Estimation of Residual Stresses in Laminated Composites by Slitting Method Utilizing Eigen Strains

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The manufacturing parameters such as curing process cause residual stresses in polymeric laminated composites. Therefore, an accurate method of measurement of residual stresses is essential for the design and analysis of composites structures. The slitting method is recently used for measurement of the residual stresses in laminated composites. However, this method has some drawbacks such as high sensitivity to noise of measurements and high scattering in the final results, which necessitate using of normalization techniques. Moreover, the form of polynomials, used in the conventional slitting method for calculation of the stiffness matrix, has a significant effect on final results. In this paper, it is shown that the major reason of the drawbacks of the slitting method in calculating the residual stresses is a direct use of the elastic released strains recorded by strain gages. In the present study, instead of direct calculation of residual stresses from the elastic released strains, eigen strain distribution as a constant and invariant field has been calculated from the recorded elastic strains. Then, by using the calculated eigen strain field in a finite-element model, the residual stress filed was obtained. Also, instead of using polynomials to calculate the compliance, a superposition method was used. The results show that the new method decreases the sensitivity of the final results to noise and scattering of the experimental data. It means that the normalization methods are not needed any more. [DOI: 10.1115/1.4033374]

Keywords: residual stress, slitting method, eigen strain, composite laminate, finite-element method

1 Introduction

Residual stresses are self-equilibrate stresses that exist in unloaded structures. The residual stresses can be created at any step of fabrication, finishing, and assembling. The sources of these stresses in laminated composites are the existence of several phases (e.g., fiber, matrix, etc.) with different thermal expansion coefficients and the stacking sequence of different plies with different angles. The first ply failure stress degradation, fiber and matrix debonding, matrix cracking and delamination are some results of existence of residual stresses in laminated composites [1–3]. Therefore, it is necessary to investigate new methods or improve the accuracy of previous ones to consider the effect of residual stresses in designing of composites.

The methods for measurement of residual stress have been divided in two main groups: destructive and nondestructive methods. The slitting method is the most used destructive approach after the hole drilling method [4,5]. This method was first introduced in 1987 [6] and in recent years, this method has been modified [7,8]. In this method, creating a slit and increasing its depth in several steps cause change in the elastic strain field of the body. By sticking a strain gage on the surface of the specimen, the change of elastic strain can be recorded in a certain point. By using this recorded elastic strain in analytical or numerical methods, distribution of residual stress could be found. The schematic of the slitting method has been shown in Fig. 1.

Shokrieh and Akbari [9] developed a new approach called simulated slitting method to estimate residual stresses in laminated composites. They established their methods for laminated composites, while the earlier analytical methods for calculation of the compliance matrix [10] were limited to isotropic materials. Prime [11] used the eigen strain method to estimate fiber-scale residual stress in a unidirectional lamina. Equation (1) shows the relation between the elastic strains and residual stresses perpendicular to the slit face, when the depth of the slit is a_i

$$\varepsilon_{yy}(a_i) = \frac{1}{E'} \int_0^{a_i} G(x, a_i)\sigma_{yy}(x)\, dx \tag{1}$$

$G(x, a_i)$ as a Kernel function is equal to the elastic strain recorded by the strain gage because of the unit residual stress in the depth x, and when the depth of the slot is a_i. E' has been defined as below

$$\begin{cases} E' = E & \text{when } \dfrac{B}{t} \le 0.5 \text{ (Plane Strain)} \\ E' = \dfrac{E}{1-\upsilon^2} & \text{when } \dfrac{B}{t} \ge 2 \text{ (Plane Stress)} \end{cases}$$

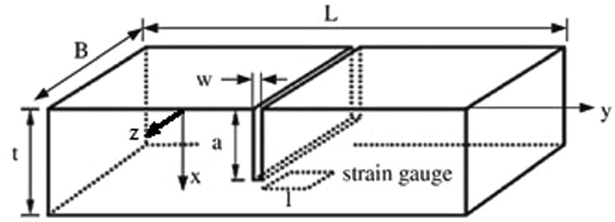

Fig. 1 A schematic figure of slitting method

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E and υ are elastic constants [10]. Equation (1) is an inverse relation, because the residual stress σ_{yy} as the unknown parameter of Eq. (1) is on the right side of the equation and the known parameter (ε_{yy}) has been appeared on the left side of the equation. So, a small error in the measurement of elastic strain causes higher-order deviation in calculation of the residual stress σ_{yy} [9]; and this results in a bad conditioning compliance matrix [12]. The least square method and the Tikhonov regularization method [13,14] are used for stabilization of the final solution and decreasing the effect of measurement errors on the final calculated stresses.

To solve Eq. (1), in the conventional slitting method, the distribution of the residual stress has been considered as a polynomial

$$\sigma(x) = \sum_{j=0}^{n} A_j P_j(x) \tag{2}$$

$P_j(x)$ is a basic polynomial and A_j is the domain factor for $P_j(x)$. Although the form of $P_j(x)$ is arbitrary, it can affect the final results. By substituting Eq. (1) in Eq. (2), Eq. (3) is achieved

$$\begin{aligned}\varepsilon(a_i) &= \frac{1}{E'}\int_0^{a_i} G(x,a_i)\sum_{j=0}^{n} A_j P_j(x)\,dx \\ &= \frac{1}{E'}\sum_{j=0}^{n} A_j \int_0^{a_i} G(x,a_i)P_j(x)dx = \sum_{j=0}^{n} A_j C_{ij}\end{aligned} \tag{3}$$

where C_{ij} is an element of the compliance matrix and is equal to the recorded elastic strain, when the stress on the surface of the slot with a depth of a_i is $P_j(x)$.

The selected polynomial must satisfy force and moment equilibrium equations. For instance, by elimination of the first and second terms, the Legendre polynomials satisfy these conditions. The other limitation for selection of a polynomial is satisfying of compatibility equations. It should be noted that a polynomial that simultaneously satisfies all those conditions has not been found yet. According to the best knowledge of the present authors, in most of the published studies, the equilibrium equations were just satisfied.

