

Time-Dependent Crack Initiation and Growth in Ceramic Matrix Composites

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Matrix cracking in ceramic matrix composites with fine grained fibers at high temperatures will be governed by fiber creep, as relaxation of the fibers eliminates crack tip shielding. Using a time dependent bridging law that describes the effect of creeping fibers bridging a crack in an elastic matrix, crack growth initiation and history have been modeled. For a stationary crack, crack tip stress intensity factors as a function of time are presented to predict incubation times before subcritical crack growth. Two crack growth studies are reviewed: a constant velocity approximation for small-scale bridging, and a complete velocity history analysis which can be used to predict crack length as a function of time. The predictions are summarized and discussed in terms of identifying various regimes of crack growth initiation, subcritical growth, and catastrophic matrix cracking.

1 Introduction

High strength ceramic fibers can be achieved by decreasing fiber grain size, which limits the flaw size in the fibers. Ceramic matrix composites (CMCs) are made by combining these fibers with relatively coarse-grained matrices. Since the role of the matrix is generally to provide ductility to the composite, the decrease in matrix strength due to large flaw sizes is not considered detrimental. The low temperature behavior of such composites has been studied extensively both experimentally and analytically, and is generally well understood (Evans and Zok, 1994).

The primary motivation for using such composites, however, is the high temperature capabilities of the ceramic constituents. At high temperatures, predicting composite performance becomes more complicated due to oxidation and creep (Heredia et al., 1995; Lamouroux et al., 1994). The fine-grained fibers are particularly susceptible to creep, as small grain sizes increase avenues for grain boundary diffusion. (The coarser grained matrix can be considered to behave elastically, as creep rates are significantly lower.) Crack bridging by intact fibers becomes time-dependent at high temperatures, as fiber creep causes crack closure forces to decay over time (Begley, 1997; Begley et al., 1997; Begley et al., 1995a; Begley et al., 1995b; El-Azab and Ghoniem, 1995; Henager and Jones, 1994; Henager and Jones, 1993; Nair and Gwo, 1993). Furthermore, cracks that can be considered benign at low temperatures may cause significant problems at high temperatures, as they may provide pathways for oxidation to occur in the interior of the composite or degrade the components ability to contain gases (Evans et al., 1996). The question of crack stability at room temperature thus changes at elevated temperatures to questions about when and how fast cracks grow.

This paper is intended to provide a summary of some recent work on predicting time dependent crack growth in CMCs caused by fiber creep at high temperatures. A time dependent bridging law has been developed to describe the effect of creeping fibers bridging a matrix crack in a composite whose matrix can be considered elastic (Begley et al., 1995a). This bridging law has been used to estimate the time needed to initiate crack growth from both fully bridged and partially bridged stationary

cracks (Begley, 1997). The issue of crack growth rate has been addressed in both the small-scale and large-scale bridging regimes (Begley et al., 1997; Begley 1995b). Representative results are presented and used to discuss the issues raised by time dependent crack growth in both unidirectional and laminated CMCs.

In the crack behavior studied in this work, a finite crack tip stress intensity factor is assumed to exist. For analysis of initiation times, it is assumed that the crack geometry and loading is such that the crack tip stress intensity factor, K_{tip} , is less than the toughness of the matrix, K_c (adjusted for matrix volume fraction). For crack growth studies, it is assumed that crack growth occurs under the condition that $K_{tip} = K_c$. The analyses and results are often similar to a cohesive zone approach in different materials (Knauss, 1993; Fager et al., 1991); however, it should be emphasized that the assumption of a finite stress intensity factor at the crack tip (based on the fact that the matrix remains elastic) leads to significant differences.

2 Time Dependent Bridging

The crack tip shielding provided by creeping fibers bridging a matrix crack can be analyzed by determining the relationship between the crack opening rate and bridging traction for a representative bridged section of the crack. Such cell models can then be integrated with traditional fracture mechanic relations to develop an integral equation which is solved for the closure forces in the bridged section of the crack. These closure forces are then used in the usual manner to predict the reduction in crack tip stress intensity factor.

