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A Boundary Element Method for Dynamic Analysis of Elastic Structures Subjected to Axial Flow

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Abstract

A boundary element method is presented to investigate the dynamic behavior of elastic structures partially or completely in contact with uniform axial flow. In the analysis of the linear fluid-structure interaction problem, it is assumed that the fluid is ideal and its motion is irrotational. Furthermore, the elastic structure is assumed to vibrate in relatively high-frequencies, so the infinite frequency limit condition is imposed for fluid free surface, which is satisfied implicitly by using method of images. When in contact with the flowing fluid, the structure is assumed to vibrate in its *in vacuo* eigen-modes that are obtained by using a finite element software. The wetted surface of the structure is idealized by using appropriate hydrodynamic panels and a boundary element method is formulated for velocity potential function, which is taken as linearly varying over the panels. Using the Bernoulli's equation, the dynamic fluid pressure on the elastic structure is expressed in terms of potential function, and the fluid–structure interaction forces are calculated as generalized added mass, hydrodynamic damping and hydrodynamic stiffness coefficients, due to the inertia, Coriolis and centrifugal effects of fluid, respectively. Solution of the eigenvalue problem associated with the generalized equation of motion gives the dynamic characteristics of the structure in contact with fluid. As an application of the method, the dynamics of a simply supported cylindrical shell subjected to internal flow is studied. The predictions compare quite well with the previous results in the literature.

INTRODUCTION

This paper presents a boundary element method for investigating the dynamic response behavior of elastic structures in contact with a quiescent or axially flowing fluid. The method is general and can be applied to any shape of elastic body. The numerical method has been successfully used for different kinds of structures such as a cantilever plate vibrating partially submerged in a quiescent fluid [1], liquid storage tanks filled with water [2], and a circular cylindrical shell in contact with internal and/or external axially flowing fluid [3] by the authors.

The response of elastic structures immersed in or conveying flowing fluid has been extensively studied, and general reviews of the literature have been given by Païdoussis [4] and Païdoussis and Li [5]. Recent books by Païdoussis [6, 7] provide a comprehensive treatment of the subject as well as a complete bibliography of all important work in the field. Moreover, Amabili and Garziera [8] presented a study on the linear dynamic analysis of cylindrical shells with flowing fluid. They investigated the influence of various complicating effects, such as non-uniform edge boundaries; internal, external and annular flows, etc. On the other hand, a three-part study, investigating the dynamics of cantilever cylinders in axial flow, has been reported. In the first part, Païdoussis *et al* [9] presented some old and new experimental results, and a comparison with linear theory was made. In the second part, Lopes *et al* [10] derived a weakly nonlinear equation of motion. The fluid dynamic forces were introduced in terms of virtual work expressions. The results of the calculations based on this theoretical model were presented in Semler *et al* [11]. Lakis and Selmane [12] present a hybrid finite element method to investigate the large amplitude vibrations of orthotropic cylindrical shells subjected to flowing fluid, in which only the linear effects of the fluid were taken into account. In a study based on the finite element method, refined shell theories and linear fluid dynamic theory, Toorani and Lakis [13] presented the flow-induced vibration characteristics of the anisotropic laminated cylindrical shells partially or completely filled with a quiescent liquid or subjected to a flowing fluid. Recently, Uğurlu and Ergin [3] studied the dynamic response behavior of a simply supported circular cylindrical shell subjected to axially flowing fluid by using a boundary element method in conjunction with the method of images. They presented the non-dimensional wet frequencies as a function of the non-dimensional flow velocities, and a comparison with the results in open literature was provided. Furthermore, they investigated the dynamic characteristics of the shell partially in contact with the internal and/or external flowing fluid.

In this investigation, it is assumed that the fluid is ideal, i.e., inviscid, incompressible and its motion is irrotational. It is assumed that the flexible structure vibrates in its *in vacuo* eigenmodes when it is in contact with flowing fluid, and that each mode gives rise to a corresponding surface pressure distribution on the wetted surface of the structure. The *in vacuo* dynamic analysis entails the vibration of the structure in the absence of any external force and structural damping, and the corresponding dynamic characteristics (e.g., natural frequencies and mode shapes) of the structure were obtained by using a standard finite element software.

