Impedance Matching and Power Division Algorithm Considering Cross Coupling for Wireless Power Transfer via Magnetic Resonance

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***Abstract*—Wireless power transfer via magnetic resonant coupling has been extensively researched for various applications. Impedance matching is implemented so that the transfer system is resilient to positional changed and load changed. Recently powering multiple loads simultaneously is becoming the focus in wireless power transfer researches and designs. Apart from maintaining high efficiency, controllable power division is also an important feature in multi-receiver system. This is due to ratio of power division depends not only on the load but also on the gap of each receiver to the transmitter. On the other hand, cross coupling exists when receivers are placed relatively near to each other. In this paper, novel algorithm is proposed for impedance matching and power division considering cross coupling in between two receivers. The wireless power transfer system is first represented by equivalent circuit and the algorithm for impedance matching and power division is derived. The new method is then verified using simulations.**

I. INTRODUCTION

Since the introduction of wireless power transfer via magnetic resonant coupling in year 2007 [1], the technology has been researched for various potential applications. Paper [2] and [3] proposed charging electric vehicles, [4] on charging electronics portable devices, [5] and [6] on powering implantable medical devices and [7] discussed on position sensing using this technology. Other promising applications include home, manufacturing line and robotic industry to increase mobility and convenience in these places.

The optimal transfer efficiency occurs only when the resonators are critically coupled [8]. Impedance matching [9], [10] and other methods [11], [12] are implemented to maintain the efficiency when the system is moving away from this optimal operating point. Recently sending power to multiple receivers from a single transmitter simultaneously has also been researched extensively [4], [10], [13]-[15]. However these papers only focus on efficiency analysis but not on power distribution.

Assuming identical loads, the receiver nearer to the transmitter tend to absorb most of the power [16]. On the other hand, cross coupling between receivers changes the impedance viewed by the high frequency power supply and thus may reduce the efficiency [17]. If cross coupling is not put into consideration, the receivers cannot be placed near each other and consequently reduce the flexibility of the system.

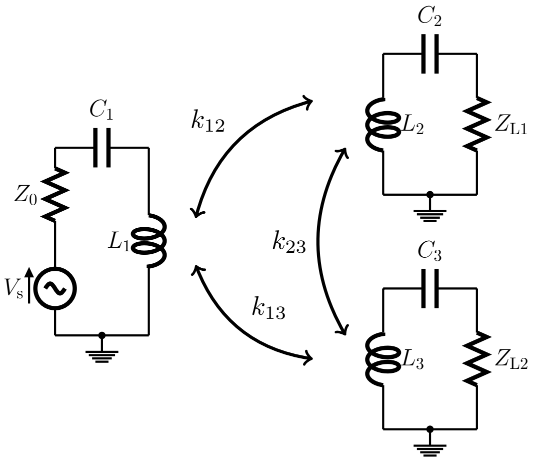

Fig. 1. Equivalent Circuit of a Two-Receiver Wireless Power Transfer.

In this paper, impedance matching and controllable power division method considering cross coupling is proposed. Using this method, the receivers can be placed anywhere in the strongly coupled region and the optimal efficiency can be maintained. Section II of this paper presents the derivation and the method. The method is then validated mathematically using two example cases in Section III. The same example cases are simulated and the result is presented in Section IV.

II. DERIVATION AND METHOD

The equivalent circuit of Fig. 1 represents a wireless power transfer with two receivers and cross coupling exists in between the receivers. In the circuit diagram, Z_0 is the power supply's termination resistor. Term k_{12}, k_{13}, and k_{23} are the coupling coefficients. L_1 and C_1 pair represents the transmitter antenna. L_2 and C_2, L_3 and C_3 pairs are the receiver antennas. Finally, Z_{L1} and Z_{L2} are the loads connected to respective receiver antennas.

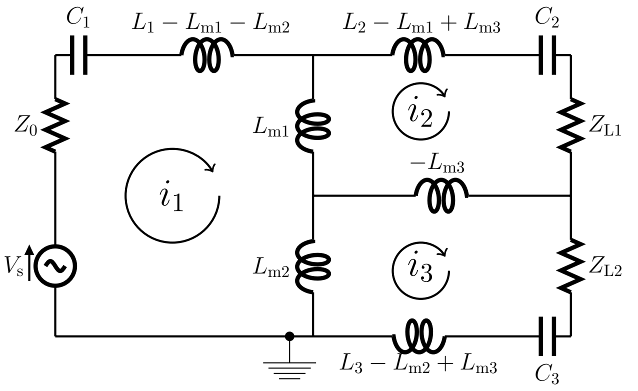

Fig. 2. Equivalent Circuit with Couplings Represented by Mutual Inductance.