It is usual to implement a finite-element method (FEM) to calculate the compliance matrix ($\boldsymbol{C}$) in the slitting method. Shokrieh and Akbari [9] obtained this matrix for laminated composites. The condition number has been defined as below to show the stability of the compliance matrix

$$\mathrm{Cond}([\bar{C}]) = \|[\bar{C}]\|\|[\bar{C}]^{-1}\| \tag{4}$$

$$[\bar{C}] = [C]^{\mathrm{T}}[C] \tag{5}$$

$$\|[\bar{C}]\| = \underset{1\le i\le n}{\mathrm{Max}} \sum_{j=1}^{n} |\bar{C}_{ij}| \tag{6}$$

When the condition number converges to 1, the compliance matrix will be more stable. According to Fig. 2, increasing the number of terms in a polynomial (used in Eq. (2)) strongly affects the matrix stability and so the results. In this, figure n is the highest order of polynomials and m is the number of slot depth increasing. The final results deviation, due to small errors in measurements, occurs when the compliance matrix is unstable.

It is undeniable that, decreasing the number of terms in a polynomial decreases the accuracy of the estimation. It should be noted that the type of polynomials (Legendre, Chebyshev, Power, etc.) and their continuity (continuous or noncontinuous) affect the final results. Therefore, the main drawbacks of the conventional slitting method are: unsatisfying of compatibility equations and effects of polynomial type on the instability of the compliance matrix and final estimation of the residual stress. The present study tries to develop a new computational method based on the eigen strain concept to eliminate these drawbacks.

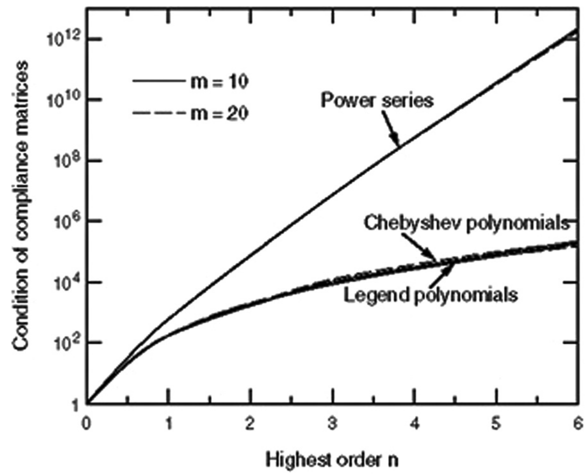

Fig. 2 Effect of number of terms in a polynomial on the condition number [9]

2 Modified Simulated Slitting (MSS) Method

In the conventional slitting method, the compliance matrix relates the elastic strains recorded by the strain gage to the distribution of residual stress in depth of the specimen. It means that in the conventional slitting method, the residual stress was directly related to the elastic strain. By increasing the slitting depth, the elastic strain and stress fields are changed and cause ill-conditioning for the compliance matrix. In the present method, this problem has been solved by calculating the eigen strain as a permanent and invariable field from the recorded elastic strain. Then, the residual stress field is obtained by inducing the eigen strain field in a finite-element model. The new model called MSS method. Since the eigen strain is a permanent and invariable field, even during the slitting process, less errors will be induced in the calculation process. First the laminate is modeled using finite-element software and by simulating the slitting process, the $\boldsymbol{C}$ matrix is obtained. Then, the $\boldsymbol{A}$ matrix could be calculated. Furthermore, the strain is recorded (by the strain gage) using experimental slitting method and then eigen strains are calculated. Then, these eigen strains are induced in the finite-element model to calculate the residual stresses. More detail of the present method will be explained in Sec. 2.8.

2.1 Eigen Strains. In micromechanics, the eigen strain generally refers to inelastic strains such as thermal strains, plastic strains, phase transform strains, initial strains, and nonhomogeneity strains [15]. In a continuous region, the total strain is equal to summation of the elastic and eigen strains

$$\varepsilon^T = \varepsilon^e + \varepsilon^* \tag{7}$$

where ε^T is the total strain and ε^e and ε^* are the elastic and eigen strains, respectively. Using the linear stress–strain relation

$$\sigma_{ij} = L_{ijkl}\varepsilon^e_{kl} \tag{8}$$

Substituting Eq. (7) in Eq. (8), the following equation is achieved:

$$\sigma_{ij} = L_{ijkl}(\varepsilon^T_{kl} - \varepsilon^*_{kl}) \tag{9}$$

Mura [15], by considering the eigen strain effect as a source of creation of the external force in the equilibrium equation and solving it by the Fourier transform method, obtained the stress, elastic strain, and displacement fields. He introduced a part of the eigen strain field that did not satisfy the compatibility equation as the source of the residual stress and then solved some problems in this field. Some researchers followed his idea and estimated the residual stress field using eigen strain concept. Using this method, Korsunsky et al. [16–19] estimated the residual stress for different cases such as a crack tip, for the shot pinning process, for a welded nickel plate, and for a friction weld. Some other researchers such as Wang and coworkers [20] and Lokhov et al. [21] have published some papers on this topic. In all of these papers, the eigen strain field is known. Therefore, stress, elastic strain, and displacement fields have been obtained in a closed form solution. Mura's equations are complicated and finding a closed form solution for composite materials to find the elastic strain field, eigen strain field, and residual stresses are difficult. Residual stresses are induced in a composite laminate, because of the curing process and the material mismatch of layers. In this case, the eigen strain field is unknown and a method should be developed to find it. In the following, a simple method using a FEM is developed to solve this problem.

2.2 Eigen Strain in Laminated Composites. The source of the residual stress is the eigen strain field which does not satisfy the compatibility equations. So, this type of the stains has been called as the incompatible strain field. In other words, the existence of an incompatible strain field will cause a residual stress field. In fact, for a deformed body by removing the external forces, only the residual stresses can keep the body in the deformed shape.

Slitting of a composite laminate release some of the stresses and will change the residual stress field. It is a disadvantage of the conventional slitting method. During the slitting process, the eigen strain field remains constant and invariable. In other words, the eigen strain is a permanent field. Therefore, in the slitting process, more accurate residual stresses can be calculated by using the eigen strain field instead of the released elastic strain.