At the fluid-structure interface, continuity considerations require that the normal velocity of the fluid is equal to that of structure. The normal velocities on the wetted surface of the structure are expressed in terms of modal structural displacements and their derivatives. By using a boundary element formulation, the fluid pressure is eliminated from the problem, and the fluid-structure interaction forces are calculated in terms of the generalized hydrodynamic added mass coefficients (due to the inertial effect of fluid), generalized fluid damping coefficients (due to the Coriolis acceleration of fluid) and generalized fluid stiffness coefficients (due to the centrifugal effect of fluid). However, when the structure is in contact with an ideal, quiescent fluid, the fluid-structure interaction forces are only associated with the inertial effect of fluid, i.e., the fluid pressure on the wetted surface of the structure is in phase with the structural acceleration.

During the numerical analysis, the wetted surface is idealized by using appropriate boundary elements, referred to as hydrodynamic panels, over which a linear distribution is assumed for the potential function. The potential values were calculated by satisfying the necessary boundary conditions on the wetted surface.

The generalized structural mass matrix is merged with the generalized hydrodynamic mass matrix, and the structural stiffness matrix with the generalized fluid stiffness matrix. Then, the total generalized mass and stiffness matrices are used together with the generalized fluid damping matrix in solving the eigenvalue problem for the elastic structure immersed in and/or containing flowing fluid. To assess the influence of the flowing fluid on the dynamic response behavior of the elastic structure, the non-dimensional eigenfrequency for the fundamental mode is presented as a function of the non-dimensional flow velocity. The associated eigenmode is also presented for different flow velocities. The predictions of this study were compared with the results found in the open literature, and a very good comparison was obtained.

MATHEMATICAL MODEL

The Generalized Equation of Motion

The equation of motion describing the response of a flexible structure to external excitation may be written as in Ergin *et al* [14]

$$\mathbf{M}\,\ddot{\mathbf{U}} + \mathbf{C_V}\,\dot{\mathbf{U}} + \mathbf{K}\,\mathbf{U} = \mathbf{P}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{C_V}$, $\mathbf{K}$ denote the mass, structural damping and stiffness matrices respectively. The vectors $\mathbf{U}$, $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ represent the structural displacements, velocities and accelerations, respectively, and the column vector $\mathbf{P}$ denotes the external forces.

In an *in vacuo* analysis, the structure is assumed to vibrate in the absence of any structural damping and external forces reducing equation (1) to the form

$$\mathbf{M}\,\ddot{\mathbf{U}} + \mathbf{K}\,\mathbf{U} = \mathbf{0}. \tag{2}$$

The form of equation (2) suggests that one can express the trial solution as

$$\mathbf{U} = \mathbf{D}\mathrm{e}^{\mathrm{i}\omega t}. \tag{3}$$

Using equation (3) in equation (2) and canceling the common factor $\mathrm{e}^{\mathrm{i}\omega t}$, one obtains the equation

$$(-\omega^2\,\mathbf{M} + \mathbf{K}\,)\,\mathbf{D} = \mathbf{0}. \tag{4}$$

This equation describes the simple harmonic oscillations of the free undamped structure and the *in vacuo* principle modes and natural frequencies are determined from the associated eigenvalue problem.

The distortions of the structure may be expressed as the sum of the distortions in the principal modes,

$$\mathbf{U} = \mathbf{D}\,\mathbf{p}\,(t)\,, \tag{5}$$

where $\mathbf{D}$ is the modal matrix whose columns are the *in vacuo*, undamped mode vectors of the structure. $\mathbf{p}$ is the principal coordinates vector. By substituting equation (5) into equation (1) and pre-multiplying by $\mathbf{D}^{\mathrm{T}}$, the following generalized equation in terms of the principal coordinates of the structure is obtained:

$$\mathbf{a}\,\ddot{\mathbf{p}}\,(t) + \mathbf{b}\,\dot{\mathbf{p}}\,(t) + \mathbf{c}\,\mathbf{p}\,(t) = \mathbf{Q}\,(t). \tag{6}$$

Here **a**, **b**, **c** denote the generalized mass, damping and stiffness matrices, respectively, and are defined as follows:

$$\mathbf{a} = \mathbf{D}^T\mathbf{M}\,\mathbf{D},\ \ \mathbf{b} = \mathbf{D}^T\mathbf{C}_V\,\mathbf{D},\ \ \mathbf{c} = \mathbf{D}^T\mathbf{K}\,\mathbf{D},\ \ \mathbf{Q} = \mathbf{D}^T\mathbf{P}. \tag{7}$$

The generalized force matrix, **Q**(t) represents the fluid-structure interaction and all other external forces, and it may be expressed as follows:

$$\mathbf{Q}(t) = -(\mathbf{A}\ddot{\mathbf{p}}(t) + \mathbf{B}\dot{\mathbf{p}}(t) + \mathbf{C}\mathbf{p}(t)) + \Xi(t), \tag{8}$$

where **A**, **B** and **C** are the generalized added mass, generalized fluid damping, and generalized fluid stiffness matrices, respectively, and Ξ (t) denotes the generalized external force vector caused by waves, mechanical excitation, etc.

