Proceedings of the ASME 2009 International Mechanical Engineering Congress & Exposition
IMECE2009
November 13-19, Lake Buena Vista, Florida, USA

IMECE2009-12186

APPLICATION OF THE EXTENDED KANTOROVICH METHOD TO THE VIBRATIONAL ANALYSIS OF ELECTRICALLY ACTUATED MICROPLATES

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ABSTRACT

This paper presents an extended Kantorovich approach to investigate the vibrational behavior of electrically actuated rectangular microplates. The model accounts for the electric force of the excitation and for the applied in plane loads. Starting from a one term Galerkin approximation and following the extended Kantorovich procedure, the partial differential equation governing the microplate vibration, is discretized to two ordinary differential equation with constant coefficients. These equations are then solved analytically and iteratively with a rapid convergence procedure for finding microplate natural frequencies and modeshapes. Results in some specific cases are validated against other theoretical results reported in the literature. It is shown that rapid convergence, high precision and independency of initial guess function make the EKM an effective and accurate design tool for design optimization.

KEYWORDS: Microplate, Vibration, Electrostatic actuation, Fundamental natural frequency.

INTRODUCTION

Technology can touch our daily lives in so many different ways, but the role of miniature devices and systems is not immediately apparent [1]. Technology of micro electro mechanical systems has experienced a lot of progress in testing and fabricating new devices recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them even more attractive [2, 3].

Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3]. So simulation of micromachined systems and sensors is becoming increasingly important. Before prototyping a device, one wishes to virtually build the device and predict its behavior. This allows the optimization of various design parameters according to the specifications [4].

Typical MEMS devices employ a parallel plate capacitor with variable capacity in which one plate is actuated electrically and its motion is detected by capacitive changes [5]. These electrically actuated microplates form the most important actuation component part of many MEMS devices such as micropumps, micromirrors, microphones and microresonators [6-10].

Analysis of MEMS devices is challenging because the classical structural dynamic methodology is not easily applicable to the types of forcing and nonlinearities encountered in MEMS. A common approach in the literature is to assume a linear relationship between the excitation force and the plate deflection. The linear plate equation is then solved by numerical methods, such as Galerkin method, the Rayleigh Ritz method and the finite element method [5].

There has been extensive research in to the behavior of micromachined systems. Researchers have used variety of methods, such as analytical methods, numerical methods, reduced order modeling and perturbation methods to achieve this goal. Hu [11] suggested three analytical models, namely the full order, the fourth order and the third order models and the corresponding closed form solutions for the pull-in voltages of micro curled beams, subjected to electrostatic loads. Chao et al [3] present a novel method to predict the pull-in voltage in a closed form for microplates actuated by a distributed electrostatic force.

Many researchers have used numerical methods for the analysis of MEMS devices. Abdel-Rahman et al [12] presented a nonlinear model for electrically actuated microbeams. They solved the boundary value problem describing the static deflection of the microbeam under electrostatic force due to a

DC polarization voltage numerically. They also used numerical approach to solve the eigenvalue problem describing the vibration of the microbeam around its statically deflected position for the natural frequencies and modeshapes. Faris et al. [13] have investigated the nonlinear modeling of annular plates. They determine the static deflection using a numerical shooting technique.

Reduced order models have been paid close attention in analysis of MEMS devices. Zhao et al [5] proposed a reduced order model for electrically actuated microplate based MEMS. They found the linear undamped vibrational modes numerically and used those modeshapes in a Galerkin approximation to reduce the partial differential equations of motion into a finite dimensional system of nonlinearly coupled second order ordinary differential equations. They validated their model against experimental findings.

Many researchers have used perturbation methods to simulate MEMS behavior. Nayfeh and Younis [14] used perturbation methods in conjunction with the finite element method to model and simulate flexible microstructures under the effect of squeeze film damping. Abdel-Rahman and Nayfeh [15] investigate secondary resonances of electrically actuated resonant microsensors using the multiple scales method. Nayfeh and Younis [16] utilized perturbation methods to present analytic expressions for the quality factors of microplate due to thermoelastic damping.

Although the analysis of electrically actuated microplates has been the subject of many researches, the problem when a high precision response and low computational time is required still remains uninvestigated. In this paper an extended Kantorovich approach is implemented to model the vibration of microplates under electrostatic actuation.