If the coupling coefficients in Fig. 1 are expressed in terms of mutual inductance as in (1), the equivalent circuit can be redrawn as in Fig. 2.

$$k_{12} = \frac{L_{\mathrm{m1}}}{\sqrt{L_1 L_2}} \qquad k_{13} = \frac{L_{\mathrm{m2}}}{\sqrt{L_1 L_3}} \qquad k_{23} = \frac{L_{\mathrm{m3}}}{\sqrt{L_2 L_3}}. \quad (1)$$

In order to simplify the analysis for power division, the system is assumed to be in near perfect resonance and therefore the impedance of the antennas can be ignored. Furthermore, the internal resistance of the antennas can also be ignored compared to the load impedances. The current loop equations for Fig. 2 are given as:

$$\left\{ \begin{array}{rcl} V_s & = & i_1 Z_0 - i_2 j\omega_0 L_{\mathrm{m1}} - i_3 j\omega_0 L_{\mathrm{m2}} \\ 0 & = & -i_1 j\omega_0 L_{\mathrm{m1}} + i_2 Z_{\mathrm{L1}} + i_3 j\omega_0 L_{\mathrm{m3}} \\ 0 & = & -i_1 j\omega_0 L_{\mathrm{m2}} + i_2 j\omega_0 L_{\mathrm{m3}} + i_3 Z_{\mathrm{L2}} \end{array} \right\} \quad (2)$$

where ω_0: resonant angular frequency.

External coupling coefficient is the ratio of the resonator's termination resistance to the resonator's "reactance slope parameter" [18], [19]:

$$k_{\mathrm{t}} = \frac{Z_0}{\omega_0 L_1} \qquad k_{\mathrm{r1}} = \frac{Z_{\mathrm{L1}}}{\omega_0 L_2} \qquad k_{\mathrm{r2}} = \frac{Z_{\mathrm{L2}}}{\omega_0 L_3}. \quad (3)$$

Where

k_{t} : external coupling coefficient of the transmitter;
k_{r1}: external coupling coefficient of the first receiver;
k_{r2}: external coupling coefficient of the second receiver.

Solving current i_2 and i_3 in terms of i_1 and replace the load impedances and mutual inductances with (1) and (3):

$$\begin{aligned} \frac{V_{\mathrm{s}}}{i_1} = {} & k_t \omega_0 L_1 + \frac{{k_{12}}^2 k_{\mathrm{r2}} - j k_{12} k_{13} k_{23}}{k_{\mathrm{r1}} k_{\mathrm{r2}} + {k_{23}}^2} \omega_0 L_1 \\ & + \frac{{k_{13}}^2 k_{\mathrm{r1}} - j k_{12} k_{13} k_{23}}{k_{\mathrm{r1}} k_{\mathrm{r2}} + {k_{23}}^2} \omega_0 L_1. \end{aligned} \quad (4)$$

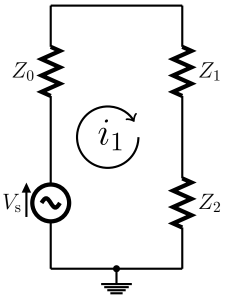

Fig. 3. Simplified Equivalent Circuit.

Equation 4 indicates that the two-receiver wireless power transfer equivalent circuit can be simplified into Fig. 3 with Z_0 the power supply's termination impedance and:

$$\begin{aligned} Z_1 &= \frac{{k_{12}}^2 k_{\mathrm{r2}} - j k_{12} k_{13} k_{23}}{k_{\mathrm{r1}} k_{\mathrm{r2}} + {k_{23}}^2} \omega_0 L_1 \\ Z_2 &= \frac{{k_{13}}^2 k_{\mathrm{r1}} - j k_{12} k_{13} k_{23}}{k_{\mathrm{r1}} k_{\mathrm{r2}} + {k_{23}}^2} \omega_0 L_1. \end{aligned} \quad (5)$$

