

A Multi-Objective Coordinated Operation Model for Supply Chain with Uncertain Demand Based on Fuzzy Interval

Jiawang Xu, Bo Jiang, Liang Tang and Ye Yuan

School of Economics and Management, Shenyang Aerospace University, Shenyang 110136, China

Abstract: The coordinated operation process of a class of supply chain with uncertain demands is considered. The supply chain consists of a manufacturer, a supplier and several customers. The semi-finished products of the supplier are raw materials of the manufacturer; demands of customers are uncertain and can be described as fuzzy intervals. A multi-objective programming model for coordinated operation of the supply chain is constructed and a numerical example is proposed. The results of the numerical example shows that decision makers can obtain an optimal operation strategy by using the model proposed in this study according to the level of uncertainties of demands and the operation strategy possesses robustness in some ways.

Keywords: Coordinated operation, fuzzy interval, multi-objective programming, supply chain

INTRODUCTION

Uncertainty is one of the inherent attribute of supply chain which affects supply chain performance. It is the main source that leads to supply chain risk also. In addition, it is one of the main reasons for the difficulty and complexity of supply chain management. So it is regarded as an important goal to reduce uncertainty in supply chain management (Wu, 2010). There are mainly three factors causing the uncertainty of supply chain. They are the supply process, manufacturing process and customer requirement process. Those uncertain factors will lead to the problems of production planning, scheduling, controlling and hence affect the supply chain performance (Zhang and Sun, 2005).

Whether the uncertainty in the supply chain can be reduced or eliminated effectively, to a great extent, relies on our description of the uncertainty in supply chain systems. There exist a lot of methods for quantitative description of supply chain uncertainty, generally involving interval analysis (Stefano *et al.*, 2011), statistical method (Wang and Ma, 2011), fuzzy sets method (Ruan and Fu, 2011), scenario analysis method (Sha *et al.*, 2011), etc.

Recently, Chen *et al.* (2010) set up a fuzzy programming model of supply chain with uncertain supply and demand. In this model fuzzy sets were used to indicate the uncertainty in the supply and demand of supply chain. Cardona-Valdés *et al.* (2011) developed a multi-objective stochastic optimization model under demand uncertainty, where the inherent risk is modeled by scenarios. Lin and Wang (2011) studied a supply

chain network design problem under supply and demand uncertainty with embedded supply chain disruption mitigation strategies, postponement with downward substitution, centralized stocking and supplier sourcing base. Bidhandi and Yusuff (2011) proposed an integrated model and a modified solution method for solving supply chain network design problems under uncertainty. The stochastic supply chain network design model was provided as a two-stage stochastic program where the two stages in the decision-making process correspond to the strategic and tactical decisions. The inputs to supply chain planning models are subject to environmental and system uncertainties. Kabak and Ülengin (2011) proposed a fuzzy set theory-based model to deal with those uncertainties and used a possibilistic linear programming model to make strategic resource-planning decisions using fuzzy demand forecasts and fuzzy yield rate as well as other inputs such as costs and capacities. Paksoy *et al.* (2012) applied fuzzy sets to integrate the supply chain network of an edible vegetable oils manufacturer. A fuzzy multi-objective linear programming model attempts to simultaneously minimize the total transportation costs was proposed. The approach incorporated all operating realities and actual flow patterns at production/distribution network with reference to demands of warehouses, capacities of tin and pet packaging lines. The model was formulated as a multi objective linear programming model where data were modeled by triangular fuzzy numbers.

Here, the coordinated operation process for a class of supply chain consisting of a single manufacturer, a single supplier and several customers is considered. In