The Kantorovich method occupies a position intermediate between the exact solution of a given problem and solution which is obtained by means of methods of Ritz and Galerkin [17]. Results from extended Kantorovich method are even more accurate. This method is based on Variational principle and reduces the partial differential equation governing the system behavior to a set of uncoupled ordinary differential equations which are solved iteratively with a rapid convergence and the final solution would be independent of the initial guess function.

The Kantorovich method was suggested by Kantorovich and Krylov [18]. Kerr [19] and Kerr and Alexander [20], extended the Kantorovich method by using it as a first step of an iterative procedure and showed that the implemented procedure converges very rapidly to a final form, irrespective of the initial guess function. They [20] used the extended Kantorovich method to analyze a clamped rectangular plate subjected to a uniform lateral load. Cortinez and Laura [21] used the same method for the vibrational analysis of rectangular plates of discontinuously varying thickness. Dalaei and Kerr [17] analyzed clamped rectangular orthotropic plates subjected to a uniform lateral load. Since there was no exact analytical solution for that problem, they tried to derive a closed-form approximate solution of high accuracy which was achieved by the extended Kantorovich method. They found that the convergence of the procedure is very rapid and that the final form of the generated solution is independent of the initial choice.

Kerr [22] presented an extended Kantorovich procedure for the solution of the eigenvalue problems. His specific examples were the vibration of rectangular membrane and stability of an elastic rectangular plate compressed in its plane. He showed that for the membrane problem, the generated expressions for the eigenvalues and eigenfunctions are identical with the corresponding exact solution and for the clamped plate compressed uni-axially or bi-axially, the generated eigenvalues, based on a one term expression for the eigenfunction agree very closely with the relevant results available in the literature.

Jones and Milne [23] applied the extended Kantorovich method to the vibration analysis of clamped rectangular plates and presented closed-form solutions for the plate mode shapes with high accuracy. They found that the process converges so rapidly that usually two iterates is sufficient to achieve a precise response. Dalaei and Kerr [24] extended what Jones and Milne [23] did and used the Extended Kantorovich method to analyze free vibration of clamped rectangular orthotropic plates. They derived closed-form solutions for system mode shapes and corresponding natural frequencies for the problem which had no exact solution.

The current paper makes use of the advantages of the extended Kantorovich method, to model the static deflection of electrically actuated microplates under electrostatic force. It was found that the convergence of the procedure is very rapid. The effect of various design parameters such as applied in plane loads and microplate aspect ratio on the vibrational response of the microplate has also been studied. Presented model in some specific cases has been validated with theoretical findings reported in the literature.

Problem Formulation

The capacitively actuated microplate shown in figure (1) is being studied. Using the model of Nayfeh and Younis [14] and neglecting the pressure dependent terms one may obtain:

$$D\left(\frac{\partial^4\hat{W}}{\partial\hat{x}^4}+2\frac{\partial^4\hat{W}}{\partial\hat{x}^2\partial\hat{y}^2}+\frac{\partial^4\hat{W}}{\partial\hat{y}^4}\right)-\hat{N}_1\frac{\partial^2\hat{W}}{\partial\hat{x}^2}+\rho h\frac{\partial^2\hat{W}}{\partial\hat{t}^2}= \frac{\varepsilon_0\varepsilon_r V_p v_e}{d^2}+\frac{\varepsilon_0\varepsilon_r V_p^2}{d^3}\hat{W} \tag{1}$$

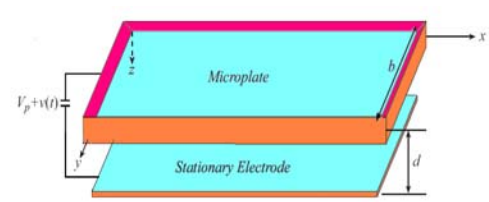

Figure (1): Capacitively actuated microplate.

