed scalar $\gamma > 0$, the closed-loop system (10) is asymptotically admissible with H_∞ performance γ, if there exist symmetric positive-definite matrices P, S, Q_i, Z_i, $i = 1, \ldots, N$, such that*

$$E^T P = P^T E \geq 0, \tag{27}$$

$$\begin{bmatrix}
\Xi_1^1 & \Xi_2^1 & 0 & \cdots & 0 & \Xi_{N+2}^1 & 0 & \cdots & 0 & 0 & \Xi_{2N+3}^1 & \Xi_{2N+4}^1 & \Xi_{2N+5}^1 & A^T & C^T \\
* & \Xi_2^2 & \ddots & \ddots & \cdots & \Xi_{N+2}^2 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 \\
* & * & \ddots & \Xi_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \Xi_N^N & \Xi_{N+1}^N & \vdots & \ddots & \ddots & \Xi_{2N+1}^N & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & \Xi_{N+1}^{N+1} & 0 & \cdots & 0 & \Xi_{2N+1}^{N+1} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & -12E^T Z_i E & 0 & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & -12E^T Z_N E & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & \Xi_{2N+2}^{2N+2} & \vdots & \vdots & \vdots & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & 0 & \vdots & \vdots & K_1^T B_1^T & K_1^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & 0 & 0 & K_2^T B_1^T & K_2^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I & B^T & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\delta & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \tag{28}$$

$$\begin{aligned}
\Xi_1^1 &= P^T A + A^T P + S_1 + Q_1 - 4E^T Z_1 E, \\
\Xi_{N+1+i}^i &= \frac{d}{N E^T R_i E} + 6E^T Z_i E, \\
\Xi_{N+1+i}^{i+1} &= -\frac{d}{N E^T R_i E} + 6E^T Z_i E, \quad i = 1, \ldots, N, \\
\Xi_{2N+3}^1 &= P B_1 K_1, \\
\Xi_{2N+4}^1 &= P B_1 K_2, \\
\Xi_{2N+5}^1 &= P B, \\
\Xi_i^i &= -Q_{i-1} + Q_i - 4E^T Z_{i-1} E - 4E^T Z_i E, \quad i = 2, \ldots, N, \\
\Xi_{i+1}^i &= -2E^T Z_i E, \quad i = 1, \ldots, N, \\
\Xi_{N+1}^{N+1} &= -Q_N - 4E^T Z_N E - S_1, \\
\Xi_{2N+2}^{2N+2} &= -E^T S_2 E, \\
\delta &= \left(\frac{d}{N}\right)^2 \sum_{i=1}^{N} E^T Z_i E + E^T S_2 E.
\end{aligned} \tag{29}$$

Proof. It is obvious that (14) implies (28) holds which, according to Theorem 6, guarantees the asymptotically admissibility of system (10) when $w(t) = 0$. Then, we will prove that system (10) satisfies the H_∞ performance γ.

Choose the same Lyapunov-Krasovskii functional given in (14), and then follow similar lines in above proof; we can get

$$\begin{aligned} & y^T(t)\, y(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(t) \\ & \quad \le \zeta^T(t) \left(\Omega + \varphi^T \sigma \varphi\right) \zeta(t), \end{aligned} \tag{30}$$

where

$$\begin{aligned} & \zeta^T(t) \\ & = \left[x^T(t) \;\; x^T\left(t - \frac{d}{N}\right) \;\; \cdots \;\; x^T(t-d) \;\; \frac{1}{N}\int_{t-d/N}^{t} x^T(s)\, ds \;\; \cdots \;\; \frac{1}{N}\int_{t-d}^{t-(N-1)d/N} x^T(s)\, ds \;\; \dot{x}^T(t-d) \;\; x^T(t-\tau(t)) \;\; \dot{x}^T(t-\tau(t)) \;\; w(t) \right], \\ & \varphi = [\varphi_1 \;\; \varphi_2], \\ & \varphi_1 = [A \;\; 0 \;\; \cdots \;\; 0 \;\; BK_1 \;\; 0 \;\; \cdots \;\; 0 \;\; BK_2 \;\; E], \\ & \varphi_2 = [C \;\; 0 \;\; \cdots \;\; 0 \;\; D_1K_1 \;\; 0 \;\; \cdots \;\; 0 \;\; D_1K_2 \;\; 0], \end{aligned}$$