Using average maximum power transfer theorem [20], impedance matching is achieved when:

$$\begin{aligned} Z_0 &= \overline{Z_1 + Z_2} \\ k_t &= \overline{\frac{{k_{12}}^2 k_{\mathrm{r2}} + {k_{13}}^2 k_{\mathrm{r1}} - j 2 k_{12} k_{13} k_{23}}{k_{\mathrm{r1}} k_{\mathrm{r2}} + {k_{23}}^2}}. \end{aligned} \quad (6)$$

Since the same current is flowing through impedance Z_1 and impedance Z_2, ratio of power division is:

$$\begin{array}{rcl} & Z_1 : Z_2 & \\ = ({k_{12}}^2 k_{\mathrm{r2}} - j k_{12} k_{13} k_{23}) & : & ({k_{13}}^2 k_{\mathrm{r1}} - j k_{12} k_{13} k_{23}) \end{array} \quad (7)$$

For controllable power division and impedance matching without having to change the antennas' positions, the external coupling coefficients should be modifiable. The required external coupling coefficients are calculable by solving (6) and (7). Impedance matching circuits can be inserted between antennas and the corresponding antennas' terminations to realize the required external coupling coefficients.

III. PROOF OF THE NEW METHOD

In order to validate the power division method, the real power across impedance Z_1 should be equal to the real power across load Z_{L1} and the real power across impedance Z_2 should be equal to the real power across load Z_{L2}.

The equations for power across the mentioned impedances are:

$$\begin{aligned} \text{Power across } Z_1,\ P_{\mathrm{Z1}} &= |i_1|^2 \times Z_1 \\ \text{Power across } Z_2,\ P_{\mathrm{Z2}} &= |i_1|^2 \times Z_2 \\ \text{Power across } Z_{\mathrm{L1}},\ P_{\mathrm{ZL1}} &= |i_2|^2 \times Z_{\mathrm{L1}} \\ \text{Power across } Z_{\mathrm{L2}},\ P_{\mathrm{ZL2}} &= |i_3|^2 \times Z_{\mathrm{L2}}. \end{aligned} \quad (8)$$

Solving current i_2 in terms of i_i and current i_3 in terms of i_1:

$$
\begin{aligned}
i_2 &= \frac{k_{13}k_{23} + jk_{13}k_{\mathrm{r1}}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2} \times \frac{\sqrt{L_1}}{\sqrt{L_2}} \times i_1 \\
i_3 &= \frac{k_{12}k_{23} + jk_{12}k_{\mathrm{r2}}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2} \times \frac{\sqrt{L_1}}{\sqrt{L_3}} \times i_1 \qquad (9)
\end{aligned}
$$

Substituting (3), (5) and (9) into (8) and by direct comparison, the power division method is proven when:

$$
\begin{aligned}
&\Re\left[{P_{\mathrm{ZL1}}}'\right] = \Re\left[{P_{\mathrm{Z1}}}'\right] \\
&\left|\frac{k_{13}k_{23} + jk_{13}k_{\mathrm{r1}}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2}\right|^2 \times \Re\left[k_{\mathrm{r1}}\right] \\
&= \Re\left[\frac{{k_{12}}^2 k_{\mathrm{r2}} - jk_{12}k_{13}k_{23}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2}\right], \qquad (10)
\end{aligned}
$$

and:

$$
\begin{aligned}
&\Re\left[{P_{\mathrm{ZL2}}}'\right] = \Re\left[{P_{\mathrm{Z2}}}'\right] \\
&\left|\frac{k_{12}k_{23} + jk_{12}k_{\mathrm{r2}}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2}\right|^2 \times \Re\left[k_{\mathrm{r2}}\right] \\
&= \Re\left[\frac{{k_{13}}^2 k_{\mathrm{r1}} - jk_{12}k_{13}k_{23}}{k_{\mathrm{r1}}k_{\mathrm{r2}} + {k_{23}}^2}\right]. \qquad (11)
\end{aligned}
$$

Where ${P_{\mathrm{Z1}}}'$, ${P_{\mathrm{Z2}}}'$, ${P_{\mathrm{ZL1}}}'$ and ${P_{\mathrm{ZL2}}}'$ are power terms from (8) normalized against $|i_1|^2\,\omega_0 L_1$. Two calculation examples are given in this section to validate (10) and (11).