Where D is a coefficient of Elastic modulus of the microplate, $\hat{W}$ is the microplate deflection, $\hat{N}_1$ is the applied in plane load per unit length in x direction, ρ is the microplate density, h is the microplate thickness, $\hat{t}$ is the time, ε_0 is the electric permittivity of the vacuum, ε_r is the electric permittivity of the ambient, V_p is the applied bias voltage, v_e is the applied AC voltage and d is the distance between two electrode. Using the nondimensionalized variables $w = \hat{W}/d$, $x = \hat{x}/a$, $y = \hat{y}/b$, $\alpha = a/b$, $N = \hat{N}_1 a^2/D$, $t = (\hat{t}/a^2)\sqrt{D/\rho h}$ and $\beta = \varepsilon_0 \varepsilon_r a^4 V_p^2 / Dd^3$, the nondimensionalized form of the equation (1) would be as equation (2).

$$\frac{\partial^4 w}{\partial x^4} + 2\alpha^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha^4 \frac{\partial^4 w}{\partial y^4} - N\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} - \beta w = 0 \tag{2}$$

For finding the natural frequencies and modeshapes, one may assume.

$$w(x,y) = \varphi(x,y)\exp(i\omega t) \tag{3}$$

Where $\varphi(x,y)$ is the microplate modeshape and ω is the corresponding natural frequency. Substituting equation (3) in to homogenous form of the equation (1), would lead to equation (4).

$$\frac{\partial^4 \varphi}{\partial x^4} + 2\alpha^2 \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \alpha^4 \frac{\partial^4 \varphi}{\partial y^4} - N\frac{\partial^2 \varphi}{\partial x^2} - \left(\beta + \omega_n^2\right)\varphi = 0 \tag{4}$$

The extended Kantorovich method would be used to solve equation (4). First of all a first order Galerkin approximation is used.

$$\begin{aligned}&\int_{-1}^{1}\int_{-1}^{1}\Big(\frac{\partial^4 \varphi}{\partial x^4} + 2\alpha^2 \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \alpha^4 \frac{\partial^4 \varphi}{\partial y^4} - N\frac{\partial^2 \varphi}{\partial x^2}\\ &-\left(\beta + \omega_n^2\right)\varphi\Big)\delta\varphi \, dx\, dy = 0\end{aligned} \tag{5}$$

Now according to the extended Kantorovich method it is assumed that:

$$\varphi(x,y) = f(x)g(y) \tag{6}$$

If $g(y)$ is a prescribed known function, then variation of φ would be just because of variation of f. That is:

$$\delta\varphi(x,y) = g(y)\delta f(x) \tag{7}$$

Substituting equations (6) and (7) in to equation (5), one can arrive at:

$$\begin{aligned}&\int_{-1}^{1}\Big(\int_{-1}^{1}\Big(g\frac{d^4 f}{dx^4} + 2\alpha^2 \frac{d^2 g}{dy^2}\frac{d^2 f}{dx^2} + \alpha^4 \frac{d^4 g}{dy^4} f - N.g\frac{d^2 f}{dx^2}\\ &\quad -\left(\beta + \omega_n^2\right) f.g\Big)g\,dy\Big)\delta f\, dx = 0\end{aligned} \tag{8}$$

Now, considering the point that δf has arbitrary values except at the boundaries and utilizing the fundamental lemma of variational calculus, one may arrive at the following ODE for $f(x)$.

$$\begin{aligned}&\left(\int_{-1}^{1} g^2 dy\right)\frac{d^4 f}{dx^4} + \left(2\alpha^2\int_{-1}^{1} g\frac{d^2 g}{dy^2}dy - N\int_{-1}^{1} g^2 dy\right)\frac{d^2 f}{dx^2} +\\ &\quad\left(\alpha^4\int_{-1}^{1} g\frac{d^4 g}{dy^4}dy - \left(\beta+\omega_n^2\right)\int_{-1}^{1} g^2 dy\right) f = 0\end{aligned} \tag{9}$$

Here it is tried to weaken some of the integrals appear in the equation (9). This incorporate some of boundary condition's and reduces the degree of derivatives which have to be evaluated. Using the integration by parts:

$$\int_{-1}^{1} g\frac{d^2 g}{dy^2} = -\int_{-1}^{1}\left(\frac{dg}{dy}\right)^2 dy \tag{10}$$

$$\int_{-1}^{1} g\frac{d^4 g}{dy^4} = \int_{-1}^{1}\left(\frac{d^2 g}{dy^2}\right)^2 dy \tag{11}$$

Using equations (10) and (11) would reduce the equation (9) to equation (12).