$$\Omega = \begin{bmatrix} \Xi_1^1 & \Xi_2^1 & 0 & \cdots & 0 & \Xi_{N+2}^1 & 0 & \cdots & 0 & 0 & \Xi_{2N+3}^1 & \Xi_{2N+4}^1 & \Xi_{2N+5}^1 \\ * & \Xi_2^2 & \ddots & \ddots & \cdots & \Xi_{N+2}^2 & \ddots & \ddots & \vdots & \vdots & 0 & 0 & 0 \\ * & * & \ddots & \Xi_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \Xi_N^N & \Xi_{N+1}^N & \vdots & \ddots & \ddots & \Xi_{2N+1}^N & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & \Xi_{N+1}^{N+1} & 0 & \cdots & 0 & \Xi_{2N+1}^{N+1} & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & -12E^T Z_i E & 0 & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & -12E^T Z_N E & 0 & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & * & \Xi_{2N+2}^{2N+2} & \vdots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & * & * & 0 & \vdots & \vdots \\ * & * & * & * & * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}. \tag{31}$$

By Schur complement, inequality (28) guarantees

$$\Omega + \varphi^T \sigma \varphi < 0. \tag{32}$$

Thus, from (30), we obtain

$$y^T(t)\, y(t) - \gamma^2 w^T(t)\, w(t) + \dot{V}(t) < 0. \tag{33}$$

Therefore, we have $\|y(t)\|_2 \le \gamma \|w(t)\|_2$ for random nonzero $w(t) \in L_2[0, \infty)$ and then establish H_∞ performance. This completes the proof. □

Now, based on Theorem 9, the H_∞ sampled-data controller (3) will be designed to guarantee the asymptotically admissibility of system (10) in the theorem as follows.

Theorem 10. *For given scalars $d > 0$ and $\gamma > 0$, the closed-loop system (10) is asymptotically admissible with H_∞ performance γ, if there exist symmetric positive-definite matrices $P = P^T, S = S^T, Q_i = Q_i^T$, and $Z_i = Z_i^T$, $i = 1, \ldots, N$, such that LMIs hold:*

