Model of brittle destruction based on hypothesis of scale similarity

A.S. Arakcheev, K.V. Lotov

Budker Institute of Nuclear Physics, Novosibirsk, Russia

Novosibirsk State University, Novosibirsk, Russia

Introduction

The dust appears in most of nuclear fusion devices due to the plasma-wall interaction. The dust particles pose potential problems to plasma confinement: decrease the plasma temperature, absorb tritium, etc. The size distribution is an important characteristic of the dust. To our notion, there is no analytical model explaining experimentally observed dust size distributions.

The power (Junge) distribution was observed in several experiments [1–3] for tungsten and carbon dust in the size range from several nanometers to tens of microns. The exponent $(-\alpha)$ of the power distribution was measured to fall between -3.3 and -2.1.

The power size distribution for the atmospheric dust was explained by the model of particle coagulation [4]. In plasma devices, the power size distribution was observed not only for agglomerations, but also for single dust particles in the size range from several nanometers to several microns [1]. Hence it follows that the coagulation model cannot explain the dust size distribution in the whole range of sizes.

Four basic mechanisms of dust formation are usually distinguished in plasma devices: flaking of redeposited layers, brittle destruction, condensation, and growth from hydrocarbon molecules [5]. If the dust grows from the gas, the size distribution differs from the power law [6]. The typical size of the dust formed by flaking is greater than one micron [7]. At smaller sizes, flaking can be thought of as brittle destruction. The brittle destruction is thus the only mechanism that can be responsible for formation of the observed dust.

The power law for single dust particles was observed in the wide range between two typical sizes: the grain size of materials and interatomic distance. This fact suggests that the law of material fragmentation is independent of the scale.

Mathematical model

The dust particles produced due to the brittle destruction are fragments of a solid body. The body is to be divided into pieces. Let us mentally remove the pieces from the body one by one in the order of decreasing size and number them sequentially by the index n. The size distribution function does not depend on the shape of the initial body only for fragments of the size much smaller than the typical size of the initial body. Hence we assume $n \gg 1$.

Denote the volume of the body remained after removal of the n-th piece by V_n, and its surface area by S_n. We can write recurrent expressions relating V_n, S_n, and the characteristic size r_n of the n-th fragment:

$$V_n = V_{n-1} - c_1 r_n^3, \qquad S_n = S_{n-1} + c_2 r_n^2, \tag{1}$$

where c_1 and c_2 are coefficients that depend on the shape of removed fragments. In what follows we consider these coefficients to be independent of the fragment size and its number due to the scale similarity assumption. The scale similarity gives the expression closing the recurrent scheme:

$$r_{n+1} = c_3 V_n / S_n, \tag{2}$$

where c_3 is a positive coefficient dependent on the shape of removed fragments.

The recurrent scheme yields the size distribution function:

$$f(r) = \left|\frac{dn}{dr}\right| \propto r^{-\alpha}, \qquad \alpha = 3 + \frac{1}{1 + c_1 c_3 / c_2}. \tag{3}$$

From the condition $3c_2 + 2c_1 c_3 > 0$ necessarily used in calculations, we find that α falls into the interval from 1 to 4. All known experimental results are in this interval.

In the case of a N-dimensional body fragmentation, similar calculations give the allowable range for the exponent between $-N-1$ and -1.

The above model of body decomposition resembles an algorithm of fractal construction and can be treated in terms of the fractal theory. The model assumes division of the body into an infinite number of domains filling the body volume completely. The same situation is realized at tiling packings. There is a relation between the exponent of fragments size distribution and the fractal dimension of packing D :

$$f(r) \propto r^{-1-D}, \qquad \alpha = 1 + D. \tag{4}$$

The connection is obviously demonstrable in the case of self-similar fractals and proved for some non-self-similar sets, particularly for osculatory packings [8].

The fractal dimension of the residual set in three-dimensional space evidently falls between 0 and 3. The interval for possible values of α thus naturally follows from the dimension of space. However, the result of the paper is more general, since fractional-dimensional nature is not assumed there and the exponent is related to the shape of fragments.

In Ref. [2], a fractal structure of separate dust particles falling on a substrate was detected. The fractal dimension of particles was measured to be 2.2 ± 0.2, which is close to the coefficient $\alpha \approx 2.3 \pm 0.1$ for the same dust. This coincidence conflicts with relation (4) that follows from the very basics of the fractal theory. Thus the used method of measuring the fractal dimension is open to question, although the observation of a fractal structure is important by itself.