2.3 Residual Stresses as a Function of Eigen Strains. In the present study, the laminated composite is assumed to be a linear elastic material. The tensorial relation between the eigen strain and the residual stress is as follows:

$$\bar{\sigma} = \bar{f}(\bar{\varepsilon^*}) \tag{10}$$

where $\bar{\sigma}$ and $\bar{\varepsilon^*}$ are first-order tensors of the residual stress and eigen strain that are functions of coordinate systems, respectively. Also, $\bar{f}$ is a function that gives residual stresses from the eigen strains. In the elasticity problems, the eigen strain contributes in the solution process using the following constitutive equation:

$$\bar{\sigma} = \bar{C}.(\bar{\varepsilon^T} - \bar{\varepsilon^*}) \tag{11}$$

where $\bar{\varepsilon^T}$ and $\bar{C}$ are the total strain and constitutive elastic tensors, respectively. It is obvious from Eq. (7) that $(\bar{\varepsilon^T} - \bar{\varepsilon^*})$ is an elastic strain tensor. Comparing Eqs. (10) and (11) shows that $\bar{f}$ is an elastic reaction function for the eigen strain.

2.4 The Linear Relation Between the Residual Stress and Eigen Strain. In this section, it will be proved that the relation between the eigen strain and residual stress is linear. In a finite-element model, a number of certain nodes, elements, and interpolation functions have been chosen. For $\bar{\varepsilon^*}$ and $\bar{\sigma}$, some basic functions can be considered. For the coordinate system shown in Fig. 3, the eigen strain distribution can be expressed as follows:

$$\varepsilon^*(x) = \sum_{k=1}^{n} \varepsilon_k^* N_k(x) \tag{12}$$

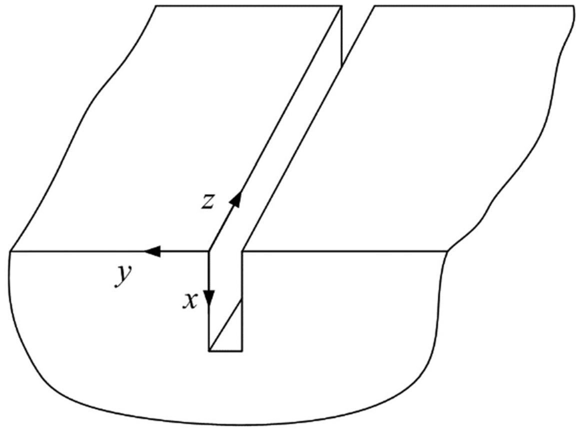

Fig. 3 The coordinate system

where ε_k^* is the amount of the eigen strain in the specified nodes, N_k is the interpolation function, and $\varepsilon^*(x)$ is the distribution of the eigen strain in the laminate. In fact, for a point located on one of the specific nodes of the finite-element model, the function gives the eigen strain on that node and for the point located between the nodes, the function uses an interpolation scheme.

As an example, for node i with the eigen strain ε_i^* and coordinate y_i, Eq. (12) converts to

$$\varepsilon^*(y_n) = \varepsilon_1^* \underbrace{N_1(y_1)}_{=0} + \varepsilon_2^* \underbrace{N_2(y_2)}_{=0} + \cdots + \varepsilon_i^* \underbrace{N_i(y_i)}_{=1} + \cdots = \varepsilon_i^* \tag{13}$$

Similarly, the following equations are valid for the stress and elastic reaction functions:

$$\sigma(x) = \sum_{k=1}^{n} \sigma_k N_k(x) \tag{14}$$

$$f(x) = \sum_{k=1}^{n} f_k N_k(x) \tag{15}$$

If in the finite-element model, the eigen strain for the ith node is equal to one and zero for other nodes. By performing a finite-element analysis, the stress distribution in all nodes based on the above distribution of the eigen strain is found

$$M_{ij} = f_j(\varepsilon^* = \varepsilon_i^* N_i | \varepsilon_i^* = 1) \tag{16}$$

where the M_{ij} is stress in jth node because of the unit eigen strain in ith node and zero for other nodes. Therefore, the superposition principle simplifies Eq. (10) to

$$\sigma_i = \sum_j M_{ij} \varepsilon_j^* \tag{17}$$

For example, in a model with three nodes, Eq. (17) will be as below

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{Bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{Bmatrix} \begin{bmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \varepsilon_3^* \end{bmatrix} \tag{18}$$

When the eigen strain is equal to one for node 2 and zero for other nodes

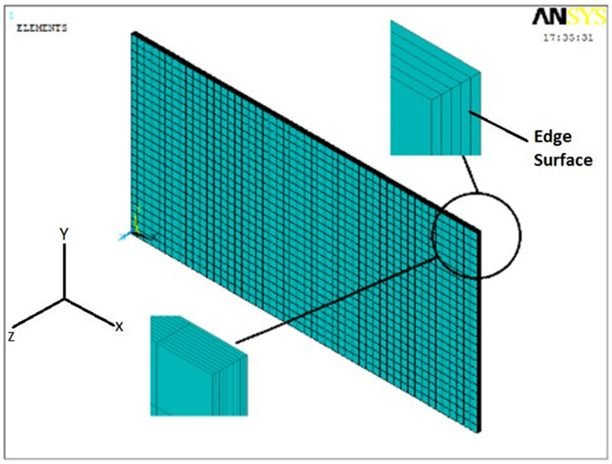

Fig. 4 Finite-element model of a five-layer laminate model

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{Bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{Bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} M_{12} \\ M_{22} \\ M_{32} \end{bmatrix} \quad (19)$$

According to Eq. (19), when the eigen strain in jth node of a model is equal to unit and is zero for other nodes, a finite-element analysis calculates the stresses in all nodes and this procedure gives jth column of the matrix $\boldsymbol{M}$. Repeating this process gives all columns of the matrix.

2.5 Evaluation of the Procedure for Calculation of Matrix *M*. To verify the procedure for calculation of the matrix $\boldsymbol{M}$, the APDL programming language of ANSYS has been implemented to analyze five-layer composites as shown in Fig. 4. Mechanical properties of all layers are the same (shown in Table 1) and the stacking sequence was [0/90/0/90/0]. SOLID46 element has been used in this analysis. All nodes on the edge surface, shown in Fig. 4, have been constrained in X direction. The nodes on the center line of this surface have been constrained in Y direction and finally the node that located exactly at the center of this surface has been constrained in Z direction. It should be mentioned here that this type of element and boundary condition have been used for all models in this study. There are 50 elements in the length direction and 25 in the width direction of each layer, totally 12,500 elements for the model.