$$E^T P^T = PE \geq 0, \quad (34)$$

$$\begin{bmatrix}
\bar{\Xi}_1^1 & \bar{\Xi}_2^1 & 0 & \cdots & \bar{\Xi}_{N+1}^1 & \bar{\Xi}_{N+2}^1 & 0 & \cdots & 0 & \bar{\Xi}_{2N+2}^1 & \bar{\Xi}_{2N+3}^1 & \bar{\Xi}_{2N+4}^1 & \bar{\Xi}_{2N+5}^1 & \bar{P}A^T & \bar{P}C^T \\
* & \bar{\Xi}_2^2 & \ddots & \ddots & \cdots & \bar{\Xi}_{N+2}^2 & \ddots & \ddots & \vdots & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \ddots & \bar{\Xi}_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \bar{\Xi}_N^N & \bar{\Xi}_{N+1}^N & \vdots & \ddots & \ddots & \bar{\Xi}_{2N+1}^N & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & \bar{\Xi}_{N+1}^{N+1} & 0 & \cdots & 0 & \bar{\Xi}_{2N+1}^{N+1} & \bar{\Xi}_{2N+2}^{N+1} & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & -12\bar{E}^T Z_i \bar{E} & 0 & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & -12\bar{E}^T Z_N \bar{E} & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & \bar{\Xi}_{2N+2}^{2N+2} & \vdots & \vdots & \vdots & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & 0 & \vdots & \vdots & \bar{K}_1^T B_1^T & \bar{K}_1^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & 0 & 0 & \bar{K}_2^T B_1^T & \bar{K}_2^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I & B^T & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \bar{\delta} - 2\bar{P} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned}
\bar{\Xi}_1^1 &= A\bar{P} + \bar{P}A^T + \bar{S}_1 + \bar{Q}_1 - 4E^T\bar{Z}_1 E, \\
\bar{\Xi}_{N+1+i}^i &= \frac{d}{NE^T\bar{R}_i E} + 6E^T\bar{Z}_i E, \\
\bar{\Xi}_{N+1+i}^{i+1} &= -\frac{d}{NE^T\bar{R}_i E} + 6E^T\bar{Z}_i E, \quad i = 1, \ldots, N, \\
\bar{\Xi}_{2N+3}^1 &= B_1\bar{K}_1, \\
\bar{\Xi}_{2N+4}^1 &= B_1\bar{K}_2, \\
\bar{\Xi}_{2N+5}^1 &= B, \\
\bar{\Xi}_i^i &= -\bar{Q}_{i-1} + \bar{Q}_i - 4E^T\bar{Z}_{i-1}E - 4E^T\bar{Z}_i E, \\
&\qquad i = 2, \ldots, N, \\
\bar{\Xi}_{i+1}^i &= -2E^T\bar{Z}_i E, \quad i = 1, \ldots, N, \\
\bar{\Xi}_{N+1}^{N+1} &= -\bar{Q}_N - 4E^T\bar{Z}_N E - \bar{S}_1, \\
\bar{\Xi}_{2N+2}^{2N+2} &= -E^T\bar{S}_2 E, \\
\bar{\delta} &= \left(\frac{d}{N}\right)^2 \sum_{i=1}^N E^T\bar{Z}_i E + E^T\bar{S}_2 E.
\end{aligned} \quad (36)$$

Moreover, a suitable controller with H_∞ performance γ in forms of (3) is designed to satisfy the proposed conditions. And the control gain matrices K_1 and K_2 are given by

$$\begin{aligned}
K_1 &= \bar{K}_1\bar{P}^{-1}, \\
K_2 &= \bar{K}_2\bar{P}^{-1}.
\end{aligned} \quad (37)$$

Proof. By noticing that $\bar{\delta} > 0$, we have

$$-\bar{P}\bar{\delta}\bar{P} \leq \bar{\delta} - 2\bar{P}. \quad (38)$$

Let $\varsigma = \operatorname{diag}\{P^{-T}, P^{-T}, 0, \ldots, 0, P^{-T}, 0, \ldots, 0, 0, P^{-T}, P^{-T}, I, I, I\}$. Denoting

$$\begin{aligned}
\overline{P} &= P^{-1}, \\
\overline{K}_1 &= K_1 P^{-1}, \\
\overline{K}_2 &= K_2 P^{-1}, \\
\overline{Q}_i &= P^{-T} Q_i P^{-1}, \\
\overline{S} &= P^{-T} S P^{-1}, \\
\overline{R}_i &= P^{-T} R_i P^{-1}, \\
\overline{Z}_i &= P^{-T} Z_i P^{-1}
\end{aligned} \tag{39}$$

and premultiplying and postmultiplying (28) by ς and ς^T, respectively, the result (35) can be obtained. This completes the proof. □

Remark 11. According to Theorem 10, sufficient conditions are provided to solve the H_∞ control problem for singular neutral system based on sampled-data, and the desired H_∞ sampled-data controller is proposed. The conditions take the form of LMI, which can be readily determined by standard numerical software. Moreover, the methods proposed in the work can be easily extended to other system.

Remark 12. When the matrix E in (1) is nonsingular, assuming that E is an identity matrix, system (1) reduces to a traditional sampled-data control system.

The regular sampled-data control system is considered as follows.

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 u(t) + Bw(t), \quad t > 0, \\
y(t) &= Cx(t) + Du(t), \quad t \in [-d, 0], \\
x(t) &= \phi(t).
\end{aligned} \tag{40}$$

Using the input delay approach and a state-derivative control law, the sampled-data control system (1) is turned into a neutral system with time-varying delays.