A full derivation for a creeping fiber embedded in an elastic matrix results in the following bridging law, which incorporates the effects of frictional slip between the fiber and the matrix (Begley et al., 1995a):

$$\begin{aligned} \dot{\delta}(t) = & \frac{(1-f)^2 E_m^2 D}{2f^2 \tau E_f} \left\{ \frac{\sigma(t)}{E_L} + \frac{f B E_f^2}{E_L^2} \int_{-\infty}^t \sigma(\bar{T}) e^{-(t-\bar{T})/T} d\bar{T} \right\} \\ & \times \left\{ \frac{\dot{\sigma}(t)}{E_L} + \frac{B E_f \sigma(t)}{2(1-f) E_m} \left[1 + \frac{f E_f (f E_f - (1-f) E_m)}{E_L^2} \right. \right. \\ & \left. \left. \times \left\{ \int_{-\infty}^t \frac{\sigma(\bar{T})}{T \sigma(t)} e^{-(t-\bar{T})/T} d\bar{T} \right\} \right] \right\}. \quad (1) \end{aligned}$$

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D is the diameter of the fibers; f is the fiber volume fraction; B is the creep coefficient of the fibers; τ is the shear sliding stress between the fibers and the matrix; E_f , E_m , and E_L are the elastic moduli of the fiber, matrix and composite, respectively; and T is the characteristic relaxation time for the intact composite, given by $E_L/B(1-f)E_mE_f$.

For a growing crack, the convolution integrals in (1) (which reflect the history dependence of the bridging stress) complicate things, and the following simplified bridging law has been used in crack growth studies (Begley et al., 1997; Begley et al., 1995b):

$$\delta(t) = 2\lambda\sigma(t)[\dot{\sigma}(t) + \beta\sigma(t)], \quad (2)$$

where λ is the rate-independent bridging coefficient, and β is a modified creep coefficient. The assumptions that justify simplifying Eq. (1) to Eq. (2) are based on comparing the relative magnitudes of terms in the full bridging law and neglecting smaller terms. In summary, Eq. (2) neglects creep of the fibers in the intact portion of the composite, which is acceptable during crack growth studies since creep in the slip region adjacent to the matrix crack, and the short time ($t \ll T$) response of Eq. (1), dominate the shielding effect of the fibers. Further details of these assumptions and their validity is discussed fully by Begley (1997), Begley et al. (1997), and Begley et al. (1995b).

3 Time to Initiate Crack Growth

For a given crack geometry and load level, the crack tip stress intensity factor may be beneath the critical stress intensity factor of the matrix, implying that some time is required to relax the shielding effect of the fibers to the point that crack growth occurs. Such a situation would arise when a matrix flaw exists that spans multiple fibers, provided the load is beneath the critical stress to propagate the bridged matrix crack. Another example is the case of an overload, where the applied load on the composite decreases; during the peak loading, the matrix crack is driven to the length where $K_{tip} = K_c$ for the peak load. If the load is subsequently decreased, K_{tip} falls beneath K_c and some time is required to decay the bridging tractions until crack growth occurs.

In the following calculations, the crack is assumed to be stationary; thus, the problem is simply to calculate the evolution of the bridging stress over time. The bridging stress profile at a given instant in time was then used to predict the instantaneous

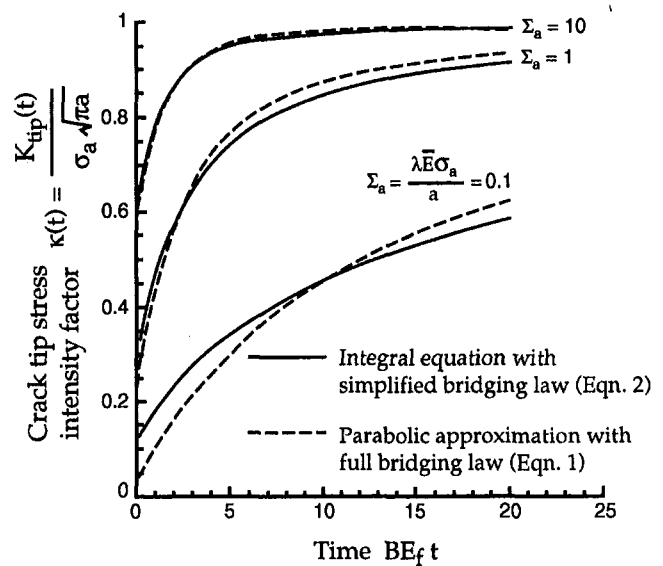


Fig. 1 Two calculations for the crack tip stress intensity factor as a function of time for three different loads and $f = 0.3$, $E_f = E_m = E_L = \bar{E}$

value of the crack tip intensity factor. The geometry is a fully bridged center crack of length $2a$ in an infinite panel. The crack tip stress intensity factor as a function of time is shown in Fig. 1 for several different cases.