The dimension of the model was $50 \times 25 \times 0.25$ mm. Since, in laminated composites, the residual stress field is usually a function of the depth coordinate, so the eigen strain field is considered as a function of this coordinate. Therefore, all nodes in a specific coordinate of Z direction (refer to Fig. 1) can be considered as a group and for each step of calculation of the matrix $\boldsymbol{M}$, the unit eigen strain is induced in all nodes of one of these groups. Thus, all nodes of the laminate composites have been divided to eleven groups. Figure 5 shows these groups. In each step of the analysis for calculating $\boldsymbol{M}$, unit value of the eigen strain was induced in all nodes of a group. In the common boundary of layers, there are two groups of nodes that are coincided; one group belongs to a layer and other group belongs to adjacent layer.

In the finite-element model, the unit value of the eigen strain is induced for the ith node group and zero value for all other nodes. Then, stresses in all groups were calculated and considered as the

Table 1 Mechanical properties of a unidirectional ply

E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	ν_{xy}
104	10	6	0.3

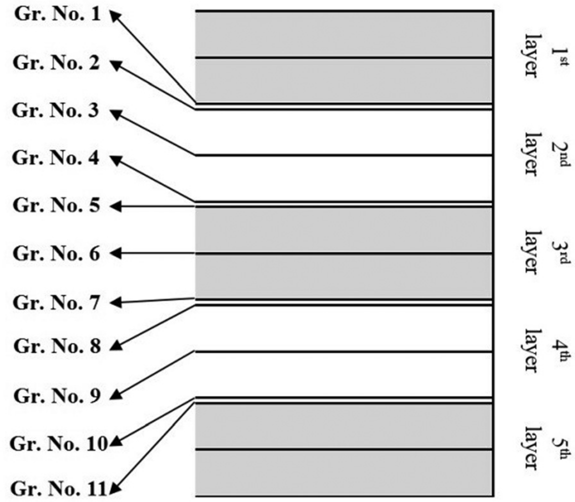

Fig. 5 The groups of nodes

ith column of the $\boldsymbol{M}$ matrix. By repeating this procedure, all columns of the $\boldsymbol{M}$ matrix are calculated.

An arbitrary distribution of the eigen strain was induced in finite-element model. Then, the finite-element analysis calculates stresses (S_{fem}) based on an arbitrary eigen strain in the model. On the other hand, when the eigen strain and $\boldsymbol{M}$ matrix are known, using Eq. (19) the stresses (S_{esm}) for the arbitrary eigen strain distribution were calculated. Figure 6 shows a comparison of S_{fem} and S_{esm}.

The excellent agreement of stresses calculated by the new method and the FE analysis shows the capability of the present method. The effect of singularity of matrix $\boldsymbol{M}$ on the results and the method of solving this problem will be explained in Sec. 2.6.

2.6 Singularity of Matrix *M*. To prevent the singularity of matrix $\boldsymbol{M}$, the eigen strain matrix has not been induced in two groups of nodes; the highest and the lowest groups. In inducing process of the eigen strain in the model for calculation of matrix $\boldsymbol{M}$, nodes near to the surface should be ignored. All inner node groups of the model are in mismatch with two groups of nodes; the upper and the lower ones. However, nodes in the top and back surfaces are in mismatch with one group. So, this group of nodes produces much less mismatch of property. The mismatch causes the eigen strains produce residual stresses in the laminate. So, the eigen strain in boundary nodes makes insignificant values of residual stresses. Inducing the eigen strain in nodes that give zero or insignificant values of residual stress makes the matrix $\boldsymbol{M}$ singular.

Utilizing a finite-element model, the effects of singularity of matrix $\boldsymbol{M}$ have been studied. Mechanical properties and dimensions of this model are the same as ones were used for the

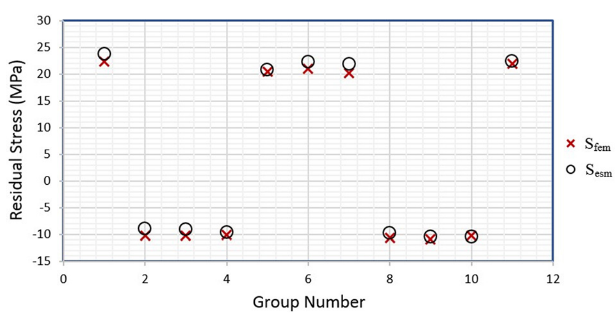

Fig. 6 A comparison of stresses calculated by the model and the FEM

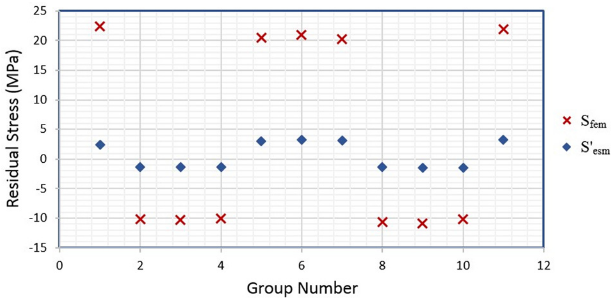

Fig. 7 Effect of singularity of matrix M on results

previous model. In this step, using a singular matrix, the residual stresses (S'_{esm}) are computed by Eq. (19). The residual stresses obtained by the FE analysis (S_{fem}) and the residual stresses obtained by the singular matrix (S'_{esm}) have been shown in Fig. 7. Because of the singularity of matrix $\boldsymbol{M}$, results converged to zero. As shown in this figure, there is a significant difference between the accurate results (S_{fem}) and result obtained by the model (S'_{esm}). So, the singularity of matrix $\boldsymbol{M}$ strongly decreases the accuracy of method.