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + A_1 x(t - \tau(t)) + G_1 \dot{x}(t - \tau(t)) \\
&\quad + Bw(t), \quad t > 0 \\
y(t) &= Cx(t) + D_1 x(t - \tau(t)) + D_2 \dot{x}(t - \tau(t)), \\
x(t) &= \phi(t), \quad t \in [-d, 0],
\end{aligned} \tag{41}$$

where $A_1 = B_1 K_1, G_1 = B_1 K_2, D_1 = DK_1$, and $D_2 = DK_2$.

According to Theorem 10, the sampled-data controller is designed for the system (41) such that the system is asymptotically stable.

Corollary 13. *For given a scalar $d > 0$ and a prescribed scalar $\gamma > 0$, the closed-loop system (41) is asymptotically stable with H_∞ performance γ, if there exist symmetric positive-definite matrices P, S, Q_i, Z_i, $i = 1, \ldots, N$, such that*

$$\begin{bmatrix}
\overline{\Xi}_1^1 & \overline{\Xi}_2^1 & 0 & \cdots & \overline{\Xi}_{N+1}^1 & \overline{\Xi}_{N+2}^1 & 0 & \cdots & 0 & \overline{\Xi}_{2N+2}^1 & \overline{\Xi}_{2N+3}^1 & \overline{\Xi}_{2N+4}^1 & \overline{\Xi}_{2N+5}^1 & \overline{P}A^T & \overline{P}C^T \\
* & \overline{\Xi}_2^2 & \ddots & \ddots & \cdots & \overline{\Xi}_{N+2}^2 & \ddots & \ddots & \vdots & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \ddots & \overline{\Xi}_N^{N-1} & 0 & 0 & \ddots & \ddots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \overline{\Xi}_N^N & \overline{\Xi}_{N+1}^N & \vdots & \ddots & \ddots & \overline{\Xi}_{2N+1}^N & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & \overline{\Xi}_{N+1}^{N+1} & 0 & \cdots & 0 & \overline{\Xi}_{2N+1}^{N+1} & \overline{\Xi}_{2N+2}^{N+1} & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & -12\overline{Z}_i & 0 & \cdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & -12\overline{Z}_N & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & * & * & * & * & * & * & \overline{\Xi}_{2N+2}^{2N+2} & \vdots & \vdots & \vdots & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & 0 & \vdots & \vdots & \overline{K}_1^T B_1^T & \overline{K}_1^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & 0 & 0 & \overline{K}_2^T B_1^T & \overline{K}_2^T D^T \\
* & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I & B^T & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \overline{\delta} - 2\overline{P} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \tag{42}$$

where

$$\overline{\Xi}_1^1 = A\overline{P} + \overline{P}A^T + \overline{S}_1 + \overline{Q}_1 - 4\overline{Z}_1,$$
$$\overline{\Xi}_{N+1+i}^i = \frac{d}{N\overline{R}_i} + 6\overline{Z}_i,$$
$$\overline{\Xi}_{N+1+i}^{i+1} = -\frac{d}{N\overline{R}_i} + 6\overline{Z}_i, \quad i = 1,\ldots,N,$$
$$\overline{\Xi}_{2N+3}^1 = B_1\overline{K}_1,$$
$$\overline{\Xi}_{2N+4}^1 = B_1\overline{K}_2,$$
$$\overline{\Xi}_{2N+5}^1 = B, \tag{43}$$
$$\overline{\Xi}_i^i = -\overline{Q}_{i-1} + \overline{Q}_i - 4\overline{Z}_{i-1} - 4\overline{Z}_i, \quad i = 2,\ldots,N,$$
$$\overline{\Xi}_{i+1}^i = -2\overline{Z}_i, \quad i = 1,\ldots,N,$$
$$\overline{\Xi}_{N+1}^{N+1} = -\overline{Q}_N - 4\overline{Z}_N - \overline{S}_1,$$
$$\overline{\Xi}_{2N+2}^{2N+2} = -\overline{S}_2,$$
$$\overline{\delta} = \left(\frac{d}{N}\right)^2 \sum_{i=1}^{N} \overline{Z}_i + \overline{S}_2.$$

Proof. Following similar lines in above proof of Theorem 10, the proof can be accomplished, so the procedure is omitted. □

4. Numerical Examples

In this section, two illustrative examples will be provided to demonstrate that the results in this paper are effective and less conservative.