Thus, equation (6) may be rewritten in the form (see, for instance, Ref. [14])

$$(\mathbf{a}+\mathbf{A})\ddot{\mathbf{p}}(t) + (\mathbf{b}+\mathbf{B})\dot{\mathbf{p}}(t) + (\mathbf{c}+\mathbf{C})\mathbf{p}(t) = \Xi(t)\,. \tag{9}$$

Formulation of the Fluid Problem

A right-handed Cartesian coordinate system, *xyz*, is adopted in the present study and it is shown in Fig.1 for the circular cylindrical shell conveying fluid. The coordinate system is fixed in space with its origin at *O*, and the x-axis coincides with the center line of the cylindrical shell in the longitudinal direction.

In the mathematical model, the fluid is assumed ideal, i.e., inviscid and incompressible, and its motion is irrotational and there exists a fluid velocity vector, **v**, which can be defined as the gradient of the velocity potential function Φ as

$$\mathbf{v}(x,y,z,t) = \nabla\Phi(x,y,z,t). \tag{10}$$

The velocity potential Φ may be written as

$$\Phi = U\,x + \phi\,. \tag{11}$$

Here the steady velocity potential $U\,x$ represents the effect of the mean flow associated with the undisturbed flow velocity U in the axial direction. Further, Φ is the unsteady velocity potential associated with the perturbations to the flow field due to the motion of the flexible body, and satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{12}$$

throughout the fluid domain.

For the structure containing and/or submerged in flowing fluid, the vibratory response of the structure may be expressed in terms of principal coordinates as

$$\mathbf{p}(t) = \mathbf{p_0}\,\mathrm{e}^{\lambda t}, \tag{13}$$

where $\mathbf{p_0}$ and λ are complex nonzero constants, and t is the time. The imaginary part of λ is the circular frequency of oscillations and its real part gives an exponential growth or decay. The velocity potential function due to the distortion of the structure in the *r-th in vacuo* vibration mode may be written as follows

$$\phi_r(x,y,z,t) = \phi_r(x,y,z)\,p_{0r}\,\mathrm{e}^{\lambda t},\ \ r = 1, 2, \ldots, M\,, \tag{14}$$

where M represents the number of modes of interest, and p_{0r} is an unknown complex amplitude for the *r-th* principal coordinate.

On the wetted surface of the vibrating structure the fluid normal velocity must be equal to the normal velocity on the structure and this condition for the *r-th* modal vibration of the elastic structure containing or/and submerged in flowing fluid can be expressed as (see, for instance, Ref. [8])

$$\frac{\partial\phi_r}{\partial\mathbf{n}} = \left(\frac{\partial\mathbf{u}_r}{\partial t} + U\frac{\partial\mathbf{u}_r}{\partial x}\right).\mathbf{n}, \tag{15}$$

where **n** is the unit normal vector on the wetted surface and points out of the region of interest.

The vector $\mathbf{u}_r$ denotes the displacement response of the structure in the *r-th* principal coordinate and it may be written as

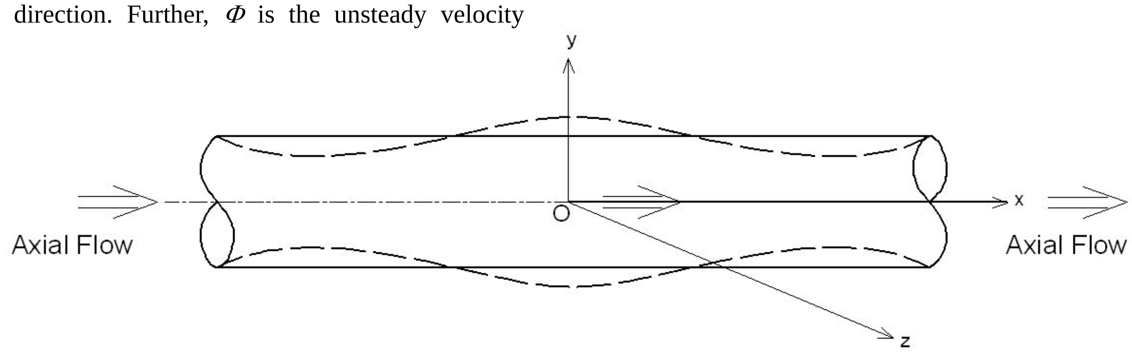

Figure 1. Fluid conveying elastic cylindrical shell

$$\mathbf{u}_r(x,y,z,t)=\mathbf{u}_r(x,y,z)\;p_{0r}\;\mathrm{e}^{\lambda t}, \tag{16}$$

where $\mathbf{u}_r$ *(x, y, z) is* the *r-th* modal displacement vector of the median surface of the elastic structure, and it is obtained from the *in vacuo* analysis.