2.7 Method of Inducing Eigen Strain in a Finite-Element Model. A simple method for inducing the eigen strain in a certain node is inducing temperature in that node. For the thermal strain

$$\varepsilon^*_{th} = \alpha T \tag{20}$$

where α, T, and ε^*_{th} are the thermal expansion coefficient, temperature changes, and the thermal strain, respectively. If α for all layers is equal to one and the initial temperature is equal to zero, then, the eigen strain field is equal to the temperature field induced in the model

$$\varepsilon^*_{th} = 1(T - T_0) = T \Rightarrow \varepsilon^*_{th} = T \tag{21}$$

In each step of calculating matrix $\boldsymbol{M}$, the temperature is equal to zero for all layers except the jth layer that is equal to 1 °C. So, for all layers (except the jth layer) the thermal eigen strain is

$$\varepsilon^*_{th} = \alpha T = 1(T_2 - T_1) = 1(0) = 0 \tag{22}$$

and for the jth layer, the thermal eigen strain is

$$\varepsilon^*_{th} = \alpha T = 1(T_2 - T_1) = 1(1 - 0) = 1 \tag{23}$$

Using a finite-element analysis, due to the mentioned eigen strain distribution in the model, the stresses in all layers can be calculated. These stresses are the members of the jth column of the matrix $\boldsymbol{M}$. Repeating this procedure gives all columns of the matrix $\boldsymbol{M}$.

It must be mentioned that Eq. (19) can be applied when the eigen strain field is known. However, for laminated composites, the eigen strain field is unknown. Therefore, to measure the residual stresses in laminated composites, a suitable estimation of the eigen strain field is necessary. A proper way of estimation of the eigen strain field in laminated composites will be explained in Sec. 2.8.

2.8 Investigation the Relation Between Elastic Strain and Eigen Strain. In the slitting method, a relation (usually an integral equation) is established that gives stresses based on the elastic strains recorded by the strain gages during the sectioning process. So, an order of errors in measurement causes higher order of errors in calculations. However, the eigen strain and elastic strain are related together by a linear function. In the following, the residual stress distribution before and after of one step of slitting are denoted by σ and σ'. These stresses are related to elastic strains before and after of one step of slitting (e and e') by matrices P and P'

$$P.\sigma = e \quad \text{and} \quad P'.\sigma' = e' \tag{24}$$

The above equations could not be combined in order to give a simple relation for estimation of residual stresses before slitting. So, there are complicated integral relations for this purpose (such as Eq. (1)). While, if the eigen strains are calculated from the elastic strains, the solution procedure will be quite different. If the slitting process is performed ideally, the eigen strain will be a constant field during the sectioning. So, for two different states of a body (for instance, before and after the ith step of slitting), relations are as follows:

$$N.\varepsilon^* = e \quad \text{and} \quad N'.\varepsilon^* = e' \tag{25}$$

By subtracting these relations

$$(N - N').\varepsilon^* = (e - e') \tag{26}$$

where $(N - N')$ can be considered as the matrix C

$$C = (N - N') \tag{27}$$

and $(e - e')$ is the elastic strain recorded by the strain gage during the slitting progress in a certain step could be noted by ε^e

$$\boldsymbol{\varepsilon}^{\boldsymbol{e}} = \boldsymbol{C}.\boldsymbol{\varepsilon}^* \tag{28}$$

where $\boldsymbol{\varepsilon}^{\boldsymbol{e}}$ and $\boldsymbol{\varepsilon}^*$ are the measured elastic strain and eigen strain vectors, respectively. Matrix $\boldsymbol{C}$ relates these two vectors and can be calculated using a finite-element analysis. If the number of components of vectors $\boldsymbol{\varepsilon}^{\boldsymbol{e}}$ and $\boldsymbol{\varepsilon}^*$ are equal, matrix $\boldsymbol{C}$ will be square. If matrix $\boldsymbol{C}$ is not singular, then the inverse of Eq. (28) can be achieved. If all elements of ε^* are effective and produce residual stress, then the matrix $\boldsymbol{C}$ is not singular and can be inversed. When the number of elements of ε^e is less than the number of elements of ε^*, there is not any solution for Eq. (28). If the number of components of eigen strain vector is less than that of the elastic strain vector, then the least square method is used to inverse Eq. (28). The difference between elastic strains recorded by the strain gage and real elastic strains are as follows:

$$\boldsymbol{\varepsilon}^{\boldsymbol{m}} - \boldsymbol{\varepsilon}^{\boldsymbol{e}} = \boldsymbol{k} \tag{29}$$

Substituting Eq. (28) in Eq. (29)

$$\boldsymbol{\varepsilon}^{\boldsymbol{m}} - \boldsymbol{C}.\boldsymbol{\varepsilon}^* = \boldsymbol{k} \tag{30}$$

In order to minimize the error, least square method has been used

$$\boldsymbol{\varepsilon}^* = (\boldsymbol{C}^{\mathrm{T}}\ \boldsymbol{C})^{-1}\boldsymbol{C}^{\mathrm{T}}.\boldsymbol{\varepsilon}^{\boldsymbol{m}} \tag{31}$$

or

$$\boldsymbol{\varepsilon}^* = \boldsymbol{A}.\boldsymbol{\varepsilon}^{\boldsymbol{m}} \tag{32}$$

$$\boldsymbol{A} = (\boldsymbol{C}^{\mathrm{T}}\ \boldsymbol{C})^{-1}\boldsymbol{C}^{\mathrm{T}} \tag{33}$$

Then, matrix $\boldsymbol{A}$ can be obtained when the matrix $\boldsymbol{C}$ is known. Using matrix $\boldsymbol{A}$ and the recorded elastic strains due to slitting, the distribution of the eigen strain can be calculated. Inducing the distribution of the eigen strain in a finite-element model and performing a stress analysis, the distribution of the residual stress in the component will be achieved. In the following, an easy method will presented to calculate the matrix $\boldsymbol{C}$. If the eigen strain in the