Example 1. Consider the singular system (4) with

$$A = \begin{bmatrix} -0.5 & 0.1 \\ 0.2 & -1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} -1 & 1 \\ 0 & 0.5 \end{bmatrix},$$
$$C = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.1 \end{bmatrix}, \tag{44}$$
$$D = \begin{bmatrix} 0.1 & 0.1 \\ 1 & 0.1 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 1 lists the maximum values of sampling period d which is obtained from different methods.

It can be seen from Table 1 that the sampling periods obtained from Theorem 6 ($N = 2$) and Theorem 6 ($N = 3$) are larger than those in [26–29], which shows the method in the paper improves the most previous works effectively.

Table 1: Maximum values of sampling period d.

Methods	Maximum d allowed
Theorem 3.1 [27]	1
Theorem 1 [28]	1.1547
Theorem 2 [26]	1.1954
Theorem 5 [29]	1.2060
Theorem 6 ($N = 2$)	1.2086
Theorem 6 ($N = 3$)	1.2457

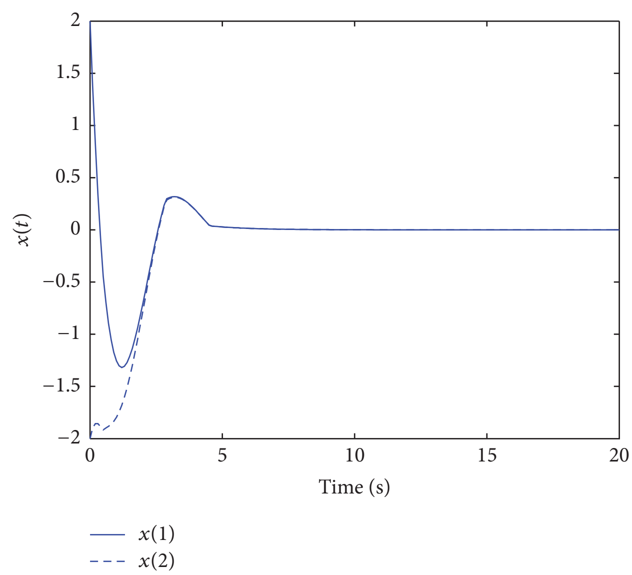

Figure 1: State response of system (4).

When the sampling period $d = 0.5$ s, the H_∞ performance $\gamma_{\min} = 0.1529$, which is achieved by Theorem 9 ($N = 3$). Then, the gain matrix of the sampled-data controller is calculated through LMI toolbox as follows:

$$K_1 = \begin{bmatrix} -3.9755 & 0.0646 \\ 2.1741 & -0.0142 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -3.7567 & 0.0651 \\ 2.070 & -0.0128 \end{bmatrix}. \tag{45}$$

Then the obtained sampled-data controllers are applied to system (4). With the initial condition $x(t) = [-2 \ \ 2]$, the response curve for system (4) is exhibited in Figure 1. From Figure 1, we can see that the state tends to zero; that is, the designed sampled-data controllers can stabilize system (4).

Example 2. According to Corollary 13, when E is an identity matrix, system (1) is turned into the traditional linear sampled-data control system. To illustrate the behavior of presented state-derivative feedback control law, we use the model of a linear dynamic positioning (DP) ship. The main parameters are referenced to the Ship Handling Simulator designed by the Institute of Navigation Jimei University (length = 175 m, beam = 25.4 m, tonnage = 2.4×10^7 kg, and draft = 9.5 m).