Substituting equations (14) and (16) into (15), the following expression is obtained for the boundary condition on the fluid-structure interface

$$\frac{\partial\phi_r}{\partial\mathbf{n}}=\lambda\,\mathbf{u}_r(x,y,z).\mathbf{n}+U\,\frac{\partial\mathbf{u}_r(x,y,z)}{\partial x}.\mathbf{n}. \tag{17}$$

In this study, it is assumed that the elastic structure vibrates at relatively high frequencies so that the effect of surface waves can be neglected. Therefore, the free surface condition (infinite frequency limit condition) for the perturbation potential can be approximated by

$$\phi_r=0\,,\ \text{ on the free surface.} \tag{18}$$

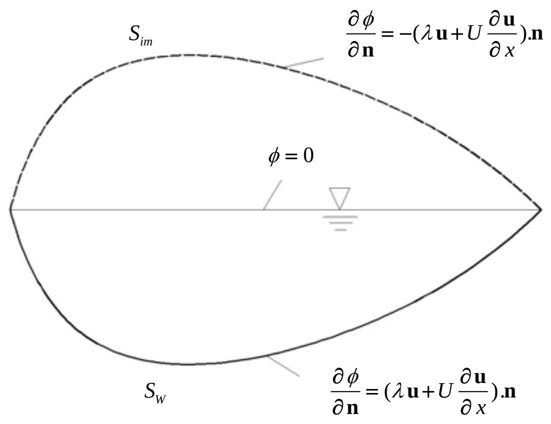

Figure 2. Wetted and imaginary surfaces of fluid domain and imposed boundary conditions

The method of images [15] may be used, as shown in Fig.2, to satisfy this boundary condition. By adding an imaginary boundary region, the condition given by equation (18) at the horizontal surface can be omitted; thus the problem is reduced to a classical Neumann case. This condition can also be satisfied directly by using appropriate Green function. It should be noted that, for the completely filled elastic structure, the normal fluid velocity can not be arbitrarily specified. It has to satisfy the incompressibility condition

$$\iint_{S_W+S_{im}}\frac{\partial\phi_r}{\partial\mathbf{n}}\,dS=0, \tag{19}$$

where S_W and S_{im} represent the wetted and image surfaces of the elastic structure, respectively.

Solution of the Fluid Problem

From Green's third identity, the boundary value problem for the perturbation potential ϕ, can be represented by the boundary integral equation,

$$c(\xi)\phi(\xi)=\int_{S_w}\left(\phi^*(s,\xi)q(s)-\phi(s)q^*(s,\xi)\right)dS, \tag{20}$$

over the fluid-structure interface, where ξ and s are the application point of the equation and a general field point on the wetted surface, respectively. Here $\phi^*(s,\xi)=1/4\pi r$ is the fundamental solution in the three dimensional inviscid flow and $q=\partial\phi/\partial\mathbf{n}$ denotes the flux where,

$$q^*(s,\xi)=-(\partial r/\partial\mathbf{n})/4\pi r^2\,. \tag{21}$$

r is the distance between source and field points and the free term $c(\xi)$ can be taken as the fraction of $\phi(\xi)$ that lies inside the domain of interest.

The fluid-structure interaction problem may be separated into two parts: (i) the vibration of the elastic structure in a quiescent fluid, and (ii) the disturbance in the main axial flow due to the oscillation of the elastic structure. Thus, defining $\phi=\lambda\,\phi_1+U\,\phi_2$, equation (17) may be divided into two seperate parts as

$$\frac{\partial\phi_1}{\partial\mathbf{n}}=\mathbf{u}(x,y,z).\mathbf{n}, \tag{22a}$$

$$\frac{\partial\phi_2}{\partial\mathbf{n}}=\frac{\partial\mathbf{u}(x,y,z)}{\partial x}.\mathbf{n}, \tag{22b}$$

where ϕ_1 denotes the displacement potential due to the vibration of the structure in a quiescent fluid, and ϕ_2 represents the disturbing effect of the term $\partial u/\partial x$ to the main axial flow field. $\mathbf{u}$ represents the *in vacuo* modal displacement vector.