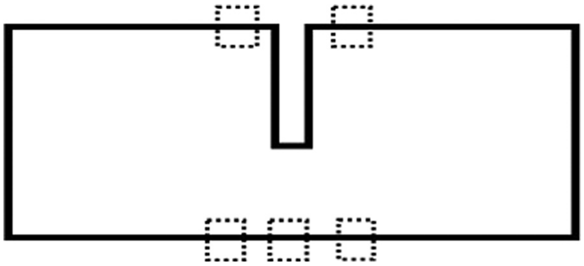

Fig. 8 The customary emplacement of strain gages in slitting method

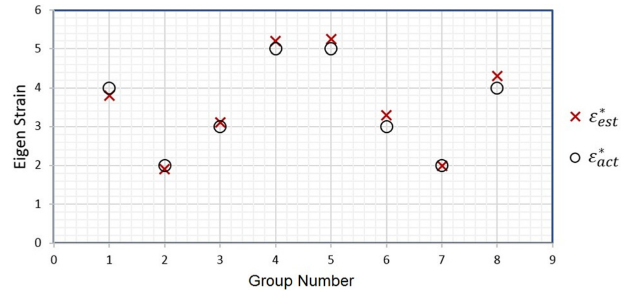

Fig. 10 Comparison of the induced and calculated eigen strains

jth layer of a laminate is equal to one and is zero in all other layers, the following equation can be written:

$$\boldsymbol{\varepsilon}^{e} = \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & & \ddots & & \vdots \\ c_{m1} & \cdots & c_{mj} & \cdots & c_{mn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \varepsilon_j^* = 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} c_{1j} \\ c_{2j} \\ \vdots \\ c_{mj} \end{bmatrix} \quad (34)$$

In fact, c_{ij} is equal to the recorded elastic strain in the ith increase in the slot depth, when the eigen strain is equal to one in the jth layer and is zero in all other layers of the laminate. The number of rows and columns in matrix $\boldsymbol{C}$ is equal to the number of the slot depth progress and the number of layers with induced eigen strain, respectively.

For evaluation of the accuracy of Eq. (32), a five-layered laminated composite was considered. Mechanical properties and dimensions are the same as the previous model, but different meshing was used. The size of elements near the slot and strain gage is smaller than other elements. The possible emplacements of strain gage in slitting method are shown in Fig. 8. Shokrieh and Akbari proved the best place for emplacement of the strain gage is the back face of the laminate; exactly in front of the slot. In this study, this place has been used for the emplacement of the strain gage.

The number of elements in this model was 2560. Figure 9 shows the front and back surfaces of the model, respectively. The elements with different colors show the place of the strain gage. Schajer [22] used the displacement of nodes in the boundary of strain gage emplacement and the initial distance between them to simulate strain gage in the finite-element model. This procedure has been used in this study. To simulate the slitting process, the elements in the slotted part removed at first and then the corresponding volume was removed from the model.

To investigate the accuracy of Eq. (32), an arbitrary distribution of the eigen strain was induced in the model ($\boldsymbol{\varepsilon}^*_{\text{act}}$). By simulating the slitting process, the elastic strain was recorded at the place of the strain gage ($\boldsymbol{\varepsilon}^{\boldsymbol{m}}$). The $\boldsymbol{C}$ matrix was calculated by the method explained at the beginning of this section. When the $\boldsymbol{C}$ matrix is known, the $\boldsymbol{A}$ matrix was obtained from Eq. (33). When the $\boldsymbol{A}$ matrix and $\boldsymbol{\varepsilon}^{\boldsymbol{m}}$ are known, Eq. (32) can be used to obtain the eigen strain distribution ($\boldsymbol{\varepsilon}^*_{\text{est}}$). This estimated eigen strain and the induced strain fields have been compared in Fig. 10. They are matched very and this result shows that the present procedure is accurate and reliable.

2.9 The Effect of Slot Width on the Linearity of the Relation Between the Eigen and Elastic Strains. To investigate the linearity of Eq. (32), two fields of eigen strain were induced in two separate models with the same mechanical properties, meshing and dimensions. The values of the eigen strain in the first model was two times of those in the second model. The models were slotted (slot width was equal to 4 mm) and elastic strains were recorded in the place of the strain gage. This procedure was repeated for different slot widths, 2 mm, 1 mm, 0.5 mm, and 0.25 mm. If Eq. (32) is linear, then the ratio of the recorded elastic strain due to the slitting in two models should be equal to the ratio of the eigen strains in the two models. Figure 11 shows that by decreasing of slot widths (0.25 and 0.5 mm) this ratio approaches to two. It can be concluded that increasing the width of the slot makes a nonlinear relation between the released elastic strains and eigen strains. Therefore, Eq. (32) is linear when the slot width is less than a certain magnitude. This magnitude must be found for each laminate before performing the experimental slitting procedure.

3 Evaluation of the Accuracy of the Present Method

To investigate the accuracy of the present method (MSS), a 16-layer laminated composite with a stacking sequence of $[90_4/0_4]_s$ was modeled in ANSYS software. Dimensions, meshing, and mechanical properties were similar to the previous models. The number of elements was 8192 (Fig. 12). At first, the $\boldsymbol{C}$ matrix was calculated from this model. The place of the strain gage was on the back face and exactly in cross of the slot. There are two elements through the thickness of each layer; totally 32 group of elements in the depth direction. By ignoring two groups of elements located in the upper and lower surfaces, the eigen strains were induced in 30 groups of elements to calculate the $\boldsymbol{C}$ matrix. The

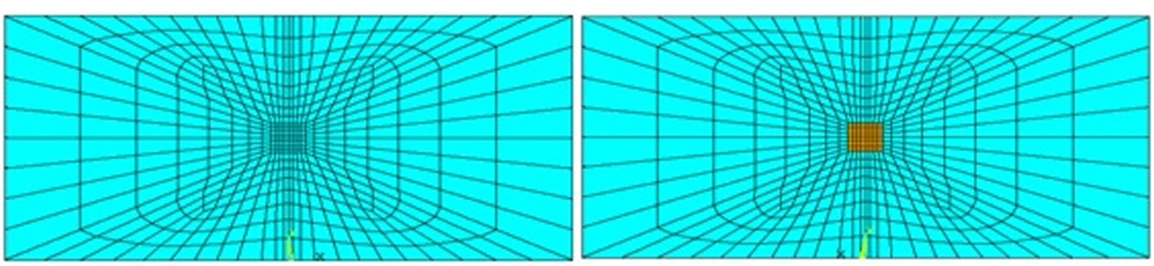

Fig. 9 The front and back surfaces view of the model (left and right)