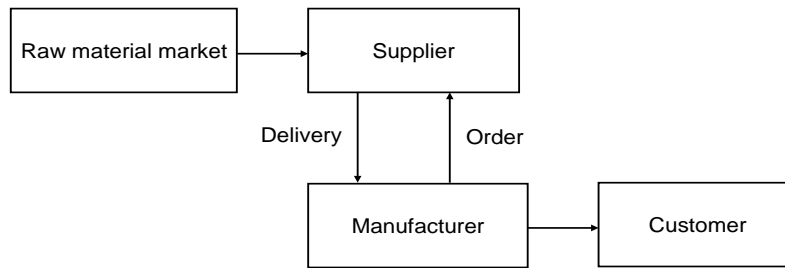


Fig. 1: The framework of the supply chain

this supply chain, the semi-finished products of the supplier produced are raw materials of the manufacturer. Product demands of customers are uncertain and can be denoted as fuzzy intervals. In the process of supply chain operation, the manufacturer decides how many semi-finished products make orders from the supplier at each phase. Similarly, the supplier also decides how many semi-finished products should deliver to the manufacture in order to maximum its profits. In each phase, the supplier's delivering quantities is equal to the manufacturer's ordering quantities. That is, the operation of the supply chain is coordinated, which is the top-priority objective for the supply chain operation. Moreover, the manufacturer and the supplier must decide his optimal operation strategy under respective market environments. The framework of the supply chain we considered is shown as Fig. 1. By describing uncertain demands as fuzzy intervals and using the approaches of fuzzy programming and multi-objective programming, we propose a multi-objective coordinated operation model of the supply chain.

NOTATIONS AND HYPOTHES

Notations:

Indexes:

- h = Index of raw material (i.e., the raw material of supplier), $h \in \{1, \dots, H\}$
- i = Index of semi-finished product (i.e., the product of supplier and the raw material of manufacturer) $i \in \{1, \dots, I\}$
- j = Index of ultimate product (i.e., the product of manufacturer), $j \in \{1, \dots, J\}$
- t = Index of phase, $t \in \{1, \dots, T\}$

Parameters:

- d_{jt} = Demand of ultimate product j at phase t
- p_{jt} = Price of ultimate product j at phase t
- q_{it} = Price of semi-finished product i at phase t

- r_{ht} = Price of raw material h at phase t
- c_j^z = Variable cost of ultimate product j
- c_i^x = Variable cost of semi-finished product i
- h_j^z = Inventory cost of ultimate product j
- h_i^y = Manufacturer's inventory cost for semi-finished product i
- h_i^x = Supplier's inventory cost for semi-finished product i
- K^{\max} = Manufacturer's maximum production Capacity
- G^{\max} = Supplier's maximum production capacity
- α_j^k = Capacity consuming rate for ultimate product j
- α_i^g = Capacity consuming rate for semi-finished product i
- o_j^z = Occupied inventory of unit ultimate product j for manufacturer
- o_j^y = Occupied inventory of unit semi-finished product j for manufacturer
- o_i^x = Occupied inventory of unit semi-finished product i for supplier
- $Z^{L^{\max}}$ = Manufacturer's total inventory level for ultimate products
- $Y^{L^{\max}}$ = Manufacturer's total inventory level for semi-finished products
- $x^{L^{\max}}$ = Supplier's total inventory level for semi-finished products
- $z_{j0}^{L'}$ = Manufacturer's initial inventory for ultimate product j
- $y_{j0}^{L'}$ = Manufacturer's initial inventory for semi-finished product j
- $x_{j0}^{L'}$ = The supplier's initial inventory for semi-finished product i
- S_{ij}^y = The BOM coefficient of ultimate product j to semi-finished product i
- S_{hi}^y = The BOM coefficient of semi-finished product i to raw material h
- S_{ht} = The quantity of raw material h the raw material market supplies at phase t

Decision variables:

- Z_{jt} = The quantity of ultimate product j manufacturer outputs at phase t
- v_{jt} = The quantity of ultimate product j manufacturer sales to customers at phase t
- Z_{jt}^L = The quantity of ultimate product j manufacturer inventories at phase t
- b_{it} = The quantity of semi-finished product i manufacturer orders at phase t
- y_{it}^L = The quantity of semi-finished product i manufacturer inventories at phase t
- x_{it} = The quantity of semi-finished product i supplier outputs at phase t
- l_{it} = The quantity of semi-finished product i supplier delivers to manufacturer at phase t
- x_{it}^L = The quantity of semi-finished product i supplier inventories at phase t

Hypotheses: In the processing of the supply chain operation, demands are all uncertain. Under normal circumstances, uncertainty can be described as a finite interval. For the convenience of analysis, we described uncertain demand d_{jt} as $[d_{jt}, d_{jt} + \Delta d_{jt}]$. Where, Δd_{jt} is a given nonnegative constant, it's the fluctuating quantity of demand d_{jt} and denotes the uncertain degree of demand. Let $\Delta d_{jt} = \rho\% \times d_{jt}$, we can adjust the uncertain degree of demand by changing the flexibility degree ρ .

Let u_{jt} and ω_{jt} be respectively the quantity of unsatisfied demand and unit punishment of not meeting demand for product i at phase t . We consider three operation objectives in the processing of supply chain operation.