For the general solution of equation (20) with boundary conditions (22), the wetted surface can be discretized by using hydrodynamic panels and the distribution of the potential function and flux over each panel may be represented by means of the shape functions and nodal values

$$\phi_e=\sum_{j=1}^{n_e}N_{ej}\phi_{ej}\ \ ,\ \ q_e=\sum_{j=1}^{n_e}N_{ej}q_{ej}\,. \tag{23}$$

Here, n_e is the number of nodal points assigned to each panel and N_{ej} represents the shape function adopted for the distribution of potential function. For instance, in the case of a linear distribution, the shape functions for a quadrilateral panel can be expressed as (see, Wrobel [16])

$$N_1=((1-\varsigma)(1-\eta))/4, N_2=((1+\varsigma)(1-\eta))/4$$
$$N_3=((1+\varsigma)(1+\eta))/4, N_4=((1-\varsigma)(1+\eta))/4. \quad (24)$$

in the local coordinate system ζ, η. The unknown potential function values can be determined from the following sets of algebraic equations, after applying equation (20) for each nodal point, for the rth modal vibration form, (see, Ref. [17])

$$c_k\,\phi_{1k}^r+\sum_{i=1}^{m}\sum_{j=1}^{n_m}(\phi_{1ij}^r\iint_{S_i}N_j\,q^*dS)$$
$$=\sum_{i=1}^{m}\sum_{j=1}^{n_m}(\mathbf{u}_{ij}^r.\mathbf{n}_j\iint_{S_i}N_j\,\phi^*dS)\,,\ k=1,2,\ldots,m \quad (25a)$$
$$c_k\,\phi_{2k}^r+\sum_{i=1}^{m}\sum_{j=1}^{n_m}(\phi_{2ij}^r\iint_{S_i}N_j\,q^*dS)$$
$$=\sum_{i=1}^{m}\sum_{j=1}^{n_m}(\frac{\partial\mathbf{u}_{ij}^r}{\partial x}.\mathbf{n}_j\iint_{S_i}N_j\,\phi^*dS)\,,\ k=1,2,\ldots,m. \quad (25b)$$

m denotes the total number of panels in the discretization.

Generalized Fluid-Structure Interaction Forces

Using the Bernoulli's equation and neglecting the second order terms, the dynamic fluid pressure on the elastic structure due to the *r-th* modal vibration becomes

$$P_r(x,y,z,t)=-\rho(\frac{\partial\phi_r}{\partial t}+U\frac{\partial\phi_r}{\partial x}). \quad (26)$$

ρ is the fluid density. Substituting equation (14) into (26), the following expression for the pressure is obtained,

$$P_r(x,y,z,t)=-\rho(\lambda\,\phi_r+U\frac{\partial\phi_r}{\partial x})\,p_{0r}\,\mathrm{e}^{\lambda t}. \quad (27)$$

By using the definition $\phi_r=\lambda\,\phi_{r1}+U\,\phi_{r2}$, equation (27) may be rewritten in the following form:

$$P_r(x,y,z,t)=-\rho(\lambda^2\,\phi_{r1}+U\,\lambda\,(\frac{\partial\phi_{r1}}{\partial x}+\phi_{r2})+U^2\,\frac{\partial\phi_{r2}}{\partial x})\,p_{0r}\,\mathrm{e}^{\lambda t}. \quad (28)$$

The *k-th* component of the generalized fluid-structure interaction force due to the *r-th* modal vibration of the elastic structure subjected to axial flow can be expressed in terms of the pressure acting on the wetted surface of the structure as

$$Z_{kr}(t)=\iint_{S_W}P_r(x,y,z,t)\mathbf{u}_k.\mathbf{n}\,dS$$
$$=-p_{0r}\,\mathrm{e}^{\lambda t}\iint_{S_W}\rho(\lambda^2\,\phi_{r1}+U\,\lambda(\frac{\partial\phi_{r1}}{\partial x}+\phi_{r2})+U^2\frac{\partial\phi_{r2}}{\partial x})\mathbf{u}_k.\mathbf{n}\,dS$$
$$=-\lambda^2\,p_{0r}\,\mathrm{e}^{\lambda t}\rho\iint_{S_W}\phi_{r1}\mathbf{u}_k.\mathbf{n}\,dS-\lambda\,p_{0r}\,\mathrm{e}^{\lambda t}\rho\,U\iint_{S_W}(\frac{\partial\phi_{r1}}{\partial x}+\phi_{r2})\mathbf{u}_k.\mathbf{n}\,dS$$
$$-\,p_{0r}\,\mathrm{e}^{\lambda t}\rho\,U^2\iint_{S_W}\frac{\partial\phi_{r2}}{\partial x}\mathbf{u}_k.\mathbf{n}\,dS. \quad (29)$$