Consider the following linearized equations of DP ship.

$$\dot{x}(t) = Ax(t) + B_1 u(t) + Bw(t),$$
$$y(t) = Cx(t) + Du(t), \tag{46}$$

where

$$A = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}D \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$
$$C = \begin{bmatrix} I & 0 \end{bmatrix}, \tag{47}$$
$$M = \begin{bmatrix} 2.6415 \times 10^7 & 0 & 0 \\ 0 & 3.345 \times 10^7 & 1.492 \times 10^7 \\ 0 & 1.492 \times 10^7 & 6.52 \times 10^9 \end{bmatrix},$$
$$D = \begin{bmatrix} 2.22 \times 10^4 & 0 & 0 \\ 0 & 2.22 \times 10^5 & -1.774 \times 10^6 \\ 0.2 & -1.774 \times 10^6 & 7.151 \times 10^8 \end{bmatrix},$$

where $x(t) = [x \;\; y \;\; \psi \;\; p \;\; v \;\; r]$ represents the position, heading, and velocities of ship; $w(t)$ represents the external disturbance input of the system like waves, wind, and ocean currents; $u(t)$ is control input vector of forces and moments; $y(t)$ is the controlled system output. The initial position and heading of the ship are $\eta_i = [0\,\text{m} \;\; 0\,\text{m} \;\; 2.5^\circ]$, and body-fixed velocities $v_i = [2\,\text{m/s} \;\; 3\,\text{m/s} \;\; 0.2^\circ/\text{s}]$. The final desired state is $\eta_f = [10 \;\; 10 \;\; 0^\circ]$ and $v_f = [0\,\text{m/s} \;\; 0\,\text{m/s} \;\; 0^\circ/\text{s}]$. When the $\gamma = 0.5$, the sampling interval $d = 3.15$ s, which is achieved by Theorem 9 ($N = 3$). Then, the control gain matrix is

$$K_1 = \begin{bmatrix} -0.0928 & 0 & 0 & -1.3715 & 0 & 0 \\ 0 & 0.6782 & -0.186 & 0 & 1.245 & 1.5821 \\ 0 & -0.186 & 0.753 & 0 & -0.046 & 0.4923 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.1025 & 0 & 0 & 0.0103 & 0 & 0 \\ 0 & 0.0329 & -0.5412 & 0 & 0.0103 & 0.235 \\ 0 & -1.3751 & 2.1672 & 0 & 1.421 & -3.392 \end{bmatrix}. \tag{48}$$

Comparing the result obtained from Corollary 13 with [34, 35] adopting the same state-derivative feedback, the calculated result of upper bound delay d for different values of γ is shown in Table 2. It can be seen that the proposed H_∞ sampled-data controller in the paper gives larger delay bound than those in [34, 35], which illustrates that the technique proposed in the paper is more effective and the stability criteria are less conservative.

Then the obtained sampled-data controllers are applied to system (46). From Figures 2–9, we can see that the proposed sampled-data controller guaranteed H_∞ control performance and the ships can maintain desired position, heading, velocity, and acceleration under the external disturbance like waves, wind, and ocean currents.

Table 2: The upper bounds of d for various of γ.

Method	0.5	1.0	10.0
Lu et al. [34]	1.56	3.61	5.65
Zhao et al. [35], Theorem 9 ($N = 2$)	2.86	6.16	10.16
Zhao et al. [35], Theorem 9 ($N = 3$)	3.07	6.27	10.28
Theorem 9 ($N = 2$)	2.93	6.23	10.27
Theorem 9 ($N = 3$)	3.21	6.39	10.41

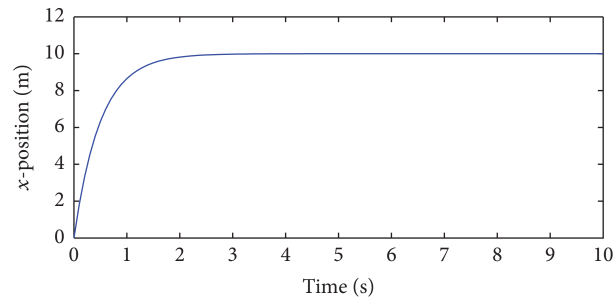

Figure 2: Position of the x direction of the ship.