A. Calculation Case I: Equal Power Division

The element values chosen are listed as below:

$$
\begin{aligned}
\omega_0 &= 2\pi \times 13.56\,\mathrm{MHz} \\
L_1 &= L_2 = L_3 = 10\,\mu\mathrm{H} \\
C_1 &= C_2 = C_3 = 13.8\,\mathrm{pF} \\
k_{12} &= 0.05 \\
k_{13} &= 0.1 \\
k_{23} &= 0.08 \\
Z_0 &= 50.
\end{aligned}
$$

The transmitter's external coupling coefficient,

$$
k_{\mathrm{t}} = \frac{Zo}{\omega_o L_1} = 0.0587,
$$

is chosen to be fixed. Solving for receivers' external coupling coefficients using (6) and (7):

$$
\begin{aligned}
k_{r1} &= \frac{50}{587} - \frac{j}{25} \\
k_{r2} &= \frac{200}{587} - \frac{j4}{25}.
\end{aligned}
$$

Inserting all values into (8) and the normalized power against $|i_1|^2\,\omega_0 L_1$ are:

$$
\begin{aligned}
{P_{\mathrm{Z1}}}' &= 0.0294 \\
{P_{\mathrm{ZL1}}}' &= 0.0294 - j0.0138 \\
{P_{\mathrm{Z2}}}' &= 0.0294 \\
{P_{\mathrm{ZL2}}}' &= 0.0294 - j0.0138.
\end{aligned}
$$

The results satisfy (10) and (11) indicating that the real power distributed to impedance Z_1 in (5) will absorbed by load Z_{L1}. Similarly, real power distributed to impedance Z_2 will absorbed by load Z_{L2}.

B. Calculation Case II: Ratio 7:3 Power Division

The element values are the same as calculation case I except that ratio in (7) is set to be 7:3. Similar to case I, solving for receivers' external coupling coefficients:

$$
\begin{aligned}
k_{\mathrm{r1}} &= \frac{250}{4109} - \frac{j3}{175} \\
k_{\mathrm{r2}} &= \frac{1000}{1761} - \frac{j28}{75}.
\end{aligned}
$$

Inserting all values into (8) and the normalized power against $|i_1|^2\,\omega_0 L_1$ are:

$$
\begin{aligned}
{P_{\mathrm{Z1}}}' &= 0.0411 \\
{P_{\mathrm{ZL1}}}' &= 0.0411 - j0.0116 \\
{P_{\mathrm{Z2}}}' &= 0.0176 \\
{P_{\mathrm{ZL2}}}' &= 0.0176 - j0.0116.
\end{aligned}
$$

Again the results satisfy (10) and (11). Furthermore, the real power absorbed by Z_{L1} is 7/3 times the real power absorbed by Z_{L2} in this case.

From both cases, the calculated powers across loads contain imaginary values. These reactive powers are balanced by other reactances present in the circuit. For example the reactive powers across the loads will be cancelled by the reactive powers across the mutual inductance, L_{m3} between the receiver loops and $L_{\mathrm{m3}}s$ that appear in both receiver loops. This shows that both receivers are also exchanging energy.

IV. SIMULATION RESULT

The example cases given in Section III are simulated to validate the new power division method. The circuit in Fig. 1 with parameters listed in both example cases is inputted in LTspice and the simulation results are plotted.

From the calculated receivers' external coefficients, the required load impedances for case I equal power distribution are calculated from (3):

$$
\begin{aligned}
Z_{\mathrm{L1}} &= (73 - j34)\,\Omega \\
Z_{\mathrm{L2}} &= (290 - j136)\,\Omega.
\end{aligned}
$$

Fig. 4(a) shows the simulation result before applying the method. Due to impedance mismatched, the reflection ratio, η_{11} is around 28.1% at the resonant frequency 13.56 MHz.