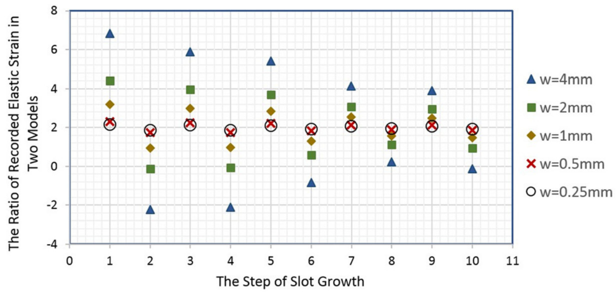

Fig. 11 Ratio of recorded elastic strain for different slot width

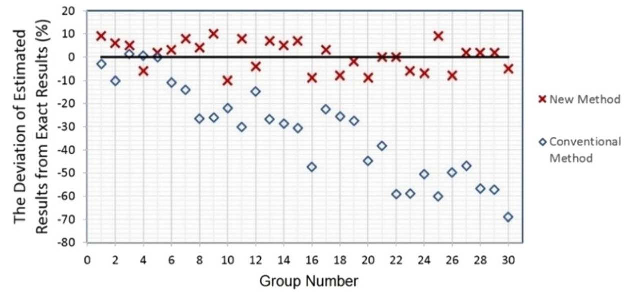

Fig. 13 The deviation of estimated results from the exact results

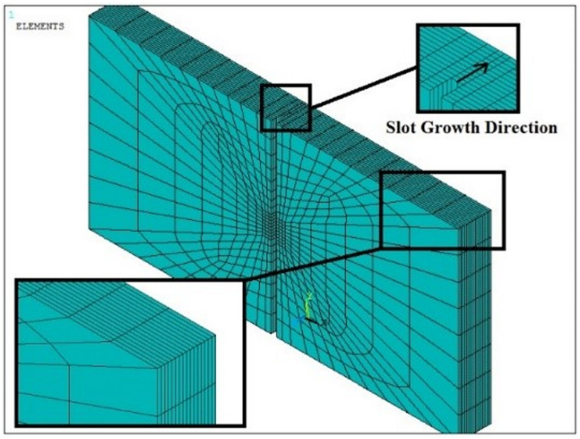

Fig. 12 A view of 16-layer model

Fig. 14 Laminated composite samples

slot was progressed in 31 steps. So, the $\boldsymbol{C}$ matrix consists of 31 rows and 30 columns (i.e., 930 elements).

In next step, an arbitrary distribution of the eigen strain has been induced in the model. The distribution of the residual stresses due to induced eigen strains were obtained from a finite-element analysis (σ_{exa}). Then, the slitting procedure simulated and the elastic strain was recorded in the place of the strain gage ($\boldsymbol{\varepsilon}^{\boldsymbol{m}}$). When the $\boldsymbol{A}$ matrix and $\boldsymbol{\varepsilon}^{\boldsymbol{m}}$ are known, Eq. (32) gives the distribution of the eigen strain ($\boldsymbol{\varepsilon}^*_{\text{est}}$). This distribution of the eigen strain that obtained from Eq. (32) ($\boldsymbol{\varepsilon}^*_{\text{est}}$) induced in a similar model and a finite-element analysis calculates the distribution of the residual stresses (σ_{est}).

Recorded elastic strains due to slitting were used in the conventional slitting method to estimate the residual stress distribution. The results of conventional slitting method have been normalized (σ'_{est}). To investigate the sensitivity of the conventional slitting method and the present MSS method to the measurement errors and noise, the recorded elastic strains were shifted randomly between -10 and 10% from the original values. To estimate the residual stresses, the shifted strains were used in both methods. Figure 13 shows that the present MSS method is less sensitive to error and noise than the conventional method. If the residual stresses are obtained by a finite-element analysis, then σ_{exa} is considered as the reference result. For the conventional method, due to the integral form of Eq. (1), errors of each step affect the results of the next steps. In the present method, this problem has been solved.

4 A Comparison of the MSS and Conventional Method

To compare the MSS and the conventional methods, carbon/epoxy (T-300/ML-506) laminated composites were manufactured (Fig. 14). The stacking sequence was $[0_4/90_4]_s$ and mechanical properties of the sample were shown in Table 2. The total thickness of the sample was 4.4 mm and 3.9 mm of the thickness was slotted in 39 steps. The recorded strain during the slitting has been shown in Fig. 15. This strain was used to calculate the residual strain in the sample by both methods and the results were compared in Fig. 16.

The results of the two methods coincide with each other for the first half of the thickness. However, for the second half of the

Table 2 Mechanical properties of the a unidirectional T-300/ML-506 ply

E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	ν_{xy}
104.6	7.5	3.8	0.31

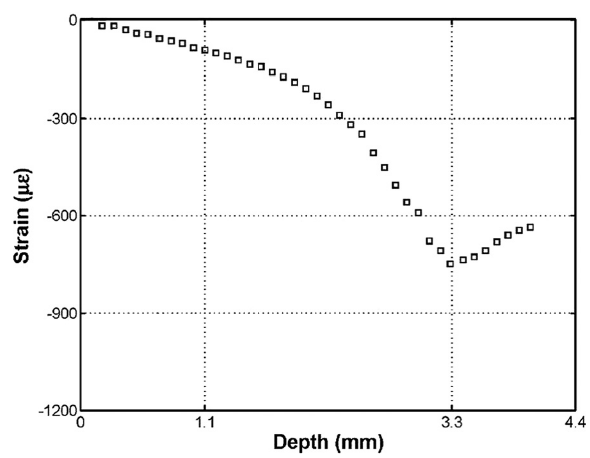

Fig. 15 The recorded strain during the slitting

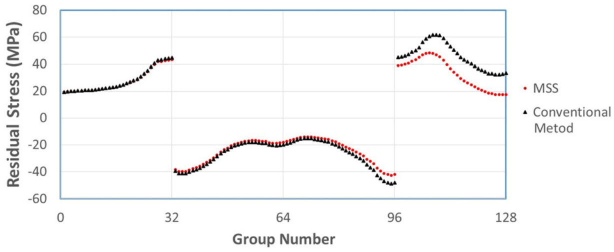

Fig. 16 A comparison between the results obtained by the MSS and conventional methods

thickness of the sample, there are some differences between the results of the two methods. These differences become strongly significant at the last quarter of the thickness. Due to the symmetric stacking sequence, the symmetry of the residual stress distribution is anticipated and it is almost observed from results obtained by the MSS method. But the results obtained by the conventional method do not show such symmetry. It can be concluded that this deviation in the conventional method is due to the accumulation of errors in calculation by the integral relation.