The curves compare the effects on the crack tip stress intensity factor of two different simplifications, for three different load levels. The solid lines were generated using the bridging stress profile generated by the solution of an integral equation derived using Eq. (2), the simplified bridging law. The dashed lines were calculated using the full bridging law and imposing a parabolic approximation for crack opening. (At high loads or for short cracks, the crack opening will be nearly parabolic; assuming the form of the profile allows the differential form of the integral equation to be reduced to a simple one-degree-of-freedom differential equation.) It can be seen that at high loads the effect of the convolution integrals is negligible. As discussed by Begley (1997), even at lower loads the neglect of the convolutions is usually not significant. The discrepancy in Fig. 1 between the two calculations for the lowest load is mostly a

Nomenclature

a , d = crack half length, crack tip velocity
 $a_m = \pi/4 (\lambda \bar{E} K_c)^{2/3}$ = characteristic length scale for the composite
 B = creep coefficient of the fibers ($\dot{\epsilon}_f = (\dot{\sigma}_f/E_f) + B\sigma_f$)
 D = fiber diameter
 E_f , E_m = Young's modulus of the fibers and matrix
 $E_L = fE_f + (1-f)E_m$ = rule of mixtures composite Young's modulus
 \bar{E} = composite modulus which accounts for orthotropy
 $\bar{\epsilon} = \lambda \bar{E}^2 \bar{\epsilon} / l$ = normalized total fiber strain at the crack plane
 $\bar{\epsilon}_{cr}$ = normalized critical total strain to fiber failure

f = fiber volume fraction
 K_c = critical stress intensity factor for matrix crack extension
 K_∞ = far-field applied stress intensity factor
 $\bar{K}_\infty = \lambda \bar{E} K_\infty / l = K_\infty / K_c$ = normalized applied stress intensity factor
 $(K_\infty)_{asym}$ = asymptotic applied stress intensity factor at low crack speeds
 $l = (\lambda \bar{E} K_c)^{2/3} = \lambda \bar{E} \sigma_{mc}$ = characteristic length scale for the composite
 $T = E_L / (1-f) B E_f E_m$ = characteristic relaxation time of intact composite
 $v_{ss} = d/\beta l$ = normalized steady-state crack velocity
 α_{asym} = asymptotic bridge length at low crack speeds

$\beta = B E_f E_L / 2(1-f) E_m$ = modified creep coefficient of the fibers
 δ , $\dot{\delta}$ = total crack opening, total crack opening rate
 $\delta_{mc} = \lambda \sigma_{mc}^2$ = total crack opening at steady-state matrix cracking stress (for the rate-independent case)
 $\Delta_{cr} = \lambda \bar{E}^2 \delta_{cr} / l^2 = \delta_{cr} / \delta_{mc}$ = normalized critical total crack opening governing fiber failure
 $\lambda = D(1-f)^2 E_m^2 / 4 f^2 \tau E_f E_L^2$ = rate-independent bridging coefficient ($\delta = \lambda \sigma^2$)
 σ , $\dot{\sigma}$ = bridging stress, bridging stress rate
 $\sigma_{mc} = [12 f^2 \tau E_f E_L^2 K_c^2 / D(1-f) E_m^3]^{1/3}$ = steady-state matrix cracking stress
 τ = shear sliding stress at the fiber-matrix interface

result of the inaccuracy of the parabolic form used to reduce the integral equation.

4 Constant Velocity Crack Growth: Small-Scale Bridging

In situations where the length of the bridging zone is much smaller than the crack length, small-scale bridging is said to apply and the governing equations can be simplified from the general case. An example of such a scenario may occur in laminated CMCs, where cracks first appear in the 90 deg layers with fibers perpendicular to the loading direction. These cracks will advance into the adjacent 0 deg plies and arrest at room temperature; at loads much smaller than the matrix cracking stress, the bridging zone is much smaller than the total crack length.