The generalized added mass A_{kr}, generalized fluid damping (due to the Coriolis effect of fluid), B_{kr}, and generalized fluid stiffness (due to the centrifugal effect of fluid), C_{kr}, terms can be defined as

$$A_{kr}=\rho\iint_{S_W}\phi_{r1}\,\mathbf{u}_k.\mathbf{n}\,dS, \quad (30)$$

$$B_{kr}=\rho U\iint_{S_W}(\frac{\partial\phi_{r1}}{\partial x}+\phi_{r2})\mathbf{u}_k.\mathbf{n}\,dS, \quad (31)$$

$$C_{kr}=\rho U^2\iint_{S_W}\frac{\partial\phi_{r2}}{\partial x}\mathbf{u}_k.\mathbf{n}\,dS. \quad (32)$$

Therefore, the generalized fluid-structure interaction force component, Z_{rk}, can be rewritten as

$$Z_{kr}(t)=-A_{kr}\,\lambda^2\,p_{0r}\,\mathrm{e}^{\lambda t}-B_{kr}\,\lambda\,p_{0r}\,\mathrm{e}^{\lambda t}-C_{kr}\,p_{0r}\,\mathrm{e}^{\lambda t}$$
$$=-A_{kr}\,\ddot{p}_r(t)-B_{kr}\,\dot{p}_r(t)-C_{kr}\,p_r(t). \quad (33)$$

Calculation of Eigenvalues and Eigenvectors

The generalized equation of motion for the elastic structure in contact with axial flow assuming free vibrations with no structural damping is

$$\left[\lambda^2(\mathbf{a}+\mathbf{A})+\lambda(\mathbf{B})+(\mathbf{c}+\mathbf{C})\right]\mathbf{p}=0, \quad (34)$$

where **a** and **c** denote the generalized structural mass and stiffness matrices respectively. The matrices **A**, **B** and **C** represent the generalized added mass, generalized fluid damping and generalized fluid stiffness matrices, respectively.

It should be noticed that the eigenvalue λ is generally complex. It was observed from the solution of the eigenvalue problem that, before the onset of the instability, the eigenvalues have zero real part, and therefore, the fluid-structure interaction system is conservative. On the other hand, the eigenvectors **p** have both real and imaginary parts, which are different from zero. Therefore, the eigenvectors (modes) are complex. However, when the axial mean flow velocity is zero the eigenvectors only have real parts

NUMERICAL RESULTS AND COMPARISONS

A finite length circular cylindrical shell is chosen to demonstrate the applicability of the aforementioned theory to structures containing and/or submerged in flowing fluid. The shell structure adopted in this study is simply supported at both ends. The cylindrical shell under consideration was analytically investigated by, Selmane and Lakis [18], Amabili *et al* [19] and Amabili and Garziera [8]. The cylindrical shell adopted has the geometric and material properties: length-to-radius ratio $L/R = 2$, thickness-to-radius ratio $h/R = 0.01$, Young's modulus $E = 206$ GPa, Poisson's ratio $\upsilon = 0.3$, and mass density $\rho_s = 7850$ kg/m^3. Fresh water is used as the contained fluid with a density of $\rho_f = 1000$ kg/m^3.

A right-handed Cartesian coordinate system, *xyz*, is adopted in this study, and it is shown in Fig.1 for the cylindrical shell subjected to axial flow. The coordinate system is fixed in space with its origin at *O*. The *x*-axis lies along the length *L*, and coincides with the centerline of the cylindrical shell.

For convenience, the following non-dimensional parameters are introduced:

$$V = U / \left\{ (\pi^2 / L) \left[D / (\rho h) \right]^{1/2} \right\}$$
$$\Omega = \lambda / \left\{ (\pi^2 / L^2) \left[D / (\rho h) \right]^{1/2} \right\}. \tag{35}$$

Here, V and Ω denote the non-dimensional axial fluid velocity and non-dimensional eigenfrequency, respectively, and λ is the corresponding complex eigenvalues of the cylindrical shell conveying and/or submerged in flowing fluid. Furthermore, D is the flexural rigidity, and it is defined as $D = Eh^3 / 12\,(1-\upsilon^2)$.