$$\begin{aligned}&\left(\int_{-1}^{1} g^2 dy\right)\frac{d^4 f}{dx^4} - \left(2\alpha^2\int_{-1}^{1}\left(\frac{dg}{dy}\right)^2 dy + N\int_{-1}^{1} g^2 dy\right)\frac{d^2 f}{dx^2} +\\ &\quad\left(\alpha^4\int_{-1}^{1}\left(\frac{d^2 g}{dy^2}\right)^2 dy - \left(\beta+\omega_n^2\right)\int_{-1}^{1} g^2 dy\right) f = 0\end{aligned} \tag{12}$$

Or in a more simplified manner:

$$\frac{d^4 f}{dx^4} - I_1\frac{d^2 f}{dx^2} + \left(I_2 - \omega_n^2\right) f = 0 \tag{13}$$

Where

$$I_1 = \frac{2\alpha^2\int_{-1}^{1}(dg/dy)^2 dy}{\int_{-1}^{1} g^2 dy} + N \tag{14}$$

$$I_2 = \frac{\alpha^4 \int_{-1}^{1} \left(\frac{d^2 g}{dy^2} \right)^2 dy}{\int_{-1}^{1} g^2 dy} - \beta \tag{15}$$

If it was assumed that $f(x)$ is known, one would arrive at a similar ODE in terms of $g(y)$. These two eigenvalue ODEs can be solved iteratively for finding $f(x)$, $g(y)$ and microplate natural frequencies. In order to this the initial guess function $g(y) = (y^2 - 1)^2$ is chosen. Using this assumption, I_1 and I_1 would be obtained as follows.

$$I_1 = 6\alpha^2 + N \tag{16}$$

$$I_2 = \frac{63}{2}\alpha^4 - \beta \tag{17}$$

Now, with solving equation (13), one can arrive at equation (18).

$$f(x) = C_1 \sinh(\rho x) + C_2 \cosh(\rho x) + C_3 \sin(\chi x) + C_4 \cos(\chi x) \tag{18}$$

Assuming a fully clamped microplate, the necessary boundary condition's for f and g are equations (19) and (20).

$$f(1) = f(-1) = f'(1) = f'(-1) = 0 \tag{19}$$

$$g(1) = g(-1) = g'(1) = g'(-1) = 0 \tag{20}$$

Imposing boundary condition's (19) on equation (18) would lead to the following set of algebraic eigenvalue, eigenvector equations.

$$C_1 \sinh(\rho) + C_2 \cosh(\rho) + C_3 \sin(\chi) + C_4 \cos(\chi) = 0 \tag{21}$$

$$-C_1 \sinh(\rho) + C_2 \cosh(\rho) - C_3 \sin(\chi) + C_4 \cos(\chi) = 0 \tag{22}$$

$$C_1 \rho \cosh(\rho) + C_2 \rho \sinh(\rho) + C_3 \chi \cos(\chi) - C_4 \chi \sin(\chi) = 0 \tag{23}$$

$$C_1 \rho \cosh(\rho) - C_2 \rho \sinh(\rho) + C_3 \chi \cos(\chi) + C_4 \chi \sin(\chi) = 0 \tag{24}$$

Now adding equations (21) and (22) and subtracting equation (24) from equation (23) would lead to the following system of algebraic equation in terms of C_2 and C_4.

$$C_2 \cosh(\rho) + C_4 \cos(\chi) = 0 \tag{25}$$

$$C_2 \rho \sinh(\rho) - C_4 \chi \sin(\chi) = 0 \tag{26}$$

In a similar manner by subtracting equation (22) from equation (21) and adding equations (23) and (24) one would arrive at equations (27) and (28) in terms of C_1 and C_3.

$$C_1 \sinh(\rho) + C_3 \sin(\chi) = 0 \tag{27}$$

$$C_1 \rho \cosh(\rho) + C_3 \chi \cos(\chi) = 0 \tag{28}$$

Equations (25) and (26) are related to symmetric modeshapes and equations (27) and (28) are related to anti-symmetric modeshapes. Equating the determinant of coefficients of equations (25) and (26) would lead to the equation (29) and equating the determinant of the coefficients of equation (27) and (28) would lead to equation (30).