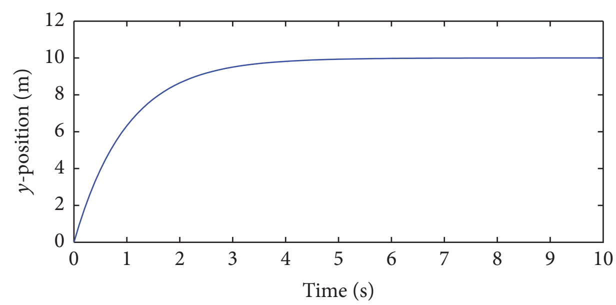

Figure 3: Position of the y direction of the ship.

5. Conclusion

The H_∞ control problem of singular neutral system with sampled-data has been studied in the paper. By input delay approach and the state-derivative control law, the singular system has been converted to a singular neutral system with time-varying delay. Less conservative results are derived for the resultant system by incorporating the delay decomposition technique, Wirtinger-based integral inequality, and an augmented Lyapunov-Krasovskii functional. The H_∞ sampled-data controller is designed to guarantee the asymptotically admissibility of the resultant system. Finally, two simulation examples have illustrated the improvement of the

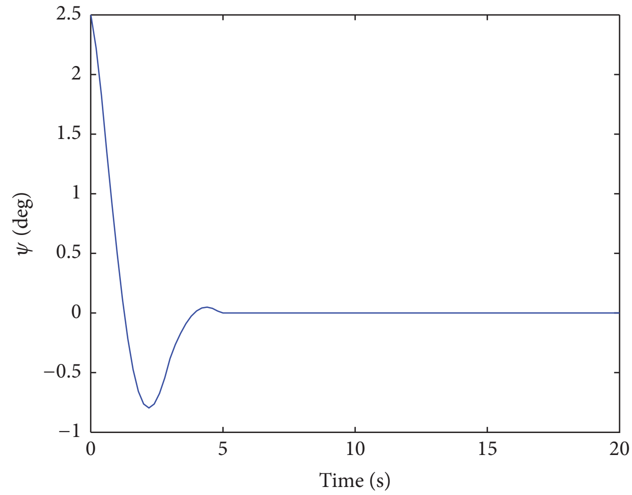

Figure 4: Heading ψ of the ship.

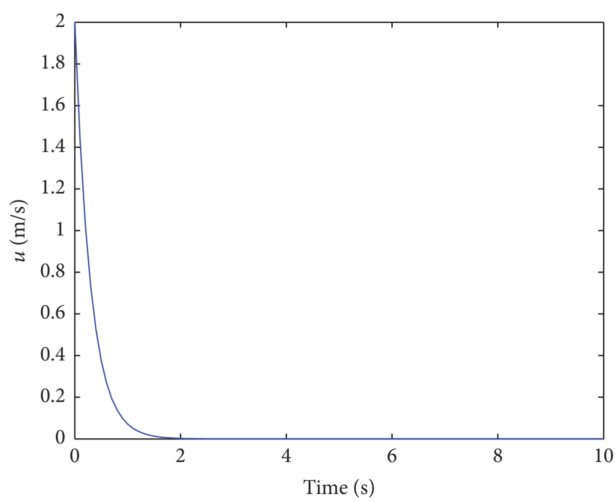

Figure 5: Surge velocity u of the ship.

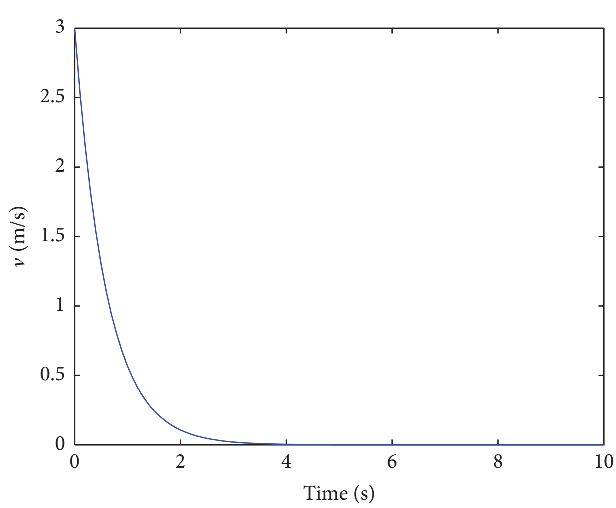

Figure 6: Sway velocity v of the ship.