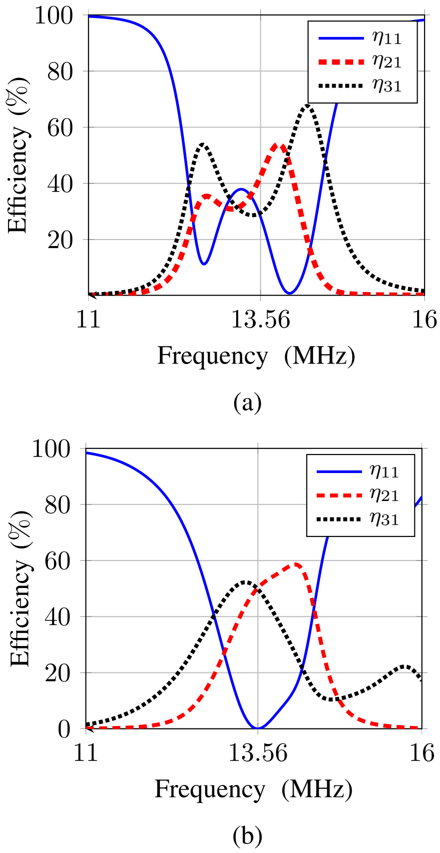

Fig. 4. Simulation result of equal power division: a) before impedance matching. b) after impedance matching.

The transmission efficiency, η_{21} to the first receiver is around 42.4%. The transmission efficiency, η_{31} to the third receiver is around 29.5%. The simulation result after inserting impedance matching circuits to realize the calculated load impedance is shown in 4(b). Reflection ratio is suppressed to almost none, and both receivers obtain almost equalized power as desired even though cross coupling exists between the two receivers.

The required load impedances for case II is:

$$\begin{aligned} Z_{L1} &= (52 - j15)\,\Omega \\ Z_{L2} &= (484 - j318)\,\Omega. \end{aligned}$$

Fig. 5 shows the simulation result after applying the method. Similar to equal power distribution case, reflection ratio, η_{11} is suppressed to almost none. Transmission efficiency, η_{21} to the first receiver is around 70% and transmission efficiency, η_{31} to the third receiver is around 30% as desired.

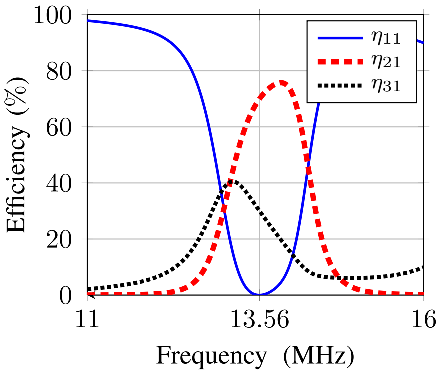

Fig. 5. Simulation result of 70%-30% power division after impedance matching.

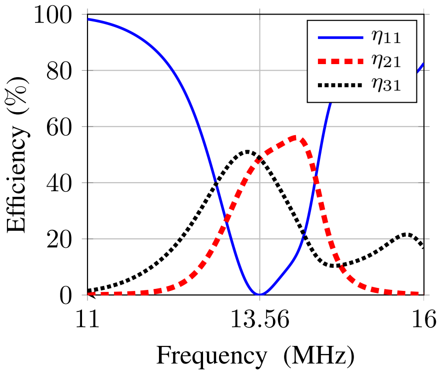

Fig. 6. Simulation result of equal power distribution considering internal resistance after impedance matching.

Above two simulation cases are performed without considering internal resistances of the antennas to validate the proposed equations. The equal power distribution case with internal resistances of all the antennas are set to $1\ \Omega$ is simulated which correspond to quality factor of 852. Fig. 6 shows that the frequency response with $1\,\Omega$ internal resistance is close to the response of Fig. 4(b) except with lower transfer efficiency. Reflection ratio, η_{11} is still close to none, however both transmission efficiencies are lower in this case with η_{21} equal to 48.5% and η_{31} equal to 48.7%. Therefore it is reasonable to omit the values in the method's equations as magnetic resonant coupling antennas should have low internal resistances [1].

V. CONCLUSION

For various applications, a wireless power transfer must be high efficiency, able to support multiple loads and selectivity. Additionally, flexibility of receiver placement is also desirable. Therefore new impedance matching and power division algorithm considering cross coupling is proposed for a two-receiver system. With cross coupling considered in the method, the receivers can be placed close to each other without affecting the efficiency. The physical system is first represented by equivalent circuit, impedance matching and power division equations are then derived from the circuit model. Finally, the algorithm is verified mathematically and by simulation.

Future work will includes performing experiments for the simulated cases, deriving generalized equations for any number of receivers and comparing different impedance matching network types for wireless power transfer.

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