$$-\chi \tan \chi = \rho \tanh \rho \tag{29}$$

$$\chi \tanh \rho = \rho \tan \chi \tag{30}$$

Since ρ and χ are functions of ω_n, solving equations (29) and (30) can find the values of microplate natural frequency. It is also obvious from equations (25), (26), (27) and (28) that for symmetric modeshapes:

$$\frac{C_2}{C_4} = -\frac{\cos \chi}{\cosh \rho} \tag{31}$$

And for anti-symmetric modeshapes:

$$\frac{C_1}{C_3} = -\frac{\sin \chi}{\sinh \rho} \tag{32}$$

In the extended Kantorovich procedure if it was assumed that $f(x)$ is a known function, then the following ODE would govern the behavior of $g(y)$.

$$\frac{d^4 g}{dy^4} - I_1' \frac{d^2 g}{dy^2} + (I_2' - \omega_n^2) g = 0 \tag{33}$$

Where:

$$I_1' = \frac{2\alpha^2 \int_{-1}^{1} \left(\frac{df}{dx}\right)^2 dx}{\int_{-1}^{1} f^2 dx} + N \tag{34}$$

$$I_2' = \frac{\alpha^4 \int_{-1}^{1} \left(\frac{d^2 f}{dx^2}\right)^2 dx}{\int_{-1}^{1} f^2 dx} - \beta \tag{35}$$

If one use the previously obtained closed form analytical function for $f(x)$, then the integrations in equations (34) and (35) can be obtained analytically as follows:

$$\begin{aligned}\int_{-1}^{1} f^2(x)dx &= C_1^2\left(\frac{\cosh\rho\sinh\rho-\rho}{\rho}\right)+C_2^2\left(\frac{\cosh\rho\sinh\rho+\rho}{\rho}\right)\\ &+C_3^2\left(\frac{-\cos\chi\sin\chi+\chi}{\chi}\right)+C_4^2\left(\frac{\cos\chi\sin\chi+\chi}{\chi}\right)\\ &-4C_1C_3\left(\frac{\chi\cos\chi\sinh\rho-\rho\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\\ &+4C_2C_4\left(\frac{\rho\cos\chi\sinh\rho+\chi\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\end{aligned} \tag{36}$$

$$\begin{aligned}\int_{-1}^{1}\left(\frac{df(x)}{dx}\right)^2 dx &= C_1^2\rho\left(\cosh\rho\sinh\rho+\rho\right)\\ &+C_2^2\rho\left(\cosh\rho\sinh\rho-\rho\right)\\ &+C_3^2\chi\left(\cos\chi\sin\chi+\chi\right)+C_4^2\chi\left(-\cos\chi\sin\chi+\chi\right)\\ &+4C_1C_3\rho\chi\left(\frac{\rho\cos\chi\sinh\rho+\chi\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\\ &+4C_2C_4\rho\chi\left(\frac{\chi\cos\chi\sinh\rho-\rho\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\end{aligned} \tag{37}$$

$$\begin{aligned}\int_{-1}^{1}\left(\frac{d^2 f(x)}{dx^2}\right)^2 dx &= C_1^2\rho^3\left(\cosh\rho\sinh\rho-\rho\right)\\ &+C_2^2\rho^3\left(\cosh\rho\sinh\rho+\rho\right)\\ &+C_3^2\chi^3\left(-\cos\chi\sin\chi+\chi\right)+C_4^2\chi^3\left(\cos\chi\sin\chi+\chi\right)\\ &+4C_1C_3\rho^2\chi^2\left(\frac{\chi\cos\chi\sinh\rho-\rho\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\\ &-4C_2C_4\rho^2\chi^2\left(\frac{\rho\cos\chi\sinh\rho+\chi\sin\chi\cosh\rho}{\rho^2+\chi^2}\right)\end{aligned} \tag{38}$$

With solving equation (33), one may obtain equation (39) for $g(y)$.

$$\begin{aligned}g(y) &= C_1'\sinh(\rho' y)+C_2'\cosh(\rho' y)+C_3'\sin(\chi' y)\\ &+C_4'\cos(\chi' y)\end{aligned} \tag{39}$$

Where:

$$\rho' = \sqrt{\frac{I_1'}{2}+\sqrt{\omega_n^2+\left(\frac{I_1'}{2}\right)^2-I_2'}} \tag{40}$$

$$\chi' = \sqrt{-\frac{I_1'}{2}+\sqrt{\omega_n^2+\left(\frac{I_1'}{2}\right)^2-I_2'}} \tag{41}$$