- The process of the supply chain operation is coordinated. That is, the quantity of semi-finished product supplier delivers to manufacturer is nicely equals to the quantity manufacturer orders in each phase
- The operation objective of manufacturer is pursuing profit maximization
- The operation objective of supplier is also pursuing profit maximization

THE MULTI-OBJECTIVE COORDINATED OPERATION MODEL OF THE SUPPLY CHAIN

For manufacturer:

Objective function: The operation objective of manufacturer is pursuing profit maximization. When demands are uncertain, the objective function of the operation model for manufacturer can be expressed as follows:

$$\max C^P = \sum_t \left[\sum_j (p_j v_{jt} - c_j^z z_{jt} - h_j^z z_{jt}^L - \omega_{jt} u_{jt}) - \sum_i (q_i b_{it} + h_i^y y_{it}^L) \right] \quad (1)$$

Constraint conditions: Restrictions of production capacity:

$$\sum_{j=1}^J \alpha_j^k z_{jt} \leq K^{\max}, \quad \forall t \quad (2)$$

Restrictions for inventory of ultimate products:

$$z_{jt}^L = z_{j,t-1}^L + z_{jt} - v_{jt} - u_{jt}, \quad \forall j, t \quad (3)$$

$$z_{j0}^L = z_{j0}^L, \quad \forall j \quad (4)$$

$$\sum_{j=1}^J \sigma_j^z z_{jt}^L \leq z^{L\max}, \quad \forall t \quad (5)$$

Restrictions for inventory of semi-finished products:

$$y_{it}^L = y_{i,t-1}^L + b_{it} - \sum_{j=1}^J s_{ij}^y z_{jt}, \quad \forall i, t \quad (6)$$

$$y_{i0}^L = y_{i0}^L, \quad \forall i \quad (7)$$

$$\sum_{i=1}^I \alpha_i^y y_{it}^L \leq y^{L\max}, \quad \forall t \quad (8)$$

Demands of ultimate products must be satisfied:

$$v_{jt} + u_{jt} = d_{jt}, \quad \forall j, t \quad (9)$$

When demand d_{jt} is uncertain ($\rho \neq 0$), in order to set up the model for supply chain operation, we convert objective function (1) into membership function as follows:

$$\mu(C^P) = \begin{cases} 1, & C^P \leq C_{\min}^P \\ \frac{C_{\max}^P - C^P}{C_{\max}^P - C_{\min}^P}, & C_{\min}^P < C^P \leq C_{\max}^P \\ 0, & C^P > C_{\max}^P \end{cases} \quad (10)$$

where, C_{\min}^P and C_{\max}^P are the minimum and maximum expected profit of manufacturer respectively and $C_{\min}^P \leq C^P \leq C_{\max}^P$. In practice, we can provide the specific estimate value of C_{\min}^P and C_{\max}^P according to the optimal solutions of corresponding mathematical programming problem under certain demands.

By converting restricted condition (9) into membership function, we can get the following equations:

$$\mu(d_{jt}) = \begin{cases} 1, & v_{jt} + u_{jt} \leq d_{jt} \\ 1 - \frac{v_{jt} + u_{jt} - d_{jt}}{\Delta d_{jt}}, & d_{jt} < v_{jt} + u_{jt} \leq d_{jt} + \Delta d_{jt} \\ 0, & v_{jt} + u_{jt} > d_{jt} + \Delta d_{jt} \end{cases} \quad (11)$$

The model we proposed above contains membership functions. For convenience, we can transform the operation model of the manufacturer into a linear programming model as follows by using the principles of fuzzy mathematics.

Objective function:

$$\min \lambda \quad (12)$$

where, $0 \leq \lambda \leq 1$.

Constraint conditions:

$$\frac{C_{\max}^P - C^P}{C_{\max}^P - C_{\min}^P} \leq \lambda \quad (13)$$

$$1 - \frac{v_{jt} + u_{jt} - d_{jt}}{\Delta d_{jt}} \leq \lambda, \quad \forall j, t \quad (14)$$

Other constraints are (2-8).