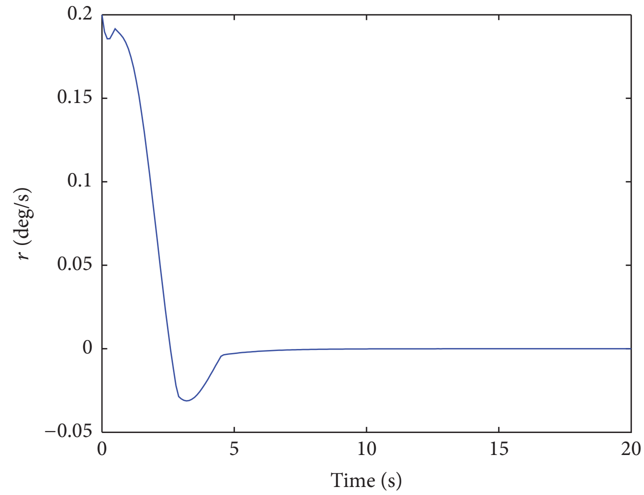

Figure 7: Yaw rate r of the ship.

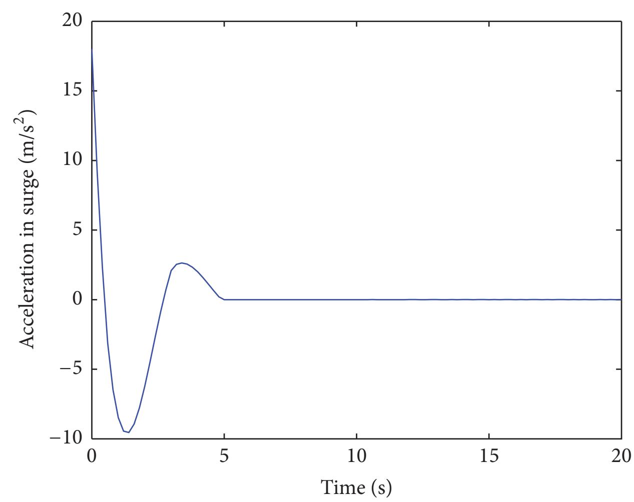

Figure 8: Acceleration of the x direction of the ship.

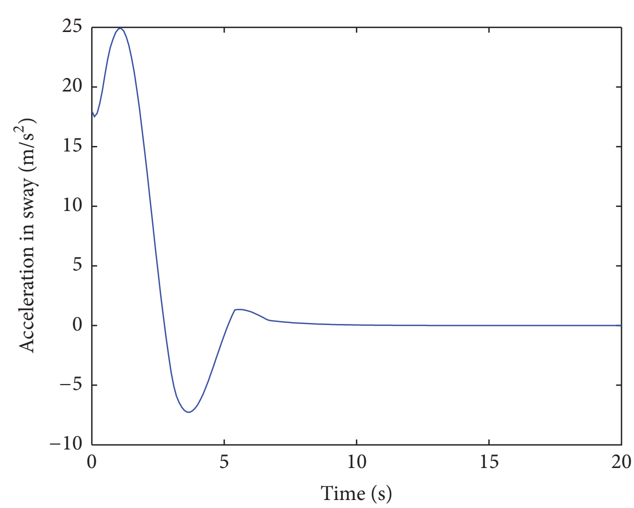

Figure 9: Acceleration of the y direction of the ship.

proposed method. Our future research topic is to investigate the fuzzy-model-based sampled-data control for nonlinear singular neutral systems.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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