Now performing the boundary condition's (20) on equation (39) one may obtain a set of four homogeneous eigenvalue, eigenvector algebraic equations. Equating the coefficients of this set of equations with zero, would lead to equations (42) and (43), where equation (42) is for symmetric and equation (43) is for anti-symmetric modeshapes.

$$-\chi' \tan\chi' = \rho' \tanh\rho' \tag{42}$$

$$\chi' \tanh\rho' = \rho' \tan\chi' \tag{43}$$

It can also be obtained that for symmetric modeshapes:

$$\frac{C_2'}{C_4'} = -\frac{\cos\chi'}{\cosh\rho'} \tag{44}$$

And for anti-symmetric modeshapes:

$$\frac{C_1'}{C_3'} = -\frac{\sin\chi'}{\sinh\rho'} \tag{45}$$

This iterative procedure can be continued to find new functions for $f(x)$ and $g(y)$. But as would be seen, the convergence of the procedure is so fast that usually two or three iterates is sufficient for finding a high precision response.

Results and Discussion

Figure (2) shows the frequency parameter ω_n at different modeshapes versus α when $\hat{N}_1 = V_p = 0$. Results of this figure are in close agreement with tabulated results of Jones and Milne [23]. Figure (2) also implements that increasing the microplate aspect ratio would increase its natural frequencies.

In order to investigate the convergence of the procedure, tables (1) and (2) has been prepared. These tables shows the values of C_2, C_4, ρ and χ for functions $f(x)$ and $g(y)$ respectively at different iterates in the first modeshape. Note that in this modeshape $C_1 = C_3 = 0$.

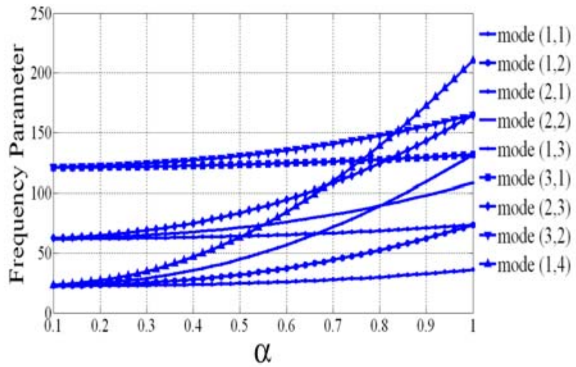

Figure (2): frequency parameter ω_n at different modeshapes versus α.

Table 1: values of C_2, C_4, ρ and χ for function $f(x)$ at different iterates when $N = V_p = 0$ and $\alpha = 1$ for modeshape $(1,1)$.

C_2	C_4	ρ	χ
0.042213065	1	3.26318507984225	2.15600947709075
0.042220101	1	3.26305421238901	2.15603343766773
0.042220103	1	3.26305417246660	2.15603345282247
0.042220103	1	3.26305417436214	2.15603345329057
0.042220103	1	3.26305417455107	2.15603345333045

Table 2: values of C_2, C_4, ρ and χ for function $g(y)$ at different iterates when $N = V_p = 0$ and $\alpha = 1$ for modeshape $(1,1)$.

C_2	C_4	ρ	χ
0.04222025392	1	3.263051335801	2.15603394396
0.04222010340	1	3.263054166668	2.15603345197
0.04222010317	1	3.263054173891	2.15603345317
0.04222010316	1	3.263054174513	2.15603345332

These tables are a simple evident of the fast convergence of the procedure. Furthermore figure (3) shows the modeshapes $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$ of the microplate at different iterates when $\hat{N}_1 = V_p = 0$. It is seen that because of the rapid convergence, visual distinguishing of different iterates is impossible.

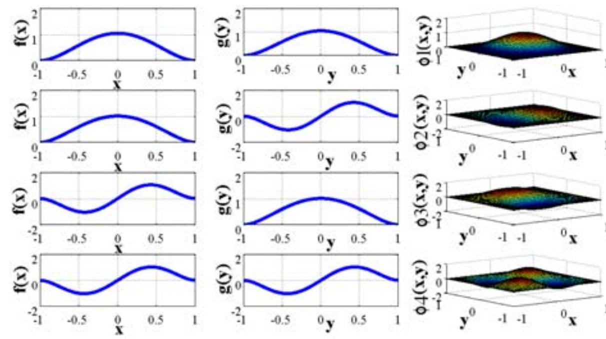

Figure (3): first Microplate modeshapes at different iterates.