Nonnegative conditions:

$$z_{jt}, z_{jt}^L, v_{jt}, u_{jt}, b_{it}, y_{it}^L \geq 0, \quad \forall i, j, t \quad (15)$$

For supplier:

Objective function: The operation objective of supplier is also pursuing profit maximization. It can be expressed as follows.

$$\max C^S = \sum_t \sum_i \left[q_{it} l_{it} - h_i^x x_{it}^L - \left(c_i^x + \sum_h r_{ht} s_{hi}^r \right) x_{it} \right] \quad (16)$$

Constraint conditions: The quantity of semi-finished product delivered to manufacturer cannot exceed to the quantity manufacturer ordered in each phase:

$$l_{it} \leq b_{it}, \quad \forall i, t \quad (17)$$

Restrictions of capacity:

$$\sum_{i=1}^I \alpha_i^g x_{it} \leq G^{\max}, \quad \forall t \quad (18)$$

Restrictions for inventory of semi-finished product:

$$x_{it}^L = x_{i,t-1}^L + x_{it} - l_{it}, \quad \forall i, t \quad (19)$$

$$x_{i0}^L = x_{i0}^L, \quad \forall i \quad (20)$$

$$\sum_{i=1}^I \alpha_i^x x_{it}^L \leq x^{L\max}, \quad \forall t \quad (21)$$

Consumption of raw material cannot exceed the supply of the raw material market:

$$\sum_{i=1}^I s_{hi}^r x_{it} \leq s_{ht}, \quad \forall h, t \quad (22)$$

Nonnegative conditions:

$$x_{it}, x_{it}^L, l_{it} \geq 0, \quad \forall i, t \quad (23)$$

Multi-objective coordinated operation model: We consider three operation objectives in the operation process of supply chain.

Objective 1: The process of the supply chain operation is coordinated. That is, the quantity of semi-finished product supplier delivered is nicely equals to the quantity manufacturer ordered in each phase.

Objective 2: The operation objective of manufacturer is pursuing profit maximization.

Objective 3: The operation objective of supplier is also pursuing profit maximization.

By using Multi-objective Programming method, the multi-objective coordinated operation model of the supply chain can be described as following.

Objective function:

$$\min P_1 \times \sum_t \sum_i (d_{it}^- + d_{it}^+) + P_2 \times (d_p^+ + d_s^-) \quad (24)$$

Constraint conditions:

$$-b_{it} + l_{it} + d_{it}^- - d_{it}^+ = 0, \quad \forall i, t \quad (25)$$

$$\lambda + d_p^- - d_p^+ = 0 \quad (26)$$

$$C^S + d_s^- - d_s^+ = M^S \quad (27)$$

$$C^S - \sum_t \sum_i \left[q_{it} l_{it} - h_i^x x_{it}^L - \left(c_i^x + \sum_h r_{ht} s_{hi}^r \right) x_{it} \right] \leq 0 \quad (28)$$

In Eq. (26), $0 \leq \lambda \leq 1$, so $0 \leq d_p^- \leq 1$ and $0 \leq d_p^+ \leq 1$. However, we can see from Eq. (27), d_s^- and d_s^+ may be much larger than 1. In order to ensure objective 2

Table 1: Supplies, demands and standard prices of market

Demands and supplies		Phase 1	Phase 2
Demand in certain case	Product 1	190	195
	Product 2	197	194
Prices of ultimate product	Product 1	180	185
	Product 2	185	180
Supply quantities of raw materials	Raw material 1	336	330
	Raw material 2	335	325
Prices of raw material	Raw material 1	30	35
	Raw material 2	35	30

and objective 3 have same priority level, we rewrite Eq. (26) as follows:

$$M^S * \lambda + d_p^- - d_p^+ = 0 \quad (29)$$

Other constraint conditions of the supply chain operation are listed as follows:

- Conditions (2-8)
- Conditions (3-14)
- Conditions (18-22)
- $0 \leq \lambda \leq 1$

Nonnegative conditions:

$$d_p^-, d_p^+ \geq 0, d_s^-, d_s^+ \geq 0, d_{it}^-, d_{it}^+ \geq 0 \quad \forall i, t$$

- Nonnegative condition (15)
- Nonnegative condition (23)