The *in vacuo* dynamic characteristics of the shell structure were obtained using a standard finite element software. This produced the information on natural frequencies and normal mode shapes of the *dry* shell structure in vacuum. In these calculations, the cylindrical shell was discretized with four-noded quadrilateral shell elements, including both membrane and bending stiffness influences. For the converged *dry* dynamic results adopted in the *wet* calculations, 64 and 32 finite elements were distributed around the circumference and along the shell structure, respectively. The mode shapes of the shell structure in vacuum are identified with the number of standing waves around the circumference, *n*, and the number of half-waves along the shell, *m*. A combination of *m* and *n* forms a particular mode shape (*m*, *n*). The results occur in pairs. That is, in general, for each natural frequency, there exists a pair of mode shapes satisfying the relevant orthogonality conditions.

By solving the associated eigenvalue problem (34), the non-dimensional eigenvalues and associated eigenmodes of the cylindrical shell containing flowing fluid are obtained as a function of the non-dimensional flow velocity. The predictions based on the proposed method are compared with the analytical calculations by Selmane and Lakis [18], Amabili *et al* [19] and Amabili and Garziera [8]. Fig.3 presents the non-dimensional wet frequency values as a function of the non-dimensional flow velocity for the first and second wet modes for the circumferential wave number, $n = 5$. In order to show the convergence of the results, two groups of the calculations were performed. In the first group of the calculations, 48 and 24 hydrodynamic panels, respectively, were distributed around the circumference and along the cylindrical shell (a total number of 1152 panels). However, 64 and 32 hydrodynamic panels were adopted around the circumference and along the shell, respectively, for the second group of the calculations (a total number of 2048 panels). To take into the coupling effects, 12 *in vacuo* modes (6 of which are symmetric and 6 antiysmmetric) are included in each analysis, so that the first three *wet* modes are obtained with sufficient accuracy. As observed in Fig.3, the calculated wet frequencies based on these two idealizations are very close to each other. Therefore, it may be said that the idealization using 1152 hydrodynamic panels produces the converged wet results. For the first and second wet mode shapes with $n = 5$, the results of the present study compares well with those found in the open literature. As seen from Fig.3, the non-dimensional wet frequency values decrease with increasing non-dimensional flow velocity. The first mode shape reaches its zero frequency value at $V = 3.32$ (for the 1152 panel idealization), and the intersection of the second mode with the axis of non-dimensional flow velocity at $V = 4.41$ is the point of restabilization.

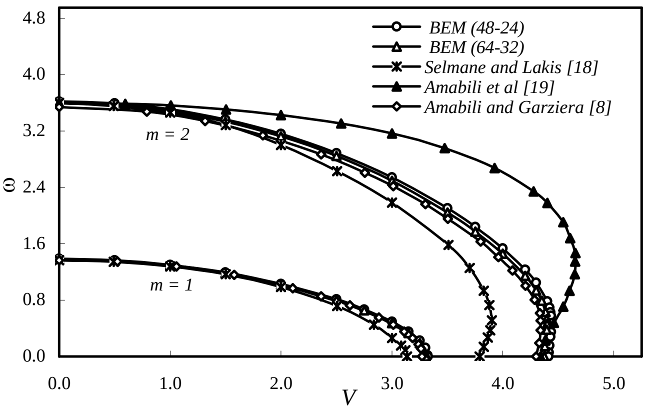

Figure 3. The variation of nondimensional frequency with respect to nondimensional flow velocity, for $n = 5$

Fig.4 (a) and (b) present, respectively, the real and imaginary parts of the non-dimensional eigenfrequency, as a function of the non-dimensional flow velocity, for the first three axial modes with the circumferential wave number, $n = 5$. As seen in Fig.4(b), the first mode reaches its zero frequency at $V = 3.50$, and the intersection of the second mode with the axis of non-dimensional flow velocity at $V = 4.56$ is the point of restabilization. Then, the first and second modes merge at $V = 4.62$ and this points corresponds to the onset of the coupled mode flutter. It should also be noted that the coupled-mode flutter cannot be properly described by the linear theory. Furthermore, the real part of the non-dimensional eigenfrequency is presented in Fig.4(a) and it is proportional to damping. It should also be noted that the system is stable when the real part of the non-dimensional eigenfrequency is zero or negative, and it is unstable when the real part is positive.