Additional assumptions

Experiments show significant diversity in values of α which, apparently, is caused by diverse fragmentation laws in different experimental setups. It is difficult to uniquely derive the fragmentation law from the exponent. Therefore we analyze several reasonable fragmentation mechanisms, calculate the parameter α for them, and correlate basic features of fragmentation with observable distribution functions of fragments.

First, we assume shape regularity of fragments. There is an inequality for the combination of coefficients in expression for α (3):

$$\left|\frac{c_1 c_3}{c_2}\right| \geq \left|\frac{V_n^*}{S_n^*} \Big/ \frac{V_n}{S_n}\right|, \quad (5)$$

where V_n^* and S_n^* is volume and surface area of the n-th fragment. If the shape of each fragment is more regular than the shape of the residual, then the right-hand side of inequality (5) is much greater than unity, as is the left-hand side. Consequently, the exponent in the size distribution function (3) differs from -3 by a small parameter. In the case of N-dimensional body, similar calculations give the exponent close to $-N$.

Second, we take into account the experimentally observed spherical shape of dust particles. Filling of space with spheres was studied in connection with osculatory packing problem. The osculatory packing in three-dimensional space gives $\alpha \approx 3.47$ [9]. The osculatory packing by disks in two-dimensional space corresponds to $\alpha \approx 2.3$.

Third, we use energy considerations. The brittle destruction is more energetically efficient than evaporation, but it also needs energy for surface tension. If this energy is a factor, then fragmentation must happen with minimal surface formation. An example of energy effective fragmentation is the Sierpinski tetrahedron with reduced remaining tetrahedrons for which the surface area of the residual decreases and the total surface area of all fragments is limited.

There is a compromise solution that meets both sphericity and energy considerations. The size of residual tetrahedrons can be optimized to make fragments as spherical as possible. The optimum corresponds to $\alpha \approx 2.4$.

Discussion

Several experiments [2] give results for α in the range from 2.2 to 2.3 with an accuracy of 0.1. Close values are given by osculatory packing by disks ($\alpha \approx 2.3$) and modified Sierpinski tetrahedron ($\alpha \approx 2.4$). This suggests that material destruction at described facilities is either energy-abundant two-dimensional or energy-scarce three-dimensional. The former variant seems more realistic since the penetration depth for particle energies involved is shorter than the smallest size of observed dust particles.

There are experiments showing greater values of the coefficient α [3]. In Ref. [3], not only size distribution was measured in the interval from $2\,\mu$m to $40\,\mu$m, but also the depth of erosion $\sim 10\,\mu$m in a single shot. These results could demonstrate the transition from the two-dimensional fragmentation to the three-dimensional one. Indeed, for small particles $\alpha \approx 3.3$, which is clearly distinguished from the value of 2.3 in [2]. Unfortunately, for particles larger than $10\,\mu$m the statistics is poor and no quantitative analysis is possible.

The exponent close to -3.47 corresponding to osculatory packing by spheres is observed for several non-fusion objects. The size distribution of interstellar grains has the power in the interval from -3.6 to -3.3 for variety of materials [10]. Brittle destructed materials (coal mine dust, crushed lead glass) has the fractal dimension corresponding to $\alpha \approx 3.5$ [11].

The observed distributions can be modified by collisions of fragments. If collisions round corners of fragments like sea waves polish pebbles through their contact, then the resulting law of fragmentation will approach the osculatory packing by spheres. Thus, the exponent close to -3.5 may bear witness to long-term collisional evolution of fragments.

This work is supported by RF President's grant NSh-7792.2010.2, Russian Ministry of Education grants RNP-2.1.1/3983 and 14.740.11.0053, and RFBR grant 11-02-00563.

References

[1] K. Koga *et al.*, Plasma and Fusion Research: Regular Articles **4**, 034 (2009).

[2] L.N. Khimchenko *et al.*, Proc.34th EPS Conf. Plasma Phys., Warsaw, 2007.

[3] A.V. Burdakov *et al.*, Transact. of Fusion Techn **35**, 1T, 146 (1999).

[4] S.K. Friedlander, Journal of Meteorogy, **17**, 5 (1960).

[5] J.Winter, Plasma Phys. Control. Fusion **46**, B583 (2004).

[6] E.M. Lifshitz, L.P. Pitaevskii, Physical kinetics. New York (Pergamon Press, 1981).

[7] J.Winter, Plasma Phys. Control. Fusion **40**, 1201 (1998).

[8] D.W. Boyd, Math. Comp. **39** (159), 249-254 (1982).

[9] M. Borkovec *et al.*, Fractals, **2** (4), 521 (1994).

[10] J.S. Mathis *et al.*, Astronomical Journal, **217**, 425 (1977).

[11] D. Avnir *et al.*, Nature, **308**, 261 (1984).