NUMERICAL EXAMPLES

Here, we consider a simple supply chain contains two ultimate products ($J = 2$), one semi-finished product ($I = 1$), two raw materials ($H = 2$) and two phases ($T = 2$). Other parameters are set as following:

$$K^{\max} = 400, G^{\max} = 600$$

$$z^{L^{\max}} = 200, y^{L^{\max}} = 200, x^{L^{\max}} = 200$$

$$z_{j0}^{L'} = y_{i0}^{L'} = x_{i0}^{L'} = 0, c_i^x = 10, c_j^z = 15 \quad \forall i, j$$

$$h_i^x = 1, h_i^y = 2, h_j^z = 3, \alpha_i^s = 1, \alpha_j^k = 1 \quad \forall i, j$$

$$s_{11}^r = 0.4, s_{21}^r = 0.6, s_{11}^y = 1, s_{12}^y = 1$$

$$o_i^x = o_i^y = o_j^z = 1, q_{it} = 90, \omega_{jt} = 200 \quad \forall i, j, t$$

Furthermore, the priority factor P_1 and P_2 are 10^5 and 100 respectively and the expected profit of the supplier is 1×10^7 . Quantities of raw materials, demands of products in certain case and the standard market prices, are listed in Table 1.

According to the data listed above, we resolved the optimal solutions by using the optimization software Lingo11.0 when demands are certain and uncertain.

- Demands of ultimate products are all certain. When demands are certain, all the optimal values of decision variables are listed in Table 2. The profits of manufacturer and supplier are 60160 and 36861 respectively and the total profit of supply chain is 97021. All demands of consumer market are satisfied commendably. For semi-finished products, the delivery quantities of supplier are all equal to the order quantities of manufacturer in each phase. Therefore, the model we proposed realizes the operating cooperativeness between the members of supply chain when demands are certain.
- Demands of ultimate products are uncertain. When the flexibility degree of demand is equal to 20 ($\rho = 20$), that is, the fluctuation interval of demand is $[d_{jt}, 1.2 \times d_{jt}]$, the operation strategies for manufacturer and supplier are shown in Table 3. Profits of supply chain, manufacturer and supplier are 95380, 52320 and 43060 respectively.

Table 2: The optimal operation strategy of supply chain with certain demands

Manufacture (profit: 52320)								Supplier (profit: 36864)			
Phase	Production (z_{jt})		Seals (v_{jt})		Inventory ($z^{L_{jt}}$)		Semi-finished product		Delivery (l_{jt})	Production (x_{it})	Inventory ($x^{L_{it}}$)
	Prod.1	Prod.2	Prod.1	Prod.2	Prod.1	Prod.2	Inventory	Order (b_{jt})			
1	190.0	197.0	190.0	197.0	0	0	0	387.0	387.0	387.0	0
2	195.0	194.0	195.0	194.0	0	0	0	389.0	389.0	389.0	0

Table 3: The optimal operation strategy of supply chain with uncertain demands ($\rho = 20$)

Manufacture (profit: 52320)								Supplier (profit: 36864)			
Phase	Production (z_{jt})		Seals (v_{jt})		Inventory ($z^{L_{jt}}$)		Semi-finished product		Delivery (l_{jt})	Production (x_{it})	Inventory ($x^{L_{it}}$)
	Prod.1	Prod.2	Prod.1	Prod.2	Prod.1	Prod.2	Inventory	Order (b_{jt})			
1	195.9	204.1	195.9	203.1	0	1.0	400.0	0	400.0	400.0	0
2	201.0	199.0	201.0	200.0	0	0	505.4	105.4	505.4	505.4	0

Table 4: Profits of manufacturer and supplier for different ρ

Flexibility degree of demand (ρ)	0	10	20	30	40	50
The manufacturer's profit	60160	54639	52320	51546	51160	50928
Difference to the certain case	-----	9.0%	13.0%	14.3%	15.0%	15.3%
The suppliers profit	36861	41849	43060	43463	43665	43786
Difference to the certain case	-----	13.5%	16.8%	17.9%	18.5%	18.8%
The total profit of the supply chain	97021	96488	95380	95009	94825	94714
Difference to the certain case	-----	0.5%	1.7%	2.1%	2.3%	2.4%

Compared with the profits in certain case, the total profit of the supply chain decreased 1.7%, the profit of manufacturer decreased 13.0% and however, the supplier's profit increased 16.8%. This result means that the supply chain and the manufacturer may take on greater risk of excessive demands. However, in relative to the manufacturer, the supplier doesn't directly respond to the risk of excessive demands. As the order quantities of semi-finished products for manufacturer increasing when demands of ultimate products are excessive, the quantities of semi-finished products produced and delivered by the supplier increase also. Therefore, on the contrary, the profit of the supplier increases.