The case being modeled here is a semi-infinite crack loaded with a far-field applied stress intensity factor, K_∞ , growing at a constant speed, v_{ss} (Begley et al., 1997). The bridging law dictating material behavior in the bridged zone is given as Eq. (2). Fibers fail in the wake of the crack and the bridging zone size remains constant. The solution represents a steady-state configuration, as the zone of creeping fibers bridging the crack propagates with the crack tip.

The length of the bridging zone depends on the crack growth rate, and is determined by a fiber failure criterion. Two failure criteria are presented here: a critical crack opening, Δ_{cr} , and a critical total strain in the fiber, $\bar{\epsilon}_{cr}$.

Figure 2 illustrates how crack velocity varies as a function of applied stress intensity factor K_∞ for the two cases. For both cases, there is a rapid increase in crack velocity as the applied stress intensity factor is raised above the matrix toughness. The figure demonstrates the existence of upper and lower bounds on the applied loading. The assumption of a finite crack tip stress intensity factor dictates that a minimum K_∞ exists, below which no crack growth will occur; this minimum is merely the matrix toughness. For applied loads above this value, fiber creep will decrease bridging tractions to drive crack growth, though perhaps at very small velocities.

Since the simplified bridging law (given as Eq. 2) reduces to the appropriate rate independent bridging law in the limit of no creep, an upper limit exists. As the applied load increases towards the level at which bridging (without creep) is no longer effective enough to maintain $K_{tip} = K_c$, crack velocities tend toward infinity. Thus, the upper limit corresponds to load levels at which instantaneous matrix crack occurs.

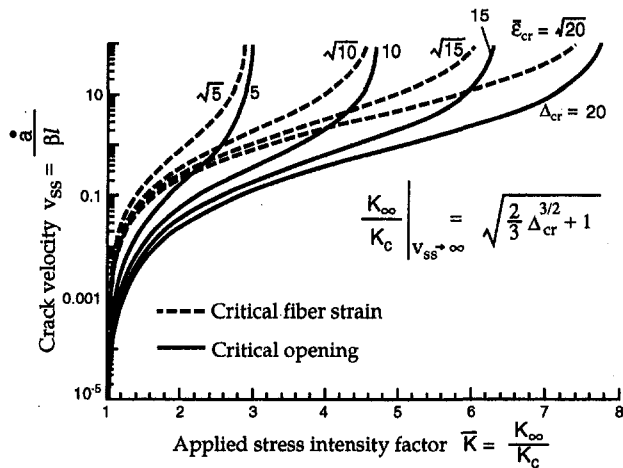


Fig. 2 Steady-state crack velocity as a function of far-field applied stress intensity factor for two fiber failure criteria and $f = 0.3$, $E_f = E_m = E_L = E$

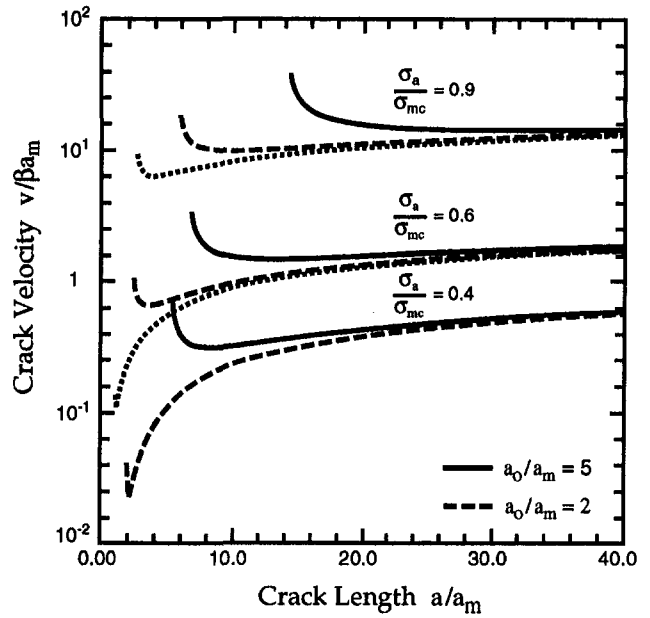


Fig. 3 Crack velocity as a function of crack length for several values of applied load and notch sizes using the simplified bridging law