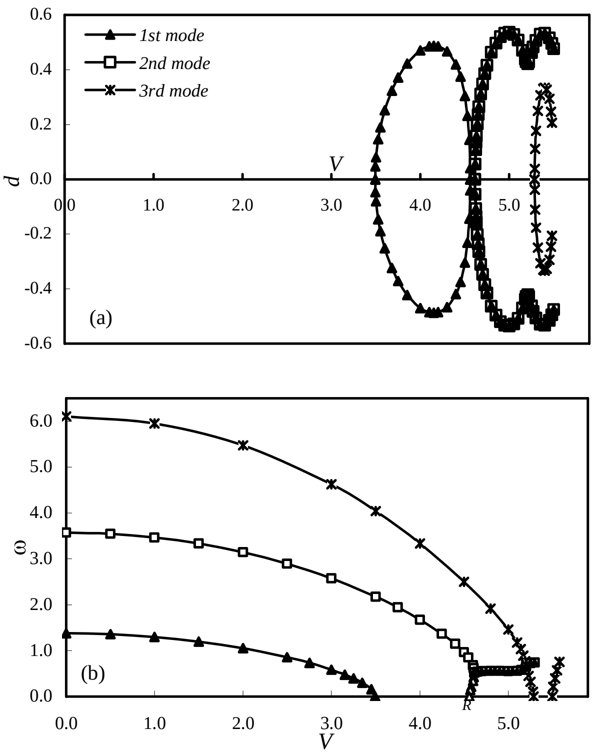

Figure 4. Variation of the first three nondimensional eigenvalues with respect to nondimensional flow velocity, for $n = 5$: (a) real part, (b) imaginary part.

The mode shapes of the cylindrical shell completely filled with flowing fluid are presented in Fig.5(a-b) at the times $t = 0$, $T/8$, $T/4$, $3T/8$ and $T/2$ (where T is the time period) for the non-dimensional flow velocities $V = 1.05$, 3.15, respectively, and in Fig.5(c) at the times $t = 0$, $T/12$, $T/8$ and $T/4$ for $V = 3.49$, just before the instability. The mode shapes presented in Fig.5 are all the first axial mode shapes with the circumferential wave number, $n = 5$. It is clear that the mode shapes are complex, and they may take different forms in a specific time period.

CONCLUSION

A method of linear analysis is presented for the dynamic response behavior of elastic structures subjected to flowing fluid, based on boundary element method and the method of images. A simply supported elastic circular cylindrical shell structure filled with flowing fluid was chosen in order to demonstrate the applicability of the method. It can be concluded from the results presented that the method proposed is suitable for the vibration analysis of flexible structures subjected to axial flow.

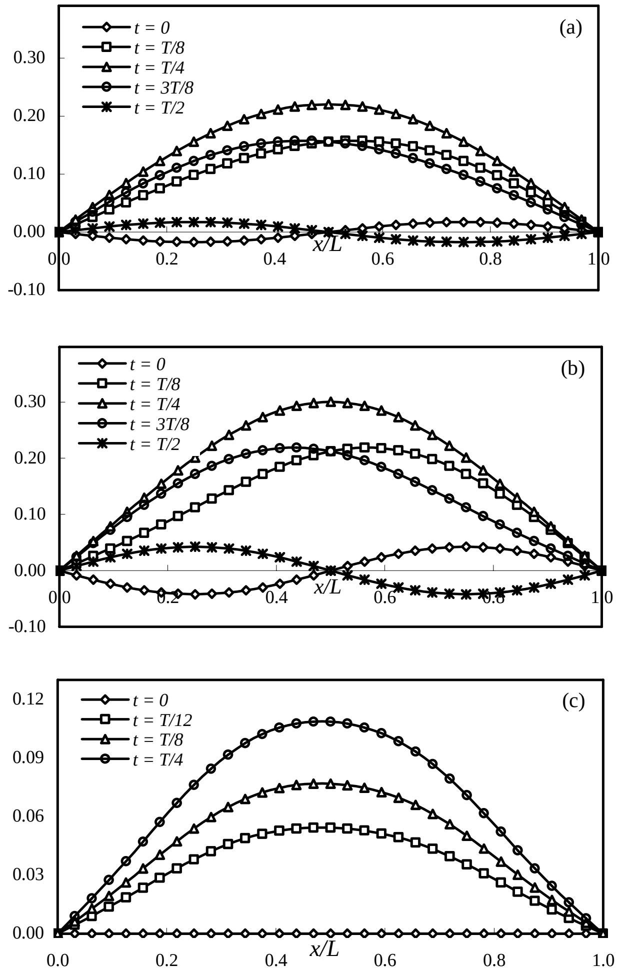

Figure 5. The wet mode shapes of cylindrical shell conveying fluid, for $n = 5$: (a) $V = 1.05$, (b) $V = 3.15$, (c) $V = 3.49$.

Acknowledgments

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