Furthermore, the data in Table 3 show that the quantity of semi-finished product delivered by the supplier is equal to the quantity ordered by the manufacturer at each phase. Therefore, just as the results of the case of certain demands, the model we proposed realizes the operation cooperativeness between the members of supply chain when demands are uncertain.

- In general, the range of demand fluctuation can be estimated according to historical data in practical application. Here, we use the model we proposed to calculate the profits and operational strategies of manufacturer and supplier together with the profit of supply chain system in the context of the flexibility degree of demand uncertainty takes different values of 10, 20, 30, 40 and 50 separately. Profits of manufacturer, supplier and supply chain as a whole for different flexibility degrees of demand are listed in Table 4. The differences to the certain case are also listed in this table.

From Table 4, we can find that the profits of supply chain and manufacturer under uncertain demand are less than which under certain demand and the decrements are increasing as the degree of demand uncertainty. Both the supply chain system and manufacturer will pay much for the risk of excess demand no matter what the range of demand fluctuation is. Furthermore, when the flexibility degrees of demand is less than 50, the decrement of the supply chain system's profit is less than 2.4% and that of manufacturer is less than 15.3%. Consequently, the

multi-objective programming model of supply chain coordinated operation with uncertain demands developed in this study is robust in some sense. The decision-maker can use the optimal strategies derive from this model to make optimal decision according to the degree of demand uncertainty.

CONCLUSION

In this study, we considered the coordinated operation process of a class of supply chain with uncertain demands. By using fuzzy programming and multi-objective programming methods, we described the uncertainties of demands as fuzzy intervals and establish a multi-objective coordinated operation model for the supply chain. Results of numerical example testified that our methods offered in this study can effectively deal with the impacts of demand uncertainty on supply chain performance, which can be referenced in the researches on operating uncertainties of supply chain.

ACKNOWLEDGMENT

This study was supported in part by MOE (Ministry of Education in China) Project of Humanities and Social Sciences under Grant No. 11YJA630165, National Nature Science Foundation of China under Grant No. 71201106, Special Major Science and Technology Projects of Zhejiang Province under Grant No. 2011C03004.

REFERENCES

Bidhandi, H.M. and R.M. Yusuff, 2011. Integrated supply chain planning under uncertainty using an improved stochastic approach. *Appl. Math. Modell.*, 35: 2618-2630.

Cardona-Valdés, Y., A. Álvarez and D. Ozdemir, 2011. A bi-objective supply chain design problem with uncertainty. *Transport. Res. Part C*, 19: 821-832.

Chen, H.P., J.B. Du and D. Mu, 2010. Analysis of fuzzy programming model of supply chain with uncertain supply and demand. *Logist. Technol.*, 2: 155-158.

- Kabak, Ö. and F. Ülengin, 2011. Possibilistic linear-programming approach for supply chain networking decisions. *Eur. J. Oper. Res.*, 209: 253-264.
- Lin, C.C. and T.H. Wang, 2011. Build-to-order supply chain network design under supply and demand uncertainties. *Transport. Res. Part B*, 45: 1162-1176.
- Paksoy, T., N.Y. Pehlivan and E. Özceylan, 2012. Application of fuzzy optimization to a supply chain network design: A case study of an edible vegetable oils manufacturer. *Appl. Math. Modell.*, 36: 2762-2776.
- Ruan, J.G. and X.H. Fu, 2011. Research on coordination decision model with fuzzy demand and fuzzy lead-time for perishable products supply chain. *Sci-Technol. Manag.*, 13(4): 73-77.
- Sha, N., J.H. Ji and X.G. Chen, 2011. An explorative study on the scenario analysis of supply chain uncertainty. *J. Intell.*, 30(2): 194-198.
- Stefano, M., S. Zeynep Alparslan Gok, R. Branzei and S. Tijs, 2011. Connection situations under uncertainty and cost monotonic solutions. *Comp. Oper. Res.*, 38: 1638-1645.
- Wang, X.L. and S.H. Ma, 2011. Research on capacity coordination in a logistics service supply chain with demand and supply uncertainties. *Oper. Res. Manag. Sci.*, 20(2): 44-49.
- Wu, X., 2010. Uncertainty of supply chain and its conceptual model of generation mechanism. *Sci. Technol. Manag. Res.*, 19: 80-83.
- Zhang, T. and L.Y. Sun, 2005. *Supply Chain Uncertainty Management: Technology and Tactics*. Tsinghua University Press, Beijing, China.