One of the attractive results of the simplifying assumptions used in this analysis is that they allow closed-form asymptotic solutions when crack speeds are low. The asymptotic dependence of the bridging zone length and crack velocity on the applied stress intensity factor are summarized below for the case where fiber failure is governed by a critical strain criterion (Begley et al., 1997):

$$\left(\frac{K_\infty}{K_c}\right)_{\text{asym}} = 1 + \frac{4}{\pi^{1/4}} \left(\frac{3E_f E_L}{4(1-f)E_m}\right)^{1/3} (\bar{\epsilon}_{cr})^{1/3} (v_{ss})^{2/3}; \quad (v_{ss} \rightarrow 0) \quad (3)$$

$$\left(\frac{\alpha}{l}\right)_{\text{asym}} = \frac{1}{4} \left(\frac{3E_L}{4(1-f)E_m}\right)^{4/3} (\bar{\epsilon}_{cr})^{4/3} (v_{ss})^{2/3}. \quad (4)$$

These closed form solutions reveal a simple power law relationship between applied stress intensity factor and crack velocities that may be useful in comparing with experiments with very small bridging zones.

5 General Time Dependent Crack Growth: Large-Scale Bridging

Obviously, cracks may not grow at constant velocity, and the bridging length may be comparable to specimen dimensions. For such cases, full large-scale bridging have been performed to predict crack length and velocity as a function in time (Begley et al., 1995b). Fiber failure was not incorporated into the analysis. The geometry being considered is a center crack of length $2a$ in an infinite panel with a 'notch', or unbridged portion of matrix crack of lengths $2a_0$. Crack growth is assumed to occur with the condition that the crack tip stress intensity factor equals the matrix toughness. The numerical procedure is summarized by Begley et al. (1995b); one attractive aspect of the technique used is that the bridging law has exactly the correct form in the limit of small time increments. Hence, the bridging behavior near the crack tip, where fibers are responding nearly elastically, is captured accurately. The results of these analyses are summarized in Figs. 3 and 4.

Crack velocities as a function of crack length are shown in Fig. 3 for several different values of applied nodes and several

different notch sizes. The curves start at different initial values of crack length, corresponding to the case where $K_{ip} = K_c$ for the rate independent case. Thus, for these cases the initiation time discussed in section 3 is zero. All cases show that the crack will initially decelerate as the rate of decay of bridging tractions decreases. After the initial transient has finished, the crack accelerates monotonically, eventually becoming asymptotically independent of initial notch size.

The crack velocity as a function of crack length can be integrated to predict crack length as a function of time. Such results are shown in Fig. 4 for one of the notch sizes in Fig. 3 and the same load levels. The curves illustrate the large effect the deceleration transient will have on the overall crack growth; the larger the dip in the crack velocity curve, the larger the dwell time in Fig. 4. For laminates, critical crack lengths at which catastrophic cracks cross the specimen may be short, i.e., equal to the ply thickness, emphasizing the importance of solutions in the transient regime.

6 Predicting Time Dependent Crack Growth in CMCs

6.1 Crack Growth Initiation. For a given loading scenario, the first question to answer is whether or not crack growth starts immediately, or whether some time is required to decrease the shielding in bridged cracks. The answers lie in the rate-independent behavior of CMCs, which has been extensively studied; provided the loading rate is high enough, the composite constituents will initially respond elastically. The upper limit on the applied loading, such that crack growth studies are applicable, is the steady-state matrix cracking stress, which represents the load level at which matrix cracks will propagate across the composite catastrophically in a rate-independent manner.

For cracks created by load histories where the specimen or component is loaded monotonically beneath the matrix cracking stress, the transition to subcritical crack growth will be instantaneous. Both cracks grown from a notch in a unidirectional composite and cracks tunneling into 0 deg plies in a laminate will arrest at the crack length at which $K_{ip} = K_c$. Therefore, any amount of fiber creep will decrease crack tip shielding and cause crack growth; initiation times for these cases are zero. The crack length at which $K_{ip} = K_c$ with rate-independent behavior will be the initial condition used in the time dependent study. The rate independent behavior of both unidirectional CMCs and

laminates is summarized by Cox and Marshall (1997), which can be used to identify relevant starting geometries for time dependent studies.

Initiation times will not be zero for cracks created by an overload, where the load applied to the composite decreases after some maximum. For these cases, the starting crack length is the length created by the maximum load, where $K_{ip}(\sigma_{max}) = K_c$. If the load drops, K_{ip} will fall below K_c , and the crack will remain stationary until fiber creep degrades shielding enough to cause crack growth. Obviously, such histories affect subsequent crack growth as fiber strains will accumulate. Further modeling and experiments are needed to discover the effect of creep prior to crack growth.