5 Conclusion

In the conventional slitting method, the residual stress field is calculated directly from the elastic strains recorded by the strain gage during the slitting process. An integral equation relates elastic strains to residual stresses, so in this method, the computational and measurement errors in previous steps affects the results obtained in the next steps. It means that a type of error accumulation occurs.

In the present study, instead of direct calculation of residual stresses from elastic strains, a new method (MSS) was presented. In the new method, first an eigen strain field was induced in a finite-element model and a stress analysis was performed to calculate the residual stress field. In fact, the new method (MSS) adds a substep to the conventional method. While, the conventional slitting method employs the polynomials to calculate compliance matrix, the present MSS method uses the principle of superposition. Therefore, in the present method, the difficulties of the singularity of the compliance matrix and scattering in the final result have been resolved.

It was shown that there is a linear relation between the eigen strain and the elastic strain in a certain range of the slot width. In this way, the induced errors in each step cannot affect the results of the next steps. The new method does not need any normalization technique. The results obtained by the present method were evaluated by a finite-element model. The results obtained by the MSS method were more realistic in comparison with those of the conventional method.

References

[1] Gascoigne, H. E., 1994, "Residual Surface Stresses in Laminated Cross-Ply Fiber-Epoxy Composite Materials," Exp. Mech., **34**(1), pp. 27–36.
[2] Ersoy, N., and Vardar, O., 2000, "Measurement of Residual Stresses in Layered Composites by Compliance Method," J. Compos. Mater., **34**(7), pp. 575–598.
[3] Nakamura, T., and Suresh, S., 1993, "Effects of Thermal Residual Stresses and Fiber Packing on Deformation of Metal-Matrix Composites," Acta Metall. Mater., **41**(6), pp. 1665–1681.
[4] Cheng, W., and Finnie, I., 1985, "A Method for Measurement of Axisymmetric Axial Residual Stresses in Circumferentially Welded Thin-Walled Cylinders," ASME J. Eng. Mater. Technol., **107**(3), p. 181.
[5] Cheng, W., and Finnie, I., 1986, "Measurement of Residual Hoop Stress in Cylinders Using the Compliance Method," ASME J. Eng. Mater. Technol., **108**(2), pp. 87–92.
[6] Vaidyanathan, S., and Finnie, I., 1971, "Determination of Residual Stresses From Stress Intensity Factor Measurements," ASME J. Basic Eng., **93**(2), p. 242.
[7] Schajer, G. S., and Prime, M. B., 2007, "Residual Stress Solution Extrapolation for the Slitting Method Using Equilibrium Constraints," ASME J. Eng. Mater. Technol., **129**(2), pp. 227–232.
[8] Ritchie, D., and Leggatt, R. H., 1987, "The Measurement of the Distribution of Residual Stresses Through the Thickness of a Welded Joint," Strain, **23**(2), pp. 61–70.
[9] Shokrieh, M. M., and Akbari, R. S., 2011, "Simulation of Slitting Method for Calculation of Compliance Functions of Laminated Composites," J. Compos. Mater., **46**(9), pp. 1101–1109.
[10] Cheng, W., and Finnie, I., 2007, "Compliance Functions for Through-Thickness Measurement: The LEFM Approach," *Residual Stress Measurement and the Slitting Method*, Springer, New York, pp. 33–52.
[11] Prime, M. B., 2004, "Measurement of Fiber-Scale Residual Stress Variation in a Metal-Matrix Composite," J. Compos. Mater., **38**(23), pp. 2079–2095.
[12] Schajer, G. S., 2010, "Relaxation Methods for Measuring Residual Stresses: Techniques and Opportunities," Exp. Mech., **50**(8), pp. 1117–1127.
[13] Schajer, G. S., and An, Y., 2010, "Residual Stress Determination Using Cross-Slitting and Dual-Axis ESPI," Exp. Mech., **50**(2), pp. 169–177.
[14] Schajer, G. S., and Prime, M. B., 2006, "Use of Inverse Solutions for Residual Stress Measurements," ASME J. Eng. Mater. Technol., **128**(3), p. 375.
[15] Mura, T., 1987, "General Theory of Eigenstrains," *Micromechanics of Defects in Solid*, 2nd ed., Martinus Nijhoff, Boston, MA, pp. 1–71.
[16] Korsunsky, A., 1995, "Fundamental Eigenstrain Solutions for Axisymmetric Crack Problems," J. Mech. Phys. Solids, **43**(8), pp. 1221–1241.
[17] Korsunsky, A. M., 2005, "The Modelling of Residual Stresses Due to Surface Peening Using Eigenstrain Distributions," J. Strain Anal. Eng. Des., **40**(8), pp. 817–824.
[18] Korsunsky, A. M., Regino, G. M., and Nowell, D., 2007, "Variational Eigenstrain Analysis of Residual Stresses in a Welded Plate," Int. J. Solids Struct., **44**(13), pp. 4574–4591.
[19] Luckhoo, H. T., Jun, T.-S., and Korsunsky, A. M., 2009, "Inverse Eigenstrain Analysis of Residual Stresses in Friction Stir Welds," Procedia Eng., **1**(1), pp. 213–216.
[20] Ma, H., Wang, Y., and Qin, Q.-H., 2011, "Determination of Welding Residual Stresses by Inverse Approach With Eigenstrain Formulations of Boundary Integral Equation," J. Phys.: Conf. Ser., **290**(1), p. 012012.
[21] Lokhov, V., Nyashin, Y., and Ziegler, F., 2009, "Statement and Solution of Optimal Problems for Independent Stress and Deformation Control by Eigenstrain," ZAMM, **89**(4), pp. 320–332.
[22] Hill, M. R., 2013, "The Slitting Method," *Practical Residual Stress Measurement Methods*, Wiley, Chichester, UK, pp. 89–108.