Figure (4) shows the variation of the normalized natural frequency with respect to natural frequency in zero voltage state with β at different values of α when $N = 0$. It is observed that with increasing the value of β the fundamental natural frequency decreases and eventually approaches zero at pull-in. it is also concluded that at every specified actuating voltage, the value of natural frequency increases as the microplate aspect ratio increase.

Figure (5) shows the values of β at pull-in versus α. It is seen that square like microplates can bear more voltage before the occurrence of pull-in.

Figure (6) shows the effect of applied inplane loads on the microplate fundamental natural frequency at different values of β and two distinct values of microplate aspect ratio. As it can be seen, applying tensile loads would increase the microplate fundamental natural frequency while applying compressive loads would decrease microplate fundamental natural frequency and it would eventually approaches zero near the pull-in instability. Figure (6) also shows that increasing the applied voltage would decrease the maximum applied inplane compressive load the structure can support.

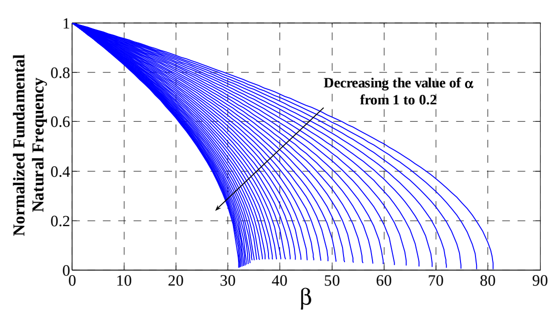

Figure (4): variation of the normalized natural frequency with β at different values of α

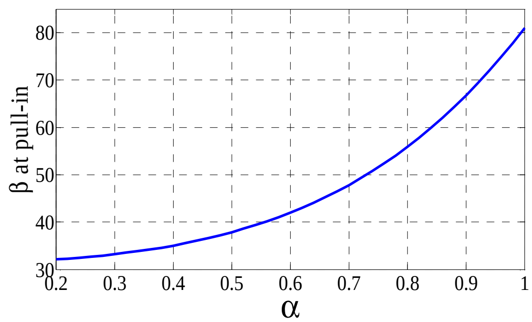

Figure (5): Values of β at pull-in versus α.

CONCLUSION

In the current paper an extended Kantorovich method is developed for the vibrational analysis of electrically actuated microplates. The rapid convergence and high precision is shown to be the power of this method. In some specific cases the model is validated against theoretical results reported in the literature. The mentioned advantages make the EKM an effective and accurate design tool, useful in design optimization and determination of the stable operation range of MEMS devices.

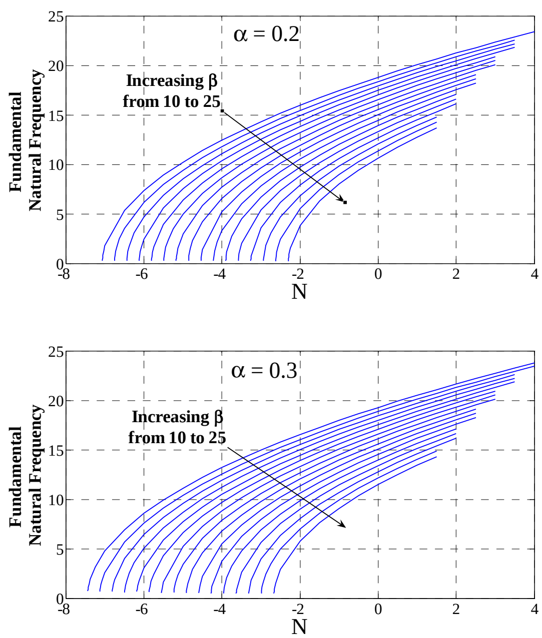

Figure (6): Effect of applied inplane loads on the microplate fundamental natural frequency at different values of β and $\alpha = 0.1$ and $\alpha = 0.2$.

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