6.2 Regime of Subcritical Crack Growth. Once crack growth has started, the relevant question to answer is when the cracks reach lengths that are undesirable. This is most likely the length at which the crack crosses the specimen, resulting in two halves of intact composite held together by creeping fibers. For laminates, the critical length at which cracks propagate unstably may be the 0 deg ply width, depending on the ratio of 0 deg/90 deg widths (Cox and Marshall, 1997).

Once the critical crack length has been identified, the models described in sections 4 and 5 can be used to estimate the amount of time required to reach this length. Noting that a_m in Fig. 3 is on the order of a tenth of a millimeter for most CMCs, it can be seen that most (if not all) of the relevant crack growth for typical laminates occurs during the transient period of crack growth. Thus, the pertinent regions in Figs. 3 and 4 are the ones where the normalized crack length a/a_m is less than 10 or so. For loads above 60 percent of the matrix crack stress, the time to reach a critical crack length is quite small, as the transient region becomes less and less pronounced. For such cases, matrix cracks quickly cross the specimen and composite rupture is governed by the rupture behavior of the fibers loaded uniformly.

For loads significantly below the matrix cracking stress, velocities will may be quite small, implying that fibers have significant time to creep. Naturally, the fibers will exhibit a finite amount of creep ductility and for slower velocities, will fail during the subcritical crack growth. A transition exists then, between subcritical crack growth characterized by fiber failure in the wake of the crack (Begley et al., 1997) and fiber failure after the crack has reached critical dimensions (Begley et al., 1995b). Experiments are needed to confirm this transition and evaluate the validity of the predictions presented here.

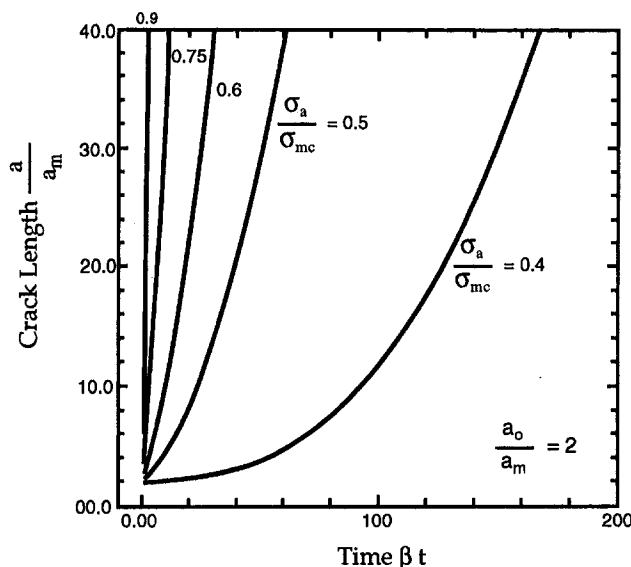


Fig. 4 Crack length as a function of time calculated by integrating the curves in Fig. 3

6.3 Appropriateness of the Bridging Law and Single Crack Model. In general, bridging laws developed from cell models fail to capture certain aspects of bridging behavior. Notably, the equation presented here does not account for the possibility of “reverse” slip, where the direction of relative sliding between the fiber and the matrix changes sign. This is most likely to happen in regions of the bridging zone that are unloading. For stationary cracks, this is most likely to be the case, and further calculations are warranted. For growing cracks, however, it has been shown that growth is dominated by the near-tip behavior of the bridging region (Begley et al., 1997; Begley et al., 1995b). Near the crack tip, bridging fibers will be loaded rapidly and will respond nearly elastically, or with the “short time” response of the bridging law. In this regime, both the possibility of “reverse” slip and the convolution integrals in Eq. (1) can be reasonably neglected.

It should be pointed out that a single crack model does not account for the stress redistribution that will occur if multiple matrix cracks occur near a stress concentration. Multiple matrix cracks are common, and a more realistic bridging law which incorporates the effect of overlapping slip zones would be more useful. Additionally, different weight functions used in developing the integral equations could be used, to account for arrays of multiple cracks.

Despite the limitations on the bridging law outlined above, the results will be qualitatively consistent with more detailed analyses incorporating reverse slip and crack interaction. The essential features of the constitutive law for the bridging zone will not be changed by considering slip zone reversal or overlap; namely, that the bridging stress increases with opening and decays with time. Most importantly, the details of appropriate bridging laws always require empirical calibration, preferably with crack-growth data rather than micromechanical tests (Cox, 1995). Regardless of the exact form of the bridging law, accurate predictions over a wide range of stress levels should be possible once the model is calibrated against experiments.

Acknowledgments

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References

- Begley, M. R., 1997, "Time Dependent Cracks in Ceramic Matrix Composites: Crack Growth Initiation," *International Journal of Solids and Structures*, Vol. 34, p. 1035.
- Begley, M. R., Cox, B. N., and McMeeking, R. M., 1997, "Creep Crack Growth With Small-Scale Bridging in Ceramic Matrix Composites," *Acta Metallurgica et Materiala*, Vol. 45, p. 2897.
- Begley, M. R., Evans, A. G., and McMeeking, R. M., 1995a, "Creep Rupture in Ceramic Matrix Composites With Creeping Fibers," *Journal of the Mechanics and Physics of Solids*, Vol. 43, p. 727.
- Begley, M. R., Cox, B. N., and McMeeking, R. M., 1995b, "Time Dependent Crack Growth in Ceramic Matrix Composites With Creeping Fibers," *Acta Metallurgica et Materiala*, Vol. 43, p. 3927.
- Cox, B. N., 1995, "Life Prediction for Bridged Cracks," *Life Prediction for Titanium Matrix Composites*, W. S. Johnson, J. Larsen, and B. N. Cox, eds., ASTM STP, ASTM, Philadelphia, PA.
- Cox, B. N., and Marshall, D. B., 1997, "Crack Initiation in Brittle Fiber Reinforced Laminates," *Journal of the American Ceramic Society*, in press.
- El-Azab, A., and Ghoniem, N. M., 1995, "Investigation of Incubation Time for Subcritical Crack Propagation in SiC-SiC Composites," *Journal of Nuclear Materials*, Vol. 219, p. 101.
- Evans, A. G., and Zok, F. W., 1994, "The Physics and Mechanics of Fibre-Reinforced Brittle Matrix Composites," *Journal of Material Science*, Vol. 29, p. 3857.
- Evans, A. G., Zok, F. W., McMeeking, R. M., and Du, Z. Z., 1996, "Embrittlement in Ceramic Matrix Composites," *Journal of the American Ceramic Society*, Vol. 79, p. 2345.
- Fager, L.-O., Bassani, J. L., Hui, C.-Y., and Xu, D.-B., 1991, "Aspects of Cohesive Zone Models and Crack Growth in Rate-Dependent Materials," *International Journal of Fracture*, Vol. 52, p. 119.
- Henager, C. H., and Jones, R. H., 1993, "High Temperature Plasticity Effects in Bridged Cracks and Subcritical Crack Growth in Ceramic Matrix Composites," *Materials Science and Engineering A*, Vol. 166, p. 211.
- Henager, C. H., and Jones, R. H., 1994, "Subcritical Crack Growth in CVI Silicon Carbide Reinforced With Nicalon Fibers: Experiment and Model," *Journal of the American Ceramic Society*, Vol. 77, p. 2381.
- Heredia, F. E., McNulty, J. C., Zok, F. W., and Evans, A. G., 1995, "Oxidation Embrittlement Probe for Ceramic Matrix Composites," *Journal of the American Ceramic Society*, Vol. 78, p. 2097.
- Knauss, W. G., 1993, "Time Dependent Fracture and Cohesive Zones," *Journal of Engineering Materials and Technology*, Vol. 115, p. 262.
- Lamouroux, F., Steen, M., and Valles, J. L., 1994, "Uniaxial Tensile and Creep Behavior in an Alumina Fiber-Reinforced Ceramic Matrix Composite: I—Experimental Study," *Journal of the European Ceramic Society*, Vol. 14, p. 529.
- Nair, S. V., and Gwo, T. J., 1993, "Role of Crack Wake Toughening on Elevated Temperature Crack Growth in a Fiber Reinforced Ceramic Matrix Composite," *Journal of Engineering Materials and Technology*, Vol. 115, p. 273.